Online VM Auto-Scaling Algorithms for Application Hosting in a Cloud

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***Abstract*—We consider the auto-scaling problem for application hosting in a cloud, where applications are elastic and the number of requests changes over time. The application requests are serviced by Virtual Machines (VMs), which reside on Physical Machines (PMs) in a cloud. We aim to minimize the number of hosting PMs by intelligently packing VMs into PMs, while the VMs are auto-scaled, i.e., dynamically acquired and released, to accommodate varying application needs. We consider a shadow routing based approach for this problem. The proposed shadow algorithm employs a specially constructed virtual queueing system to dynamically produce an optimal solution that guides the VM auto-scaling and the VM-to-PM packing. The proposed algorithm runs continuously without the need to re-solve the underlying optimization problem "from scratch", and adapts automatically to the changes in the application demands. We prove the asymptotic optimality of the shadow algorithm. The simulation experiments further demonstrate the algorithm's good performance and high adaptivity.**

I. Introduction

A Cloud data-center typically consists of a large number of physical machines (PMs). Virtualization technology enables cloud service providers to offer customers virtual machines (VMs) to run their applications, while facilitating the sharing of CPU, memory and storage resources. VMs can be configured and allocated as needed to meet hourly, daily, or weekly demand variation. When an application's demand for resources increases, additional VMs are allocated to meet the quality of service requirement; conversely, when the demand drops, resources can be freed to save cost. The cloud service providers may further wish to pack the allocated VMs into a small number of PMs so as to minimize operational costs, save energies, and/or generate higher revenues by accommodating more applications. We call by *VM auto-scaling* the dynamic selection of VM quantities and types to accommodate the applications' varying demands. The design of algorithms for the joint VM auto-scaling and VM-to-PM packing that minimize the number of hosting PMs is a challenging research problem.

Considerable prior works exist on cloud resource management with the focus on either the resource auto-scaling or VM-to-PM packing. In resource auto-scaling, the application requests are assigned to VMs to receive the service. The VM quantities and types need to be sufficient for the application requests to be serviced with satisfactory performance, i.e., satisfying the required *SLA (service-level agreement)*; yet not overly large to avoid unnecessary cost to users and resource wastage for cloud service providers. Auto-Saling is a popular feature, and is offered by many cloud service providers, e.g., Amazon [2].

The problem of efficient VM-to-PM packing, taken in isolation (i.e. disjointly from the VM auto-scaling problem), fits into the general framework of *stochastic bin packing*. Classical stochastic bin packing problems are such that the "items" (VMs) arrive exogeneously, are packed into "bins" (PMs), and never leave the system. (See e.g. [6], [7] for good reviews, and also a recent paper [10].) In the VM-to-PM packing problem, "items" (VMs) arrive in the system, are served for a random time, and then leave the system. This can be termed as a *stochastic bin packing with item departures*. The work in this direction is more recent, motivated primarily by the VM-to-PM assignment in a network cloud environment.

In this paper, we consider the joint VM auto-scaling and VM-to-PM packing problem, where the number of VMs are dynamically adjusted to meet the application needs and their performance requirements, while the VMs are configured (placed) in physical machines such that the cloud resources are efficiently utilized. In our framework, we explicitly consider (i) the varying demand load for applications, expressed in terms of request arrival rates for individual applications and the required service time to serve an application request; (ii) the VM type(s) an application may require, characterized by the computing, memory and storage resource required to handle certain number of application requests; and (iii) the physical machines and their resource configurations.

We consider a shadow routing based approach for this problem, We employ a specially constructed two-tier virtual queueing system, which in essence dynamically produces and maintains a solution to the underlying optimization problem that guides the actual auto-scaling algorithm. The advantage of shadow routing approach is that it is simple and adaptive: no need to know a priori, or explicitly measure, the application request arrival rates; if the arrival rates change, the algorithm adapts automatically. Yet, as we show, the algorithm is asymptotically optimal. All these features are confirmed by our simulation experiments.

The rest of the paper is organized as follows. The related work is presented in Section II. The formal model and the optimization problem are given in Section III. Section IV defines the Shadow routing algorithm, which is the key element of our scheme, proves its asymptotic optimality. In Section V we define our entire scheme, which is based on Shadow algorithm; here we also provide practical parameter settings, and discuss the asymptotic optimality of our scheme for large-scale systems. In Section VI we present simulation results for our scheme where we test its accuracy, adaptability and robustness in the face of varying application request loads. In Section VII we consider the special and challenging case of sudden and appreciable changes in the application request loads and demonstrate how expedited algorithm convergence can be achieved. Section VIII concludes the paper.

II. Related Work

Auto-scaling is important for clouds to efficiently utilize the virtualized resources. [13] gives an extensive literature review of proposed auto-scaling techniques, which are classified into five categories based on the underlying theory or techniques, namely threshold-based rules, reinforcement learning, queueing theory, control theory and time series analysis. We employ a shadow routing based approach which does not fall into any of the above categories.

Public cloud service providers, such as [2], often employ threshold-based techniques. They offer users access to the infrastructure's performance metrics, such as CPU/memory utilization, disk usage, bandwidth usage, etc. The users themselves make up the rules to decide when and how to conduct the scaling based on these metrics. [16] incorporates user's budget and QoS constraints to assign schedules for VM startups and shut-downs. [11] proposes a system and online algorithms for VM autoscaling based on load prediction. [8] consider an autoscaling solution which takes into account VM configuration and boot time. In this paper, we propose the shadow routing based auto-scaling algorithm that is dynamically driven by clients' requests. Thus no workload estimation/prediction is required. In addition, we aim to minimize the number of hosting physical machines. The existing work on auto-scaling does not take the VM-to-PM packing into consideration.

The work on *stochastic bin packing with item departures* includes papers [9], [12], [14], [15], [23]–[25] (and to some extent [10]), which focus on different aspects of the problem at different generality levels. There is other recent VM-placement work, e.g., [1], [17], [19], which addresses the placement problem from different angles, ranging from minimizing network traffic, to shortening the inter-VM distance/latency and statistically sharing resources. The most closely related work is [9]. The main contribution of [9] was to demonstrate how VM-to-PM packing constraints can be incorporated in the shadow routing framework. The distinctive feature of this paper is that we have a joint VM auto-scaling and VM-to-PM packing problem. We show that the shadow routing can still be applied, but the corresponding virtual queueing system is necessarily more complicated (it has two tiers). The proposed algorithms are hence more sophisticated.

III. Model and problem statement

A. Model

There are several applications, indexed by $i \in \mathcal{I} = \{1, \ldots, I\}$, and several classes of VMs, indexed by $j \in \mathcal{J} = \{1, \ldots, J\}$. At any given time, each application i employs a group of VMs, denoted by a vector $m_i = (m_{ij}, j = 1, \ldots, J)$ to provide the service, where m_{ij} is the number of VMs of class j assigned to application i. User service requests for application i arrive at the rate λ_i. A service request for application i is assigned to one of the VMs of application i and is serviced by that VM. The service time of an application i request is $1/\mu_i$. A VM of class j can service an integer number $w_{ij} \geq 0$ of concurrent application i requests without violating application i's SLA.

A PM in the data center (DC) can host multiple VMs. We call a vector $s = (s_j,\ j = 1, \ldots, J)$ a PM's *(feasible) configuration vector* if the PM can simultaneously host s_j number of class j VMs, $j = 1, \ldots, J$ with each VM obeying its SLA. There are two possible approaches to derive the feasible configuration vectors. The first approach assumes that the resources are reserved for individual VMs. Under this assumption, each class j VM needs several computing resources of different types when it is instantialized; namely, the amount $a_{jk} > 0$ of resources $k \in \mathcal{K} = \{1, \ldots, K\}$. The DC contains $\beta > 0$ physical machines (PM), each of which has the amount $A_k > 0$ of resource $k \in \mathcal{K}$. If a class j VM is instantialized in the DC, it is placed into one of the PMs, where the amounts a_{jk} of resources are allocated (if they are still available at that specific PM). This means that a PM's configuration vector is feasible if

$$\sum_j s_j a_{jk} \leq A_k, \quad \forall k \in \mathcal{K}. \tag{1}$$

If the resources allocated to a VM are not reserved and the VMs residing on the same physical machine share the resources, then the feasible configuration vector cannot be derived using (1). Rather, the applications need to be profiled, e.g., as shown in [20], [26], to determine if a configuration vector is feasible, i.e., the VMs can be co-located on the same PM with each VM obeying its SLA. In practice, the data center operators often have good intuitions on the possible feasible configuration vectors and can apply profiling to verify the feasibility. How to derive the feasible configuration vectors using profiling, however, is out of the scope this paper. Our optimization framework works as long as the feasible configuration vector set is available.

For the remainder of the paper, we assume the feasible configuration vectors are derived using the first approach for ease of presentation. Further, note that configuration vectors, which can be dominated by convex combinations of other configuration vectors, are inherently inefficient in the following sense. Suppose, for example, that $(2, 2), (4, 1), (1, 4)$ are feasible configuration vectors; then, instead of a large number K of PMs in configuration $(2, 2)$ it is more efficient to have $K/2$ PMs in configuration $(4, 1)$ and $K/2$ PMs in configuration $(4, 1)$. Since our results are asymptotic – i.e., they concern large-scale systems – such inefficient configuration vectors can be excluded a priori. We denote by S the set of configuration vectors which are not dominated by convex combinations of other configuration vectors. We also make a natural non-degeneracy assumption: for each i there exist j and s such that $w_{ij} \geq 1$ and $s_j \geq 1$ (otherwise application i cannot be served at all); for each j there exists s such that $s_j \geq 1$ (otherwise, we can simply exclude some j from the model).

B. Problem statement

Denote by $\phi_s \geq 0$ the fraction of PMs in the DC that are used in the configuration $s \in S$. We want a dynamic algorithm under which the fractions ϕ_s, and numbers of VMs, m_{ij}, are as close as possible to an optimal solution of the following linear program:

$$\min_{\{m^*_{ij}\},\{\phi^*_s\},\rho} \rho, \tag{2}$$

subject to

$$m^*_{ij} \geq 0, \ \forall (i,j), \quad \phi^*_s \geq 0, \ \forall s, \tag{3}$$

$$\lambda_i/\mu_i \leq \sum_j w_{ij} \cdot m^*_{ij}, \quad \forall i, \tag{4}$$

$$\sum_i m^*_{ij}/\beta \leq \sum_{s\in S} s_j \phi^*_s, \quad \forall j, \tag{5}$$

$$\sum_{s\in S} \phi^*_s = \rho. \tag{6}$$

In this LP the variables ϕ^*_s and m^*_{ij} are non-negative *real* numbers; they have the meaning of *average* values (of the corresponding "true," random quantities ϕ_s and m_{ij}), such that the *average* system workload can be handled. The LHS of (4) is the aggregate (average) workload for application i, while the RHS of (4) is the aggregate average service capacity allocated to this application. The constraint (5) might be easier to understand in the form $\sum_i m^*_{ij} \leq \sum_{s\in S} s_j \phi^*_s \beta$, with $\phi^*_s \beta$ being the average number of PMs used in configuration s and, therefore, $s_j \phi^*_s \beta$ being the average number of j-VMs served by PMs in configuration s. Finally, note that, although the meaning of ρ is the average fraction of the utilized PMs, the LP makes sense even if the optimal ρ is greater than 1 – if this is the case, the DC is overloaded. The key symbols used in the model and in the Shadow routing algorithm are included in Table I for your reference.

IV. Shadow routing

A. Construction of the virtual queueing system

We will now construct an algorithm, which makes variable updates upon each application request arrival, and asymptotically solves the LP (2)-(6) (in the sense that will be specified below). The algorithm maintains and updates a virtual ("shadow") queueing system, and makes "routing" and "service" decisions based on the current state of that system, depicted schematically in Figure 1.

TABLE I
Key Symbols

Symbol	Definition
i	application type; $i = 1, \ldots, I$
j	VM class type; $j = 1, \ldots, J$
k	resource type; $k = 1, \ldots, K$
m_{ij}	number of class j VMs assigned to serve application i
m^*_{ij}	avg. number of class j VMs assigned to serve application i
$1/\mu_i$	average service time of an application i request
a_{jk}	amount of resource k required by a class j VM
w_{ij}	max no. of application i's requests served by a class j VM
s	feasible configuration vector that satisfies condition (1)
S	set of undominated feasible configuration vectors
A_k	total amount of resource k in a PM
β	total number of PMs in a DC
λ_i	avg. arrival rate of application i's requests
ϕ_s	fraction of PMs being assigned with configuration s
ϕ^*_s	avg. fraction of PMs being assigned with configuration s
ρ	average fraction of the utilized PMs
Q_i	virtual queue size associated with application i
V_j	virtual queue size associated with VM j

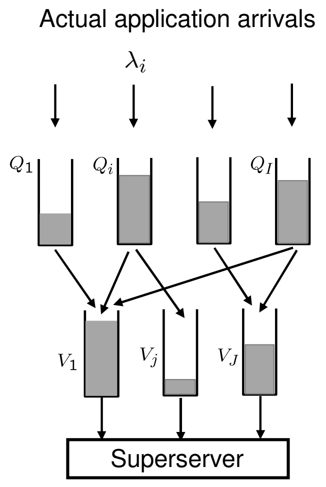


Fig. 1. Virtual (shadow) queueing system

Assume for simplicity that the arrival flow of service requests for each application i is Poisson. (This is not crucial.) Then, the sequence of request arrivals is such that the type of each arrival is determined randomly, according to probabilities λ_i/λ, where $\lambda = \sum_{i'} \lambda_{i'}$, independently of other arrivals. For each application type i there is the associated virtual queue, whose length is denoted by Q_i. For each VM class j there is the associated virtual queue, whose length is denoted by V_j. (The virtual queues are just variables maintained by the algorithm – they are not physical queues where application requests, or VMs, or anything else, wait for service.) When a request, say of type i, arrives, the following sequence of events and decisions takes place, in sequence. First, the amount $1/\mu_i$ of "work" is placed in Q_i, namely $Q_i := Q_i + 1/\mu_i$. Then, the algorithm must decide whether or not to activate a virtual server (ij), for one of the j. If (ij) server is activated, the amount $c_1 w_{ij}$ is removed from Q_i and the amount c_1/β is added to V_j, namely

$$Q_i := \max\{Q_i - c_1 w_{ij}, 0\}, \quad V_j := V_j + c_1/\beta.$$

Here $c_1 > 0$ is a parameter. Finally, the algorithm must decide whether or not to activate a "superserver" that "serves" queues V_j in one of the modes $s \in S$; if it is activated in mode s, then the amount $c_2 s_j$ is removed from each of the V_j, namely

$$V_j := \max\{V_j - c_2 s_j, 0\}, \quad \forall j.$$

Here $c_2 > 0$ is another parameter. It is easy to see that the choice of parameters c_1 and c_2 satisfying

$$c_1 > \max_i 1/\mu_i, \quad c_2 > c_1/\beta \tag{7}$$

is sufficient for the virtual queueing system to be able to "keep up" with the incoming load, in the sense that there exists an algorithm for activating servers (ij) and the superserver, so that all virtual queues remain stable (do not run away to infinity).

Now, suppose we want an algorithm for activating servers (ij) and the superserver, such that the average frequency of superserver activation is minimized, subject to the constraint that all virtual queues remain stable. The virtual queueing system and the problem we just described are within the framework of general model in [21], which gives a general *asymptotically* optimal (in the sense specified below) algorithm, called *Greedy Primal-Dual* (GPD). When we apply GPD to our virtual system we obtain the algorithm given in the next subsection. We then show in Proposition 1 that the average rates at which this algorithm activates servers (ij) and activates different modes s of the superserver, essentially solve the LP (2)-(6).

Remark. We want to emphasize that the virtual queues are *not* buffers where actual application requests or actual VM requests are placed for waiting; instead, they are no more than variables maintained by the routing algorithm. Therefore the length of virtual queues has no connection to the waiting times of actual requests.

B. Shadow algorithm

Algorithm: The algorithm maintains the virtual queues Q_i and V_j, and variables $\{b_{ij}\}, \{b_s\}, \{\bar{p}_{ij}\}$, and $\{\bar{\phi}_s\}$, $i \in \mathcal{I}, j \in \mathcal{J}, s \in S$. It also uses parameters $c_1, c_2 > 0$, as described above. In addition, there is a (small) parameter $\eta > 0$, (small) parameter $\theta > 0$, and parameter $\alpha > 0$.

Upon each new service request arrival, say of type i to be specific, the algorithm does the following (in sequence):

1. $Q_i(t) := Q_i(t) + 1/\mu_i$.
2. Set $b_{lj} := 0$ for all (lj). Compute

$$j^* \in \arg\max_j [\alpha w_{ij} Q_i - (1/\beta) V_j], \tag{8}$$

and if

$$\alpha w_{ij^*} Q_i - (1/\beta) V_{j^*} \geq 0 \tag{9}$$

do

$$Q_i := \max\{Q_i - c_1 w_{ij^*}, 0\}, \quad V_{j^*} := V_{j^*} + c_1/\beta, \quad b_{ij^*} := 1.$$

3. Set $b_s = 0$ for all $s \in S$. Find configuration vector σ such that

$$\sigma \in \arg\max_{s \in S} \sum_{j \in \mathcal{J}} s_j V_j. \tag{10}$$

If condition

$$\eta \sum_j \sigma_j V_j \geq 1 \tag{11}$$

holds, update virtual queues $V_j(t)$ as follows:

$$V_j := \max\{V_j - c_2 \sigma_j, 0\}, \quad \forall j \in \mathcal{J}.$$

4. Update variables $\bar{p}_{lj}$ and $\bar{\phi}_s$. The variables $\bar{p}_{lj}$ and $\bar{\phi}_s$ keep track of the average values of b_{lj} and b_s, respectively.

$$\bar{p}_{lj} := (1-\theta)\bar{p}_{lj} + \theta b_{lj}, \quad \forall (l,j),$$

$$\bar{\phi}_s := (1-\theta)\bar{\phi}_s + \theta b_s, \quad \forall s.$$

End of Algorithm

C. Asymptotic optimality of Shadow algorithm, as $\eta \to 0$

Proposition 1. *Suppose all system parameters and all Shadow algorithm parameters, except maybe η, are fixed rational numbers. Assume that condition (7) holds. Suppose the input flows are Poisson, with fixed rates λ_i. Consider a sequence of systems with parameter $\eta \to 0$. Then, for any η, the virtual queueing process is a positive recurrent countable discrete-time Markov chain. Moreover, stationary distributions of the processes are such that the following holds. Denote by $\bar{\phi}_s^{(\eta)}$ the steady-state probability that configuration s is chosen in (10) and condition (11) holds for it. Similarly, let $\bar{p}_{ij}^{(\eta)}$ be the steady-state probability that the arriving request is of type i, index j is chosen in (8) and condition (9) holds. Then, as $\eta \to 0$, the sequence of vectors $(\{\bar{\phi}_s^{(\eta)}\}, \{\bar{p}_{ij}^{(\eta)}\})$ is such that its any limiting point $(\{\bar{\phi}_s\}, \{\bar{p}_{ij}\})$ satisfies (using notation $\lambda = \sum_i \lambda_i$)*

$$\lambda c_1 \bar{p}_{ij} = m_{ij}^*, \quad \forall (i,j),$$

$$\lambda c_2 \bar{\phi}_s = \phi_s^*, \quad \forall\ s \in S,$$

where $(\{\phi_s^*\}, \{m_{ij}^*\})$ is an optimal solution of LP (2)-(6).

Proof of Proposition 1: The virtual queueing process, viewed as a discrete time process at the times just after request arrivals, is obviously a discrete time countable Markov chain. (Rationality of parameters implies that there is only a countable number of states.) This Markov chain is stochastically stable (see section 4.9 of [21]), which in our case means that there is a finite number of positive recurrent classes of communicating states, reachable with probability 1 from any state, and therefore a stationary distribution exists for any η. (Here we use condition (7), which guarantees that the system has sufficient capacity so that condition (55) in [21] holds.) Then, the property (AO-2) in section 4.9 of [21] can be established. In our case, it means that, as $\eta \to 0$, the algorithm solves the problem of minimizing the frequency of superserver activations, subject to stability of virtual queues. Formally, if for each η we pick a stationary distribution, then, as $\eta \to 0$, $(\{\bar{\phi}_s^{(\eta)}\}, \{\bar{p}_{ij}^{(\eta)}\})$ converges to a set of optimal solutions $(\{\bar{\phi}_s\}, \{\bar{p}_{ij}\})$ of the following LP:

$$\min_{\{\bar{p}_{ij}\},\{\bar{\phi}_s\},\bar{\rho}} \bar{\rho}, \tag{12}$$

subject to

$$\bar{p}_{ij} \geq 0, \ \forall (i,j), \quad \bar{\phi}_s \geq 0, \ \forall s, \tag{13}$$

$$(\lambda_i/\lambda)/\mu_i \leq \sum_j c_1 w_{ij} \bar{p}_{ij}, \quad \forall i, \tag{14}$$

$$\sum_i \bar{p}_{ij} c_1/\beta \leq \sum_{s \in S} s_j c_2 \bar{\phi}_s, \quad \forall j, \tag{15}$$

$$\sum_s \bar{\phi}_s = \bar{\rho}, \tag{16}$$

$$\sum_j \bar{p}_{ij} \leq 1, \ \forall i, \quad \sum_s \bar{\phi}_s \leq 1. \tag{17}$$

Using again condition (7), it is easy to see that there exists an optimal solution to the LP (12)-(16), for which condition (17) holds. This means that any optimal solution of LP (12)-(17) is also optimal for LP (12)-(16). Finally, we observe that if we rewrite LP (12)-(16) in terms of variables $m^*_{ij} = \lambda c_1 \bar{p}_{ij}$, $\phi^*_s = \lambda c_2 \bar{\phi}_s$ and $\rho = \lambda c_2 \bar{\rho}$, we obtain problem (2)-(6). The result follows. □

V. Shadow routing based scheme

We now describe the actual scheme for assigning arriving applications to VMs, and for placing VMs into PMs. The scheme is based on the Shadow algorithm, which runs continuously, driven by applications' arrivals.

A. Shadow algorithm parameter setting

First, we specify a reasonable parameter setting for the Shadow algorithm.

In view of condition (7), parameters c_1 and c_2 can be set as follows: $c_1 = 1.01 \max_i 1/\mu_i$ and $c_2 = 1.01 c_1/\beta$.

The averaging parameter θ can be set for example to $\theta = 0.001$.

"Typical value" of one V_j is

$$D_1 = (1/\eta)(1/\max_s \sum_{j \in \mathcal{J}} s_j).$$

Maximum change of a V_j is: $D_2 = \max[c_1/\beta, \ c_2 \max_{s,j} s_j]$. We choose η such that

$$D_2/D_1 = 1/\gamma, \quad \gamma = 2, 5, 10.$$

Finally, a typical scale of a Q_i is $[1/(\alpha\beta w_{ij})]$ times the typical value of V_j. (See step 2 of the algorithm.) To make sure that the typical values of Q_i are not too small compared to those of V_j, we set α so that for all (ij), $1/(\alpha\beta w_{ij}) \geq 1/3$, namely $\alpha = 3/[\beta \max_{ij} w_{ij}]$.

B. Actual application request assignment algorithm

The algorithm at any time has access to the quantities $\bar{p}_{ij}$, maintained by the Shadow algorithm. For each (ij), X_{ij} be the number of applications i currently assigned to j-VMs. This algorithm tries to drive the system state towards equalizing the ratios $X_{ij}/[w_{ij}\bar{p}_{ij}]$ across all (ij); it also tries to keep VMs that are already allocated "fully packed" by applications.

When a new i-application arrives we do the following, in sequence:
1. Map this i-application to a VM type j which minimizes $X_{ij}/[w_{ij}\bar{p}_{ij}]$.
2. This j we map into a PM configuration s according to VM-to-PM mapping algorithm (step 1) below.
3. If there are non-empty VMs of the class j, serving applications i in PMs with the configuration s, we assign the application to such a VM with the maximum number of applications in it, as long as this number is less than w_{ij} (and thus another application i still fits). Otherwise (i.e., if such non-empty VMs j do not exist or cannot fit additional aplication), we allocate a new VM j into an s-PM, according to step 2 of the VM-to-PM mapping algorithm below.

When an application departs and leaves its VM empty, we treat this as a VM departure.

C. VM-to-PM mapping algorithm

Each non-empty PM at any given time has a designated configuration $s \in S$; the designation $s = (s_1, \ldots, s_J)$ means that we will never place more than s_j class j VMs into this PM. A PM with designation s is referred to as an s-PM. Empty PMs do not have any designation. The following quantity is maintained for each $s \in S$: $z_j(s)$ – the total number of class j VMs in s-PMs. In addition, the algorithm at any time has access to the quantities $\bar{\phi}_s$, maintained by the Shadow algorithm.

This algorithm tries to drive the system state towards equalizing the ratios $z_j(s)/[s_j\bar{\phi}_s]$ across all s; it also tries to keep PMs already allocated "fully packed" by VMs.

If a VM class j needs to be mapped into a server configuration index, it is done as follows.

1. Compute configuration index

$$s \in \operatorname*{arg\,min}_{s' \in S \ : \ s'_j > 0} z_j(s')/[s'_j \bar{\phi}_{s'}].$$

If, in addition, a new VM of class j needs to be actually allocated to a PM, it is done as follows.

2. Among s-PMs (with s computed in step 1) choose a PM with the maximal number of existing VM, but such that the existing number of class j VMs is less than s_j (so that the new class j VM can still fit), and assign the VM to this PM. If no such s-PM is available, we place the VM into an empty PM and designate the PM as s-PM.

The above algorithm is identical to the VM-to-PM mapping algorithm (Algorithm B) in [9].

D. Asymptotic optimality of Shadow routing based scheme, as system scale grows

The scheme described in this section can be shown to be asymptotically optimal as the system scale grows to infinity. Specifically, suppose the application arrival rate $\lambda \to \infty$, the number of servers β grows in proportion to λ, i.e. $\beta = c_\beta \lambda$ with c_β being a constant, while all other system parameters remain constant. Then, it can be shown that, as $\lambda \to \infty$, $\phi_s/\phi_s^* \to 1$ for al s (such that $\phi_s^* > 0$) and $m_{ij}/m_{ij}^* \to 1$ for all (ij) (such that $m_{ij}^* > 0$), where ϕ_s and m_{ij} are the steady-state (random) quantities, and constants ϕ_s^* and ϕ_{ij}^* solve LP (2)-(6). The intuition for this fact is simple. The Shadow algorithm "produces" quantities $\bar{p}_{ij}$ which are close to m_{ij}^*, up to a constant factor; similarly, $\bar{\phi}_s$ are close to ϕ_s^*, up to a different constant factor. Therefore, the entire Shadow-based scheme essentially drives ratios m_{ij}/m_{ij}^* towards equalization; and similarly for the ratios ϕ_s/ϕ_s^*. However, as $\lambda \to \infty$, the total number of applications i normalized by λ becomes non-random, namely, $\sum_j X_{ij}/\lambda \to \lambda_i/\mu_i$. This can be used to show that, in the $\lambda \to \infty$ limit, the ratios m_{ij}/m_{ij}^* become non-random equal; also, in the limit, almost all allocated VMs will be fully packed; therefore, the ratios must converge to 1. By analogous argument, ratios ϕ_s/ϕ_s^* become equal to 1 in the limit. This implies that our scheme is asymptotically optimal, because LP (2)-(6) gives *lower bound on the avearge number of PMs under any scheme.*

In this paper, we do not provide a proof of asymptotic optimality in the system scale, as described in this subsection, because, despite being very intuitive, such proof would require a large amount of technical detail. (A skeptical reader can treat this form of asymptotic optimality as a conjecture.) Our simulation results, which are for sufficiently large-scale systems, do demonstrate that the system state indeed stays very close to an optimal one, given by LP (2)-(6).

VI. Evaluation

In this section we evaluate the Shadow scheme using simulations. (Here, when we refer to the Shadow scheme, or algorithm, we mean the combination of the Shadow itself as defined in Section IV-B and the actual assignment algorithms in Sections V-B and V-C.) We consider a data center with a pool of 1,000 *Physical Machines* (PMs), with each PM being configured with 42 ECUs (*Elastic Compute Units*) and 96 GBytes of memory, where one ECU is equivalent to one 1GHz single core CPU as defined by *Amazon* [3][1]. The PM configuration is roughly equivalent to a HP ProLiant SL390s G7 server. We assume there are eight *Virtual Machine* (VM) types, as listed in Table II. The *VM-size* tuple $(\#\text{ECU} : \text{MemSize})$ of these VM types are identical to the 64-bit virtual machine types supported by *Amazon*. The maximal feasible configuration set, as defined in Section III-A, contains 46 vectors.

[1]Certain commercial equipment, instruments, or materials are identified in this paper in order to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.

We assume there are four types of applications supported by the data center. Denote by w_{ij} the number of concurrent application requests of type i that can be serviced by a VM of type j. Table II lists the values of $\{w_{ij}\}$. We select the values of $\{w_{ij}\}$ such that App 1 to App 4 resemble the *CPU intensive application*, *interactive application*, *high-memory high-bandwidth application*, and *file access application*, respectively. Note that the designed algorithms are general and can handle arbitrary parameter settings.

We start with the experiments to examine the accuracy of the Shadow algorithm. We then investigate the adaptivity of the algorithms, and extend the Shadow algorithm to support *services*. The means to expedite the convergence in the face of sudden arrival rate change is studied in Section VII.

TABLE II
VM resource requirements and Number of concurrent application requests that can be serviced by a VM ($\{w_{ij}\}$)

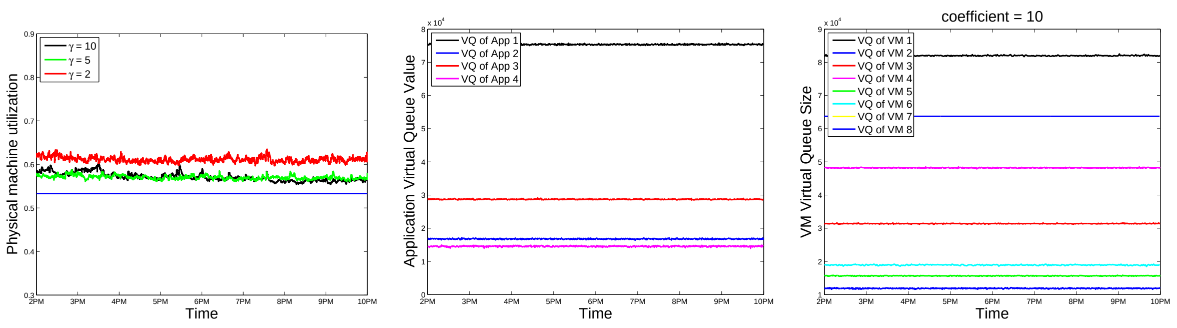
	CPU(ECU)	Mem(GB)	App1	App2	App3	App4
VM1	33.5	23	34	56	23	40
VM2	26	68.4	26	94	68	40
VM3	13	34.2	13	47	34	20
VM4	20	7	20	27	0	40
VM5	6.5	17.1	0	24	17	10
VM6	8	15	0	23	15	40
VM7	4	7.5	0	12	0	20
VM8	5	1.7	0	22	0	10

A. Datacenter PM utilizations and virtual queue sizes

In this experiment, the application requests arrive according to the Poisson process with the average arrival rate of 124 requests/sec. It resembles the scenario where all applications are equally active during the simulated time period. The service time of a request is a constant of 100 seconds. The algorithm parameters are set as described in Section V-A. While not presented, the proposed algorithms work well with different arrival rates and service times. We remove the first two hours of simulated time as the warm up period. We will examine the adaptivity/responsiveness of the algorithms in Section VI-B, and present additional procedure for expedited convergence in Section VII.

Fig. 2(a) depicts the PM utilization with different coefficient (γ) values. The larger the value of coefficient γ, the smaller the η value, and thus the Shadow algorithm is more accurate. (On the other hand, the convergence time is larger.) We also plot the optimal PM utilization value obtained using CPLEX optimizer [4] for the purpose of comparison. The optimal PM utilization is 53.4%. The PM utilizations with $\gamma = 5$ and 10 are roughly the same, around 56.5%, while the PM utilization with $\gamma = 2$ reaches 62%. The results indicate that the value of γ needs to be about 5 or greater to achieve good accuracy. We hence use $\gamma = 10$ in the following experiments.

To look into the algorithm performance, we plot the virtual queue sizes associated with different application types and different virtual machine types in Fig. 2(b) and Fig. 2(c),



(a) PM utilization ($\gamma = 2, 5, 10$) (b) Application virtual queue size ($\gamma = 10$) (c) VM virtual queue size ($\gamma = 10$)

Fig. 2. Physical machine (PM) utilizations with different coefficient values, and the virtual queue sizes for application and Virtual Machine (VM)

respectively. In Shadow algorithms, the virtual queue sizes, scaled by factor η, converge to optimal dual variables. All virtual queue sizes remain approximately constant in Fig. 2(b) and Fig. 2(c), a strong indication that the algorithm has converged to the optimal point. In addition, the values of application virtual queue sizes and the values of virtual machine virtual queue sizes are in the range of 10^4, suggesting the rescaling of virtual queue sizes using the parametesr c_1, c_2, η in Section IV-B has been effective.

B. Adaptivity of the algorithms

We next investigate the adaptivity/responsiveness of the proposed algorithms. Application request rates typically vary over time and different applications exhibit different daily patterns. For instance, companies may run more CPU intensive applications, e.g., Map-Reduce type of applications, over-night than in the day-time. Meanwhile, applications such as virtual desktop become less requested in the evening than during the working hours. In this experiment, we assume the application request rate gradually changes from 5pm to 7pm. Specifically, the request rate for application 1 increases linearly from 124 requests/second at 5pm to 208 requests/sec at 7pm, while the request rates for application 2, 3 and 4 decrease linearly from 124 requests/sec at 5pm to 35 requests/sec, 52 requests/sec, and 52 requests/sec at 7pm, respectively. The change of application request rates not only changes the optimal PM utilization, but also dynamically changes the optimal number of VMs allocated for different applications and optimal number of PMs assigned to different configurations.

Fig. 3(a) depicts the PM utilization over time. Without explicitly detecting the input rate change, the Shadow algorithm is able to automatically keep track of the new optimal operating point as it continues to run in the same manner as usual. Fig. 3(b) illustrates the number of VMs for some application types, i.e., m_{ij}. We select the ones that are *active* during the transition period from 5pm to 7pm. The number of VMs adjusts nicely based on the current application request rates. Fig. 3(c) further depicts the number of PMs assigned to some configurations. There is a total of 46 configurations in S. We selected the configurations that are assigned to PMs. The results in Fig. 3 demonstrate that the Shadow algorithm is able to promptly respond to application request rate change and successfully keep track of the new optimal operating point.

C. Providing services

In the model considered so far (see Section III), an application request is serviced by one VM. In practice, an application request can be multi-tiered and may need the services from multiple VMs. For instance, the Web application architecture consists of three tiers: *presentation tier*, *application tier*, and *data access tier*. Different tiers provide different functionalities and may require different types of VMs. The presentation tier communicates with other tiers, and displays the results to the client browsers. The application tier makes logic decisions and performs calculations. It also moves the processing data between presentation tier and data access tier. Finally, data access tier stores and retrieves the information from a data base or file system. The information is then passed back and forth to the application tier for processing, and then eventually back to the presentation tier to display to users. Different tiers require different types of computational resources.

For our purposes, let's define such request as *a service request*, where a service request employs one or multiple *applications*, as defined in Section III, to complete the service. Note that different services may employ the same underlying applications, e.g., multiple services need to access the same data base thus the data-access application can be shared among them. Without loss of generality, we assume that application requests of the same type, belonging to different services, can share a VM. (If not, these applications can be treated as different application types within our model.) Let k indexes the service types, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$, and λ_k be the arrival rate of service k. Denote by $\mathcal{I}_k \subseteq \mathcal{I}$ the subset of applications used by service k.

The optimization problem generalizes as follows:

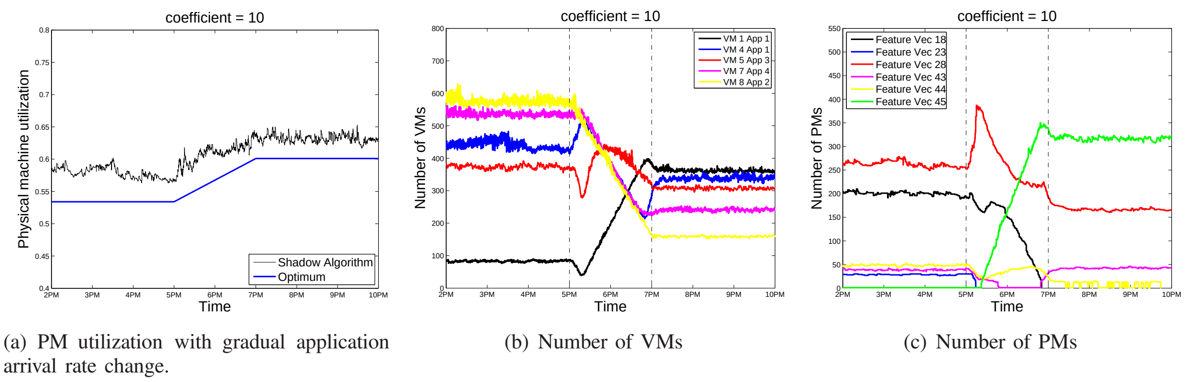
$$\min_{\{m_{ij}\},\{\phi_s\},\rho} \rho, \tag{18}$$

subject to

$$m_{ij} \geq 0, \ \forall (i,j), \quad \phi_s \geq 0, \ \forall s \tag{19}$$

$$\sum_{k:\ i \in \mathcal{I}_k} \lambda_k / \mu_i \leq \sum_j w_{ij} \cdot m_{ij}, \quad \forall i, \tag{20}$$

$$\sum_i m_{ij}/\beta \leq \sum_{s \in S} s_j \phi_s, \quad \forall j, \tag{21}$$



(a) PM utilization with gradual application arrival rate change.

(b) Number of VMs

(c) Number of PMs

Fig. 3. Datacenter physical machine utilization and optimal number of PMs and VMs with gradual application arrival rate change

$$\sum_{s \in S} \phi_s = \rho. \tag{22}$$

The problem (18) is same as the problem (2) except the constraints in (20) are more general. We note, however, that the Shadow algorithm, as described in Section IV-B, still applies as is – we simply treat each application request i as such, regardless of which service it belongs to.

We use an example to show how Shadow algorithms can solve the optimization problem (18). Suppose the cloud supports three types of services, where service 1 employs application 1, 2, and 4, service 2 employs application 2 and 4, and service 3 employs application 3 and 4. Application 4 (e.g., file access application) provides the common functionality that can be used by all three services; assume that application requests of type 4 can share a VM, regardless of which service they belong to. In contrast, assume that application requests of type 2, which belong to different services 1 and 2 *cannot* be "mixed" within same VM; therefore, the Shadow algorithm will "split" the application type 2 into two types 2.1 and 2.2, respectively.

We set the arrival rate of all three services to be 77 reqs/sec. Other parameters, such as $\{w_{ij}\}$, $\{\mu_i\}$, remain the same as in the previous experiments. Fig. 4 depicts the PM-utilization in comparison to the optimal utility. The optimal PM utility is 44.7%, while the PM utility using Shadow algorithm is 49.7% on average.

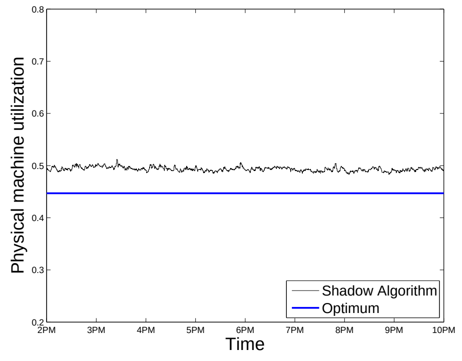


Fig. 4. PM utilization with three services

Fig. 5 further depicts five virtual queues for Constraint (20) (as compared to four virtual queues in the experiments in Section VI-A). Note that the virtual queue sizes of queue 2 and queue 5 are identical and overlap with each other. The experiment shows how to apply the Shadow algorithm to the service model, and that it performs well.

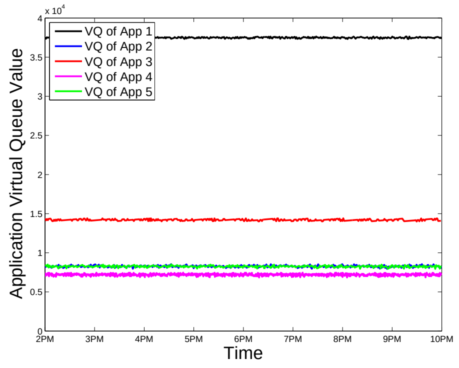


Fig. 5. Application virtual queue sizes with three services

VII. Expedited convergence

One of the salient features of Shadow algorithm is its capability to *automatically* keep track of the optimal operating point without actively measuring the request arrival rate. Different request rates correspond to different optimal points and different optimal values of $\{m_{ij}\}$ and $\{\phi_s\}$. The amount of PMs assigned to each configuration s hence needs to be adjusted accordingly so as to minimize the PM utility. While Shadow algorithm is very capable in tracking the optimal point, the VM-to-PM mapping algorithm, as described in Section V-C, also plays a pivotal role in the overall algorithm convergence speed. Below we examine how the Shadow algorithm performs in face of a sudden application request rate change. We then propose a PM redesignation algorithm that can expedite the convergence.

The experiment parameters are the same as in the experiment in Section VI-B except that the gradual transition period is replaced by a sudden trasition period, i.e., the request rates for application 1, 2, 3, and 4 are suddenly changed at 6PM. The optimal PM utilization is 53.4% from time 2PM to 6PM and 60.3% from time 6PM to 10PM. Fig. 6 depicts the value of ϕ_s, the number of PMs assigned to different configurations, and the PM utilization. The value of ϕ_s is from Shadow algorithm, while the number of PMs are the result of VM-to-PM mapping algorithm. As shown in Fig. 6(a) and Fig. 6(b), both the Shadow and VM-to-PM mapping algorithm react to the sudden rate change quickly. The value of $\bar{\phi}_s$ and the number of PMs reach new optimal point within tens of minutes.

The PM utilization, however, does suffer from a bump right after 6PM. The PM utilization shoots up to 81%, and then decreases gradually to the new optimal PM utilization point of 60.2%. This is due to the fact that optimal values of $\{\phi_s\}$ change suddenly at 6PM. Thus PMs originally assigned to a given configuration may not be needed in the new optimal solution. This is obvious as shown in Fig. 3(c). Configuration 18 is assigned to about 200 PMs in the time period before 6PM. After the sudden rate change, the number of optimal PMs assigned with configuration 18 is near zero. It takes time for the PMs assigned to the configuration 18 to leave the system, which causes the PM utilization to rise during the sharp transition period. Specifically, before 6PM, the request rate for 4 applications are 124 requests/sec. After the 6PM, the request rate for app 1 is 208 req/sec, and that of app 2, 3, and 4 are 35req/sec, 52req/sec, and 52req/sec, respectively. The minimum number of required PMs before the change is 534 PMs. Since this is a sudden change, the PMs that are used to serve the 'old' request profile won't be released immediately at 6PM. Meanwhile, the extra 208 - 124 = 84 req/sec for application 1 are extra new requests that need to be accommodated. The minimal number of PMs needed to accommodate 84 req/sec (for application 1) is 210 PMs. Hence the required number of PMs right after 6PM can be estimated as 534+210=744 PMs, or 74.4%, as shown by the red dotted line in Fig. 6(c). We next describe the *PM-redesignation algorithm* that can mitigate the above PM utilitation surge problem.

A. PM-redesignation algorithm

PM-redesignation algorithm is an additional optional procedure that can be run after a VM departs the system. Its purpose is to speed up the system transition to a new optimal regime, when sudden changes in the input flows and/or system parameters occur.

This algorithm has access to to the quantities $\bar{\phi}_s$, maintained by the Shadow algorithm. After a new VM (of any class) departs from an s-PM, we denote by $\hat{s} = (\hat{s}_1, \ldots, \hat{s}_J)$ the *actual* configuration of this PM; namely, $\hat{s}_j$ is the actual number of j-VMs this PM contains. Clearly, $\hat{s} \leq s$. If $\hat{s} = 0$, i.e., it is the empty configuration, we leave this empty PM without any designation, and stop. If $\hat{s} \neq 0$, denote by $S^*(\hat{s}) = \{s' \in S \mid \hat{s} \leq s'\}$; denote by $Y_{s'}$ the current number of s'-PMs. Compute $s^* = \arg\min_{s' \in S^*} Y_{s'}/\bar{\phi}_{s'}$. If $s^* \neq s$ and $[Y_{s^*}+1]/\bar{\phi}_{s^*} \leq [Y_s - 1]/\bar{\phi}_s$, s^* is identified as the PM-redesignation target. The PM is redesignated as an s^*-PM. In addition, every PM belonging to s is examined to see if it can be redesignated to s^*. If yes, this PM will be redesignated as an s^*-PM.

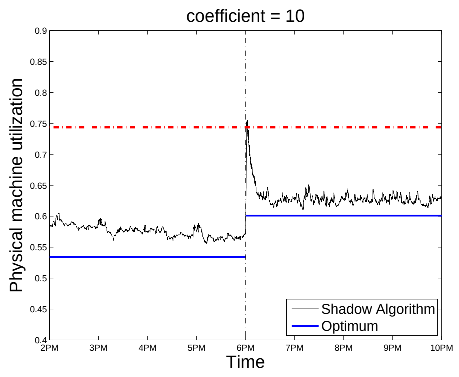


Fig. 7. PM utilization with sudden application request rates change

Fig. 7 depicts the PM utilization after running PM-redesignation algorithm. The peak PM-utilization is cut down to 75.2%, close to the required 74.4%. Fig. 8 further depicts the number of redesignated PMs over time. As expected, the majority of PM redesignations (about 200 PM redesignations) happens just after the 6PM. During the stable phase, the number of PM redesignations is minimum.

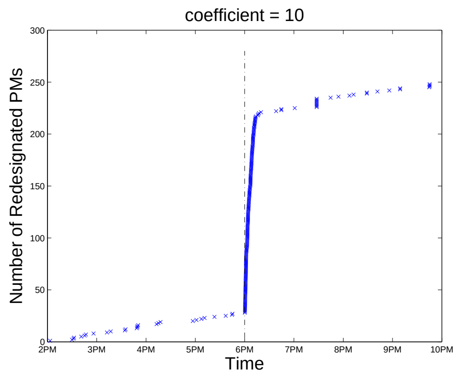
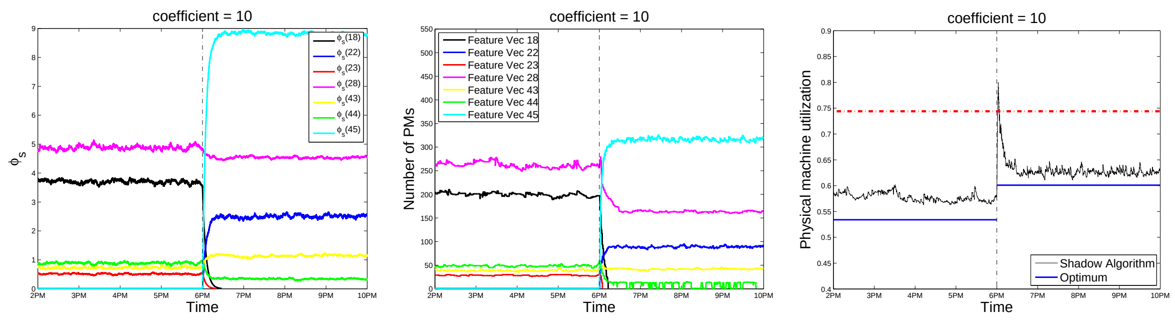


Fig. 8. Number of Redesignated PMs with sudden application request rates change

VIII. Discussion and Conclusion

In this paper we presented an optimization framework that adaptively solves the joint VM auto-scaling and VM-to-PM packing problem. Our approach integrates the interest of users in acquiring elastic resources that automatically scale with the application demands, with the interest of cloud service providers in packing VMs to efficiently utilize cloud resources. Extensive simulation results verify the proposed algorithms' efficiency, convergence, and adaptivity. Additionally, the framework is scalable to handle a large size cloud. The number of virtual queues, as shown in Fig. 1, is the sum of the number of the application types and the number of the feasible configuration vectors, and is not related to the number of PMs in a cloud.



(a) Value of $\bar{\phi}_s$ from Shadow algorithms (b) Num of Redesignated Physical Machines (PMs) (c) PM utilization

Fig. 6. Value of ϕ_s, number of PMs, and PM utilization for a suddern request rate change at time 4pm.

In the framework considered, the cloud customers do not decide on the type and number of VMs to be launched to meet the application demands. This decision is embedded in the optimization framework, and is made by the cloud provider/operator. Therefore, our framework is especially suitable for private clouds, where private cloud operators have full control over the VM auto-scaling and VM-to-PM packing. Private clouds have experienced rapid growth in recent years [5] due in part to security and privacy concerns. For public clouds, additional mechanisms are needed to allow cloud providers to steer the users' VM selection using, for example, pricing incentives. This is, however, outside the scope of this paper and is a topic of future research.

In our optimization model (2), we assume that an application can use arbitrary VM types, which is not true in practice. For instance, a CPU-intensive application needs a CPU-intensive VM type. The application profiling [26] can help in choosing the most appropriate VM type(s). For the unchosen VM types, the value of their parameter w_{ij} can simply be set to zero.

Finally, we assume in our model that the VMs can be brought up and torn down immediately. However, there is typically a boot and configuration delay which may be significant [18]. Our framework can readily incorporate the VM boot delay by inflating the job size of an application request so as to provide a sufficient resource margin. The issue of how to scale resources to reduce the number of VM boots and tear-downs may be worthy of further investigation.

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