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Josephson-based full digital bridge for high-accuracy impedance comparisons^{*}

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Abstract

This paper describes a Josephson-based full digital impedance bridge capable of comparing any two impedances, regardless of type (R-C, R-L, or L-C), over a large frequency range (from 1 kHz to 20 kHz). At the heart of the bridge are two Josephson arbitrary waveform synthesizer systems that offer unprecedented flexibility in high-precision impedance calibration, that is, it can compare impedances with arbitrary ratios and phase angles. Thus this single bridge can fully cover the entire complex plane. In the near future, this type of instrument will considerably simplify the realization and maintenance of the various impedance scales in many National Metrology Institutes around the world.

Keywords: impedance comparison, AC Josephson voltage standard, JAWS, digital bridge, AC coaxial bridge

(Some figures may appear in colour only in the online journal)

1. Introduction

Impedance metrology makes intensive use of ac coaxial bridges for the realization of the capacitance, resistance and inductance scales at audio frequencies [1]. The type and complexity of the bridge depends on the type of the comparison: ratio bridges are used to compare impedances of the same kind, quadrature bridges are used to compare capacitances to resistances, and Maxwell–Wien or resonance bridges are used to compare inductances [2, 3]. The common property of these measurement circuits is that, once the bridge is balanced, the impedance ratio to be measured is directly given by a voltage ratio. The precise and accurate generation [4–7] or measurement [8–10] of this voltage ratio is therefore the cornerstone of impedance metrology.

Prior to this work, the best voltage ratios were generated using transformers or inductive voltage dividers [11-13]. However, the main drawback of such devices is that the

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voltage ratio is set when the transformer is fabricated by choosing the number of turns of the different windings. The phase shift between the generated voltages is also limited to either 0 or 180 degrees.

Programmable Josephson voltage standards (PJVS) can generate stable and precise stepwise approximated ac waveforms and were previously used to generate an accurate voltage ratio. The first two-terminal-pair bridge based on PJVS synthesized voltages was recently demonstrated [14, 15]. This bridge was used to compare impedances of the same type (R-R and C-C) with an accuracy comparable to transformerbased bridges over a frequency range from 20 Hz to 10kHz. However, the large harmonic content of the PJVS waveform makes the comparison of impedances of different kinds (R-C, R-L or L-C) more challenging and limits the frequency range to a few kilohertz.

On the other hand, Josephson arbitrary waveform synthesizers (JAWS) are perfect digital-to-analog converters that produce quantum-accurate distortion-free voltage waveforms over frequencies between a few Hertz and 1 MHz. Combining and synchronizing two such JAWS systems enables generation of quantum-accurate, calculable voltages with arbitrary ratios and arbitrary relative phase angles.

In this work, these ideal voltage sources are implemented in a full digital bridge able to compare any impedances with arbitrary ratios and phase shifts over a large frequency range. The advantages offered by this Josephson-based full digital bridge (JB-FDB) are promising and will play a key role in simplifying impedance measurements:

- The JB-FDB currently operates up to 20kHz, a factor of two higher than the coaxial ratio bridges. However, since the maximum frequency of the JAWS systems is much higher (i.e. 1 MHz), future developments will certainly lead to impedance measurement up to a few hundred kilohertz, approaching the cutting edge of high frequency coaxial bridges [16, 17].
- Impedances of different kinds can be compared over the entire frequency range with the JB-FDB, whereas the coaxial quadrature bridge can only do this type of comparison at a single frequency.
- The bridge ratio can be set to any desired value, in particular, to 12.906 : 10. This is important for realizing the decadic scale directly from the quantized Hall resistance and will eliminate numerous steps in the traceability chain.

At present, the JB-FDB is not outperforming the traditional coaxial bridges in terms of measurement accuracy [18–22]. However, it does outperform all of them in terms of flexibility, frequency range and automation. This single fully automated JB-FDB will simultaneously replace the ratio bridge, the quadrature bridge and the Maxwell–Wien bridges. The resulting simplification in the realization of the various impedance chains is significant. In addition, the accuracy of impedance standard calibrations using the JB-FDB is not dependent on phase angle. For comparison, the present coaxial bridge technique cannot perform calibrations at arbitrary phase angles. All these new features will considerably broaden the range of possible impedance calibrations.

In section 2 of this paper, the JAWS systems are described in detail. In section 3, the full four-terminal-pair digital bridge is presented. This bridge is based on a digitally assisted bridge (fully described in [23]), in which the ratio transformer has been replaced with the dual JAWS sources. The first test of the system is presented in section 4.1, where the frequency dependence of a 12.906 k Ω resistor was measured and compared with a calibration performed using a traditional transformer-based bridge. In section 4.2, a consistency check was carried out by performing a triangular comparison of an inductor, a capacitor and a resistor. A similar although more elaborate consistency check is depicted in section 4.3 involving two capacitors and two resistors of different values. Finally, the conclusions of this work are presented in section 5.

2. The dual JAWS systems

The two voltage sources required by the JB-FDB are provided by two independent pulse-driven JAWS systems (also known as the ac Josephson voltage standard or ACJVS) operated in two separate dewars of liquid helium. Each JAWS chip consists of four arrays of 12 800 double-stacked Josephson junctions (JJs) connected in series using on-chip superconducting traces. A single channel of a custom high-frequency pulse generator [24, 25] is split by an on-chip Wilkinson divider and drives two JJ arrays at a pulse rate of $14.4 \cdot 10^9 \text{ s}^{-1}$. More system details are described in [26].

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The smallest operating current range of the systems is 1.4 mA, that is, it can withstand ± 0.7 mA of current on the input and still operate with quantum-accuracy (see [26] for details). This 1.4 mA operating current range was measured while each JAWS system was generating a 1 kHz sine wave with an rms amplitude of 950 mV. The maximum rms voltage output of each JAWS system is 1 V.

The success of the JB-FDB bridge relies on the intrinsic stability, linearity, and tunability of the two JAWS systems. The clocks of the JAWS system and the other components of the FDB are locked to a 10 MHz signal derived from a NIST primary reference frequency standard. The relative phase stability of the two JAWS pulse generators is guaranteed by having the two pulse generators share a single 14.4 GHz clock. The phase stability of the combined systems is determined by both the 10 MHz clock and by the repeatability of the response of one of the JAWS systems and the FDB to a fast rise-time trigger from the other JAWS system.

The amplitude and phase of the two JAWS output voltages are adjusted as part of the JB-FDB balancing procedure by re-calculating the required pulse pattern using a delta-sigma algorithm [27]. The phase of each synthesized waveform can be controlled to the duration of a single pulse, resulting in an approximate phase resolution of 70 ps. Since we require the output voltages to be repeatable within 1 part in 10^6 , we also do not assume that the delta-sigma algorithm provides the correct voltage. Instead, we directly extract the amplitude and phase of each output voltage by performing a Fourier transform on each pulse pattern generated by the algorithm.

Prior to the impedance ratio measurement, the relative phase (time delay) between the two JAWS systems is calibrated by measuring the phase required to cancel the output voltages from the two systems. Once adjusted, the residual voltage ('1 V'—'1 V' at 1 kHz) is less than 100 nV as measured by using a separate digitizer (see [28] for more details on this type of measurement).

Subsequent drift or instability in the trigger delay will cause an error in the relative phase of the waveforms generated by the two JAWS systems. The time delay between the trigger outputs of the two JAWS systems was monitored and found to vary by less than 1 ns. The error in the absolute voltage generated by each JAWS system is also proportional to the error in the frequency of the 10 MHz clock, but this error is small and will cancel in a ratio measurement.

The JAWS output voltage is quantum-accurate at the JJs, but the inductance and stray capacitance of the output voltage leads and the impedance of the load will affect the voltage delivered to the load [29–31]. The error due to the leads directly affects the accuracy of measurements of the synthesized JAWS signals, but a ratio measurement can be less sensitive. The two JAWS systems were constructed at the same time using the same techniques, circuits and materials, so the voltage leads are nominally identical. The effect of the voltage lead frequency response on the output voltage of the two systems is therefore expected to be similar, which typically reduces the error by an order of magnitude.

An additional voltage error is caused by the low frequency compensation current biasing the inductance of the JJ array [32]. This induced error voltage is small at low frequencies and is phase-shifted by approximately $\pi/2$ relative to the voltage generated by the JJs. Again, the symmetry of the system means that this error will mostly cancel in a ratio measurement. This and other sources of error and their stability will need to be quantified in more detail to accurately measure impedance at higher frequencies, and an additional error cancellation technique will be discussed in section 3.

3. Bridge description

Figure 1 shows a detailed schematic of the JB-FDB which was developed to make a high accuracy comparison of two impedance standards, Z_{top} and Z_{bot} . The working principle of the JB-FDB is very similar to that of the digitally assisted bridge (DAB) recently developed at METAS [23] and can be summarized as follows: the sources S_{top} and S_{bot} supply, through the 1 : 1 injection transformers, the current to the standards to be compared. The amplitude and phase of these sources are adjusted until no current is flowing in the high potential leads, i.e. $V_{top}^{HP} = V_{bot}^{HP} = 0$ (current source balances). The amplitude and phase of the source S_K is adjusted to null the voltage at the low potential port of the top standard, i.e. $V_{top}^{LP} = 0$ (Kelvin balance). Finally, the amplitude and the phase of the voltage V_{bot} is adjusted to null the voltage at the low potential port of the top standard, i.e. $V_{top}^{LP} = 0$ (Kelvin balance).

Once these balances are simultaneously reached, the fourterminal-pair definition [33] of the two impedance standards is realized and the impedance ratio is given by:

$$\frac{Z_{\text{bot}}}{Z_{\text{top}}} = -\frac{V_{\text{bot}}}{V_{\text{top}}}.$$
(1)

The main differences between the JB-FDB and the DAB are the following:

- The accurate and stable voltage ratio is generated using two JAWS instead of a ratio transformer. In other words, the amplitudes and the phases of V_{top}^{JAWS} and V_{bot}^{JAWS} can each be set to any desired value, thus making the comparison of arbitrary impedances possible using a single bridge.
- The currents flowing through the impedance standards are entirely supplied by the signal generators *S*_{top} and *S*_{bot}. In the DAB, only a fraction of this current was supplied by these sources. The remaining current was provided directly by the transformer.
- Double-screened isolation transformers are used to measure the voltage at the low-potential (LP) ports of the standards. These isolation transformers were added

to improve the galvanic isolation between the bridge itself and the electronic parts of the generators and the digitizers.

The data processing and the balancing procedure used in the JB-FDB are similar to those used by the DAB and the reader can refer to [23] for further details. The only difference lies in the main balance procedure. In the DAB, the voltage ratio of the transformer is fixed so the main balance is achieved by tuning the injected voltage S_{inj} . For the JB-FDB, on the other hand, the main balance is achieved by directly changing the voltage ratio, that is, the phase and the amplitude of V_{bot}^{JAWS} . The injection voltage S_{inj} is no longer required but the possibility of injecting a small voltage S_{inj} has been maintained so that the JB-FDB is compatible with the DAB software. Moreover, additional consistency checks can be performed on the main balance of the bridge using S_{inj} .

3.1. Cable correction and bridge offset

For a classical transformer-based bridge, the voltage ratio at the transformer's taps slightly differs from the ratio of the number of turns [34]. Therefore, the voltage ratio needs to be calibrated [11] to account for this small but significant error.

Similarly and as discussed in section 2, the quantum accurate reference voltages V_{top}^{JAWS} and V_{bot}^{JAWS} generated at the output of the JJ arrays slightly differ from the voltage V_{top} and V_{bot} in the JB-FDB due to the loading effect of the cables [29–31]. However, as shown below, this cable loading effect can be canceled by a simple repetition of the ratio measurement interchanging impedances Z_{top} and Z_{bot} [7].

The voltage V_{top} is defined along the high-potential cable (HP in figure 1) and the detection transformer is used to ensure that no current is flowing at this point when the bridge is balanced. Therefore, the JJ arrays are only loaded by the cable between the output of the JJ arrays (inside the cryostat) and the detection transformer (at room temperature). Figure 2 shows the equivalent circuit of that part of the bridge. The relation between V_{top} and $V_{\text{top}}^{\text{JAWS}}$ is then given by:

$$V_{\rm top} = \frac{V_{\rm top}^{\rm JAWS}}{1 + Y_{\rm c}[Z_{\rm o} + Z_{\rm c}/2]} = V_{\rm top}^{\rm JAWS} \left(1 + \Delta_{\rm top}^{\rm load}\right), \quad (2)$$

where Δ_{top}^{load} is a complex number that describes the loading correction to apply to the reference voltage V_{top}^{JAWS} to obtain the voltage V_{top} . The real part of the loading correction increases with the square of the frequency and the square of the cable length at frequencies below 1 MHz, with a typical values of $1 \cdot 10^{-6}$ at 10kHz [30].

A similar correction applies to the bottom voltage, and the impedance ratio (1) becomes:

$$\frac{Z_{\text{bot}}}{Z_{\text{top}}} = \Gamma = -\frac{V_{\text{bot}}^{\text{JAWS}} \left(1 + \Delta_{\text{bot}}^{\text{load}}\right)}{V_{\text{top}}^{\text{JAWS}} \left(1 + \Delta_{\text{top}}^{\text{load}}\right)},$$
(3)

where $S_{inj} = 0$ was assumed for simplicity.

When the top and bottom standards are reversed, the new impedance ratio $\tilde{\Gamma}$ is given by:



Figure 1. Schematic of the four-terminal pair JB-FDB bridge circuit. Once the bridge is balanced, i.e. once V_{top}^{HP} , V_{bot}^{LP} , V_{bot}^{LP} are nulled by adjusting the amplitude and the phase of the bottom JAWS source, as well as the voltages S_{top} , S_{bot} , and S_K , the four-terminal-pair definition of the two impedance standards is realized and the impedance ratio Z_{bot}/Z_{top} is equal to the voltage ratio $-V_{bot}/V_{top}$. V_{ref} is an auxillary measurement of the reference voltage used to normalized all the voltages in the data processing. The fundamental difference between the JB-FDB and the DAB [23] is that the accurate and stable voltage ratio is generated using two JAWS systems instead of a ratio transformer.



Figure 2. Equivalent circuit of the part of the bridge between the JJ arrays and the detection transformer where V_{top} is defined. Z_c and Y_c are respectively the series impedance and stray admittance of the cable. Z_o is the output impedance of the JJ arrays.

$$\frac{Z_{\text{top}}}{Z_{\text{bot}}} = \tilde{\Gamma} = -\frac{\tilde{V}_{\text{bot}}^{\text{JAWS}} \left(1 + \Delta_{\text{bot}}^{\text{load}}\right)}{\tilde{V}_{\text{top}}^{\text{JAWS}} \left(1 + \Delta_{\text{top}}^{\text{load}}\right)}.$$
(4)

As long as the loading corrections are the same during the direct and reverse measurements, equations (3) and (4) can be combined to obtain:

$$\frac{Z_{\text{bot}}}{Z_{\text{top}}} = \pm \sqrt{\frac{\Gamma}{\tilde{\Gamma}}} = \pm \sqrt{\frac{V_{\text{bot}}^{\text{JAWS}} \tilde{V}_{\text{top}}^{\text{JAWS}}}{V_{\text{top}}^{\text{JAWS}} \tilde{V}_{\text{bot}}^{\text{JAWS}}}},$$
(5)

where the loading corrections have been eliminated. The sign is determined by the kinds of impedances being compared.

3.2. Stability of the current sources

The realization of the four terminal-pair definition of the impedances requires that no current flows through the HP cables, i.e. $V_{top}^{HP} = 0$ and $V_{bot}^{HP} = 0$. This condition is obtained by adjusting the sources S_{top} and S_{bot} . Therefore the source stability is a critical parameter that influences the accuracy of the bridge balance. The required source stability can be greatly reduced by adding an impedance *Z* to each of the *HC* leads, as described below.

Figure 3 shows the equivalent circuit of the top part of the bridge (see figure 1), including V_{top}^{JAWS} , S_{top} , Z and Z_{top} . The impedance *z* represents the output impedance of the JJ arrays as well as the series impedance of the cable between the array and the HP port. In this particular case, the loading effect due to the cable admittance can be neglected.

When the bridge is not balanced, a small current *i* flows through the port HP and the voltage V_{top} slightly differs



Figure 3. Equivalent circuit of the part of the bridge between the JJ arrays, the current source S_{top} , and the impedance standard Z_{top} .

from the reference voltage $V_{\text{top}}^{\text{JAWS}}$ by a quantity ΔV given by $V_{\text{top}} = V_{\text{top}}^{\text{JAWS}} + \Delta V$ where:

$$\Delta V = zi = \frac{z}{Z} \left[V_{\text{top}} \frac{Z + Z_{\text{top}}}{Z_{\text{top}}} - S_{\text{top}} \right].$$
(6)

No current flows through the port HP when the bridge is balanced, and then $S_{top} \equiv S_{top}^{eq} = V_{top}(Z + Z_{top})/Z_{top}$. In this case, the potential V_{top} at this port is precisely the reference potential V_{top}^{JAWS} . The source instability and noise will add a small bias ΔS_{top} to S_{top}^{eq} that will perturb the main balance voltage by a quantity:

$$\Delta V = -\frac{z}{Z} \Delta S_{\text{top.}}$$
(7)

Equation (7) shows that the accuracy of the voltage V_{top} is limited by the stability of the source ΔS_{top} , and the perturbation is proportional to the factor |z/Z|. The series impedance z is typically smaller than 1 Ω at audio frequencies. Therefore a resistance of $Z = 12.9 \text{ k}\Omega$ was added at the output of both current sources that reduces the effects of noise and drift in the source generators S_{top} and S_{bot} by approximately a factor of 10^4 .

4. Results of validation measurements

The JB-FDB was designed and tested at METAS using semiconductor DAC voltage sources and then sent to NIST-Boulder for validation measurements with the JAWS sources. In addition, seven impedance standards were also sent to NIST-Boulder from either METAS or NIST-Gaithersburg. Table 1 lists the different impedance standards⁵ used for the validation measurements.

The main objective of this first measurement campaign was to demonstrate the flexibility and versatility of the JB-FDB bridge by comparing different types of impedances at frequencies up to 20 kHz. Although most of the standards were calibrated before the measurement campaign, it was not useful to directly compare the JB-FDB calibrations with earlier calibrations because the potential effects of transportation on the standards is significantly larger than the bridge uncertainty.

Table 1. List of the seven impedance standards used for the validation measurements.

#	Name	Туре	Nominal value
1	Z_{10k}	Vishay VHP101	10 kΩ
2	Z^{A}_{12k9}	Vishay VHP101	12.906 kΩ
3	$Z^{\mathrm{B}}_{12\mathrm{k}9}$	Vishay VHP101	12.906 kΩ
4	Z _{129k}	Vishay VHP101	129.06 k Ω
5	Z_{L1}	GenRad 1582-P	1 H
6	Z _{C1n}	GenRad 1404-A	1 nF
7	Z _{C10n}	NIST-built	10 nF

Note: Standards #2 and #3 are temperature stabilized, the others are not.

Instead, we compared the more-stable frequency dependence of the resistors and made more detailed consistency checks.

4.1. Resistance comparison

The first test performed was the measurement of the frequency dependence of the resistance standards. While the dc value of the resistances were subject to drift during transportation, their frequency dependence were stable. Therefore the measurements performed with the JB-FDB at NIST were objectively compared to those carried out at METAS using the DAB.

The frequency dependence of the resistance Z_{12k9}^{B} was measured using the resistance Z_{12k9}^{A} as a reference. The bottom part of figure 4 shows the measurements carried out between 1 kHz and 20 kHz. The symbols correspond to the values measured using the JB-FDB and the solid line is a quadratic fit to the measurements performed with the DAB.

In the top part of figure 4, the difference between values measured with the JB-FDB and the DAB are shown. The gray zone represents the combined (k = 1) uncertainties taken from the uncertainty budget of the DAB [23] while the uncertainty bars correspond solely to the type A uncertainties for the measurements made using the JB-FDB. At several frequencies, the JB-FDB measurements have been repeated a number of times over a few days and corrections for the small drift in the dc resistance have been applied. At these particular frequencies, the residual spread of the results is slightly larger than the type A uncertainty. This indicates that some systematic effects remain to be investigated. Nevertheless, the agreement between the results obtained with the JB-FDB and the DAB is better than $0.1 \mu \Omega/\Omega$. This result clearly shows the potential of the JAWS sources when implemented in an impedance bridge.

4.2. Closure of the RLC triangle

The ability of the JB-FDB to compare impedances of any kind opens the possibility of making consistency checks that are impossible to perform using transformer-based bridges. As represented in figure 5, a resistance, an inductance and a capacitance standard can be compared in a triangular manner. Their comparison, two by two, leads to three different complex numbers Γ_i (*i* = 1, 2, 3) that can be combined in the following way:

⁵ Certain commercial equipment, instruments, or materials are identified in this paper in order to specify the experimental procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology, nor is it intended to imply that the materials or equipment identified are necessarily the best available for the purpose.



Figure 4. The bottom plot (b) shows the frequency dependence of $\Delta_{ac/dc}$ of the resistance Z_{12k9}^B measured with the JB-FDB (symbols) and with the DAB (solid line). The top plot (a) shows the difference δ between the JB-FDB and DAB results. The errors bars correspond to the type A uncertainty of the JB-FDB measurement. The gray zone represents the combined (k = 1) uncertainties for the measurements made with the DAB [23].

$$\Delta_{\Delta} = \frac{\Gamma_2}{\Gamma_1} - \Gamma_3 = \frac{Z_L/Z_R}{Z_L/Z_C} - Z_C/Z_R,\tag{8}$$

where the complex number Δ_{Δ} should ideally be zero.

The main advantage of this consistency check is that the reference value of each standard does not need to be known; the standards only need to remain stable during the time required for the three comparisons which is typically less than 30 min.

This test has been realized by comparing Z_{10k} , Z_{L1} and Z_{C10n} multiple times over a period of a few days. The comparisons were carried out at a frequency of 1592 Hz where both the impedance of the 10 nF capacitor and the 1 H inductor have a nominal value equal to 10 k Ω . Therefore, the measured voltage ratios $|\Gamma_i|$ (i = 1, 2, 3) are close to unity.

Figure 6 shows the real and the imaginary components of Δ_{Δ} obtained using the different comparisons. The values are clearly not randomly distributed around $|\Delta_{\Delta}| = 0$ and a strong linear correlation between the real and imaginary components of Δ_{Δ} is observed. This correlation can be explained by the large temperature coefficient of the series resistance and inductance of Z_{L1} . Indeed, the impedance of the inductor has a temperature dependence given by: $Z_{L1} = R_s(1 + c_1\Delta T) + j\omega L_s(1 + c_2\Delta T)$. If the temperature changes by an amount ΔT between the comparisons Γ_1 and Γ_2 , then Δ_{Δ} will no longer be zero and will take the following expression:



Figure 5. Schematic of a triangular comparison allowing a consistency check of the JB-FDB.

$$\Delta_{\Delta} = \frac{\omega L_s R_s (c_2 - c_1)}{R_s^2 + \omega^2 L_s^2} \cdot \Delta T - j \frac{R_s^2 c_1 + \omega^2 L_s^2 c_2}{R_s^2 + \omega^2 L_s^2} \cdot \Delta T, \quad (9)$$

with the approximation $Z_{C10n}/Z_{10k} \approx 1/(j\omega RC) \approx -j$ at a frequency of 1592 Hz.

The solid line in figure 6 was calculated using equation (9) and a maximum temperature variation of $\Delta T = \pm 40$ mK (ends of the black line). In this calculation, the value of the different parameters were taken from the inductance standard's specification (type GenRad 1582-P): $R_s = 616 \ \Omega$, $L_s = 1 \ H$, $c_1 = 3937 \ \mu \Omega/\Omega \ /K$ and $c_2 = 30 \ \mu H/H/K$. The good agreement between the measurements and the

Data Model

2.0

1.5

1.0

0.5

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 Δ_{Δ} obtained with eight measurements of the triangular comparison depicted in figure 5 at a frequency of 1592 Hz. The uncertainty bars correspond to the type A uncertainty only. The solid line is the result of a model that includes the temperature dependence of the inductance. The ends of the line represent a variation of ±40 mK between the two comparisons Γ_1 and Γ_2 (see text for details).

calculated temperature dependence suggests that variation in the temperature of the inductance standard during the measurements is the main source of uncertainty in these measurements.

4.3. Closure of the RC square

As shown in the previous section, the accuracy of the consistency check that involves closing the RLC triangle is limited to a few $\mu \Omega / \Omega$ by the temperature stability of the inductance standard.

To circumvent this limitation, consistency check experiments have also been performed using only the more stable resistance and capacitance standards. Figure 7 graphically shows the different impedance comparisons that can be performed using an RC square that is made up of two capacitors and two resistors. Table 2 lists the different standards used for the RC square and gives the nominal impedance ratio expected at 1233 Hz. There are 1 : 1 and 1 : 10 comparisons both in phase and in quadrature. The comparison Γ'_6 uses the same standards as Γ_6 , but the JAWS systems generate different output voltages: a rms voltage of 1V is used for Γ_6 and 0.1V is used for Γ'_6 .

Inside the RC square, different consistency check loops Δ_{\Box} can be formed that combine the different measured impedance ratios Γ_i (i = 4-9). Four different loops are formed using three comparisons (triangles) and one loop is formed using four comparisons (square). Ideally, the measurements that make up these loops should be consistent and Δ_{\Box} should be zero. Any deviation from zero implies an error in the bridge and/or an impedance that is unstable during the entire RC square measurement.

Table 3 lists the real and imaginary components of Δ_{\Box} measured at a frequency of 1233 Hz. The last column contains the uncertainty component related to the accuracy of the main balance of the bridge (type A). The same uncertainty applies to both the real and imaginary components of Δ_{\Box} .



Figure 7. Graphical representation of the different impedance comparisons used in the RC square consistency check.

Table 2. List of the impedances Z_{bot} and Z_{top} used in the comparison $\Gamma_i = Z_{\text{bot}}/Z_{\text{top}}$ represented in figure 7.

#	Z _{bot} (rms voltage)	Z_{top} (rms voltage)	$\Gamma_{\rm Nominal}$
Γ ₄	$Z^{\rm A}_{12k9}(0.1{ m V})$	$Z_{129k}(1V)$	0.1 + j 0
Γ_5	Z _{C10n} (0.1 V)	$Z_{129k}(1V)$	0 - j 0.1
Γ_6	Z _{C10n} (0.1 V)	$Z^{\rm A}_{12k9}(0.1{\rm V})$	0 - j 1
Γ_7	Z _{C10n} (0.1 V)	$Z_{C1n}(1V)$	0.1 + j 0
Γ_8	$Z_{12k9}^{A}(0.1\mathrm{V})$	Z _{C1n} (1V)	0 + j 0.1
Г9	$Z_{129k} (1 \text{ V})$	$Z_{C1n}(1V)$	0 + j 1
Γ_6'	$Z_{C10n}(1V)$	$Z_{12k9}^{A}(1{ m V})$	0 - j 1

Note: The nominal rms voltage applied to the standard is given in parenthesis. The last column gives the nominal impedance ratio at a frequency of 1233 Hz.

Figure 8 shows graphically the values listed in table 3. One observes that all loops are consistent to within $0.5 \mu \Omega/\Omega$, indicating that the bridge works correctly to within this uncertainty.

For a few loops, the uncertainty is significantly larger than $0.1 \mu \Omega/\Omega$. After investigating the raw data taken during the automated measurements, it appears that the main balance for the comparison Γ_7 (the 10 nF to 1 nF comparison) was not fully reached. The uncertainty of this comparison is therefore significantly larger than the uncertainty of the other comparisons. Consequently, the uncertainty of the loops involving this comparison are also larger (by a factor of 4–8). This balancing problem will be corrected in future measurements.

At this point, it is important to note that although the closure of the RC square is a necessary condition to prove the correctness of the JB-FDB measurements, it is not sufficient to fully validate the bridge. Indeed, the different loops in the RC square involve differences of possible errors in the voltage ratios. In case these errors are independent of the relative phase between the JAWS voltages, they will cancel each other and will not be detected. Therefore, our next experiment will pursue validation of the JB-FDB by performing a direct comparison between a co-located JB-FDB and transformer-based bridge. This last RC square consistency check is instead intended to demonstrate the flexibility and versatility of the JB-FDB, it does not represent the final validation test of the system.

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Table 3. Real and imaginary components of the different consistency loops of the RC square.						
Δ_{\Box}	Shape	$\operatorname{Re}\{\Delta_{\Box}\}(\mu \Omega/\Omega)$	$\operatorname{Im}\{\Delta_{\Box}\}(\mu \Omega/\Omega)$	unc. (μ Ω/Ω)		
$\Gamma_5/\Gamma_6 - \Gamma_4$	\angle	-0.091	-0.285	±0.094		
$\Gamma_7/\Gamma_6 - \Gamma_8$	$\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	-0.087	-0.209	±0.339		
$\Gamma_8/\Gamma_9-\Gamma_4$	\bigtriangledown	+0.052	-0.364	±0.040		
$\Gamma_7/\Gamma_9 - \Gamma_5$	\bigtriangledown	+0.008	+0.066	±0.349		
$\Gamma_6 \cdot \Gamma_4 - \Gamma_7/\Gamma_9$		+0.277	-0.157	±0.341		
$\Gamma_5/\Gamma_6'-\Gamma_4$	\bigtriangleup	-0.111	-0.308	±0.093		
$\Gamma_7/\Gamma_6'-\Gamma_8$	$\ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	-0.064	-0.230	±0.339		
$\Gamma_6'\cdot\Gamma_4-\Gamma_7/\Gamma_9$		+0.300	-0.177	±0.341		

Note. The second column shows the shape of the loop in the RC square represented in figure 7. The type A uncertainty (last column) is calculated from the residuals in the main balance only.



Figure 8. Measurements of the real and imaginary components of Δ_{\Box} for the different loops listed in table 3. The solid red symbols represent the loops where the comparison Γ'_6 has been used instead of Γ_6 . The dashed circle is a guide to the eyes and corresponds to $|\Delta_{\Box}| = 0.5 \,\mu \Omega / \Omega$.

5. Conclusion

For the first time, two JAWS systems have been integrated into a four-terminal-pair bridge. This single bridge can compare impedances of different kinds (R-C, R-L or L-C) over a broad frequency range.

The first test measurement involved comparing two resistances of 12.906 k Ω at frequencies between 1 kHz and 20 kHz. The measured frequency dependence of the resistance ratio is in good agreement (<0.05 $\mu \Omega/\Omega$) with the frequency dependence measured with a classical transformer-based bridge.

Other type of internal consistency checks were carried out by checking that multiple impedance comparisons traversing an RLC triangle (at 1592 Hz) and an RC square (at 1233 Hz) were consistent.

The RLC triangle experiment showed that the JB-FDB can be used to compare any kind of impedance in a 1 : 1 ratio. However, the accuracy of this RLC measurement was limited by the temperature stability of the inductance standard.

The RC square experiment demonstrated that the bridge is also able to compare impedances in a 1 : 10 ratio, either in phase or in quadrature. The loop measurements of the RC square are consistent with an accuracy better than 0.5 $\mu \Omega/\Omega$. This accuracy is presently limited by the convergence of the main balance of the bridge.

These preliminary measurements were acquired during a two week measurement campaign; they represent a convincing demonstration of the capabilities and flexibility of the JB-FDB. The resistor frequency dependence measurements show the potential accuracy of the bridge, which is around a few parts in 10^8 at frequencies up to 20 kHz. Further measurements are necessary, both to demonstrate similar accuracy in measurements of other types of impedances and to directly compare the JB-FDB to a transformer-based bridge. These measurements must be done in a laboratory having facilities to guarantee a direct traceability of the different impedance units to the quantized Hall resistor.

The results presented in this paper clearly show the potential of this JAWS-based digital bridge. In the near future, the JAWS system will become more compact and switch to using a cryocooler instead of liquid helium. This will allow these new JAWS-based digital bridges to replace their transformer-based counterparts leading to great simplification in the establishment and maintenance of impedance scales across the world.

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