# **High-temperature Material Constitutive Models for Structural-Fire Analysis**

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## ABSTRACT

The applicability of three steel constitutive models was evaluated using finiteelement analyses and various member capacity equations. Three different hightemperature stress-strain models were compared: the model recently developed by the National Institute of Standards and Technology (NIST) [1], the Eurocode 3 model [2] and the model developed by Lie [3]. The testbed used in the analyses included twenty steel column tests and two restrained steel beam tests reported in the technical literature. The selected column tests reported buckling temperatures ranging from 500 °C to 700 °C and applied axial load ranging from 20 % to 65 % of the axial-load capacity at ambient temperature. Each reported test was analyzed in two different ways: (1) finite-element model was developed to predict the buckling temperature of the steel columns and response of the restrained steel beams in fire condition. (2) member capacity equations prescribed in Eurocode 3 and ANSI/AISC-360-10 [4] were used to compute the buckling temperature of the steel columns. Overall, the results indicate that all investigated material models give acceptable prediction of the buckling temperature of the steel columns and the behavior of restrained beams. The finite-element model with the NIST and the Lie material models predict the buckling temperature more accurately than that with the EC 3 material model. When the Eurocode column capacity equations were used, the buckling temperatures calculated using the NIST and the EC 3 models are more comparable with test results than those using the Lie model. It was also found that the current ANSI/AISC 360-10 Appendix 4 equation conservatively estimate the buckling temperature of the tested column specimens with difference of 20% on average. When the standard column equation in the Chapter E of ANSI/AISC 360-10 was used, both the EC 3 and the NIST models accurately predict the buckling temperature of the tested column specimen with difference less than 5% on average.

## **INTRODUCTION**

Calculation methods are often adopted to determine the fire protection for steel structures as opposed to conducing costly experiments. Accurate high temperature constitutive models are required to reasonably predict the structural performance

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under fire conditions. As part of the investigation on the collapse of the World Trade Center, the National Institute of Standards and Technology (NIST) characterized the steels recovered from the collapse site to analyze the failure induced by the air-craft impact and fire. In the investigation [5], the high-temperature tensile testing was conducted following ASTM E21 [6]. With the test data in the investigation and the data found in the technical literature, a new constitutive model, referred to as the NIST steel stress-strain model or NIST model in this paper, was developed to predict the high-temperature behavior of structural steels [1,7]. This paper compares the NIST model with the two widely used constitutive models, the Eurocode 3 model [2] and the TTLie model [3], for predicting the behavior of steel components under fire conditions. The constitutive models are used to predict the buckling temperature of steel columns and the response of restrained steel beams under uniform fire condition. In this paper, buckling temperature is defined as the steel temperature at the onset of buckling.

#### **STEEL STRESS-STRAIN MODELS**

#### **Mathematical formulation**

Detailed description of the NIST model can be found in Ref.[8]. The stress-strain expressions for the NIST model is given in Eq.1,

$$\sigma = \begin{cases} \varepsilon E_T & (\varepsilon \le \frac{f_{yT}}{E_T}) \\ f_{yT} + (k_3 - k_4 f_{y20}) \exp[(\frac{T}{k_2})^{k_1}] (\varepsilon - \frac{f_{yT}}{E_T})^n & (\varepsilon > \frac{f_{yT}}{E_T}) \end{cases}$$
(1)

where  $k_1$ =7.82,  $k_2$ =540°C,  $k_3$ =1006 MPa,  $k_4$ =0.759, and n=0.503. The elastic modulus and yield strength at elevated temperature are calculated by

$$\frac{E_T}{E_{20}} = \exp\left[-\frac{1}{2}\left(\frac{T-20}{639}\right)^{3.768} - \frac{1}{2}\left(\frac{T-20}{1650}\right)\right]$$
(2)

and

$$\frac{f_{yT}}{f_{y20}} = \exp\left[-\frac{1}{2}\left(\frac{T-20}{590}\right)^{5.7} - \frac{1}{2}\left(\frac{T-20}{919}\right)\right]$$
(3)

where  $E_{20}$ ,  $E_{\rm T}$  are elastic modulus of steel at ambient and elevated temperatures, respectively; and  $f_{y20}$ ,  $f_{y\rm T}$  are yield strength of steel at ambient and elevated temperatures, respectively.

The expressions for the Eurocode 3 model and the TTLie model can be found in Refs.[2] and [3], respectively.

#### Compare with material test data

Figure 1 compares the calculated reduction factors for elastic modulus and yield strength with the test data collected by Luecke et al. [1]. The NIST model shows good agreement with the test data.



Figure 1. Calculated reduction factors for elastic modulus and yield strength against the test data collected by Luecke et al. [1]. The test labels are the same as in Ref. [1].<sup>1</sup>

## **CALCULATION APPROACHES**

#### **Eurocode 3 design approach**

Simple analytical approaches given in the design codes are mostly used in daily design work. The simple approach developed by Franssen et al. [9] is recommended in the Eurocode 3 [2] for calculating the buckling resistance of axially loaded steel columns in fire, which is given by

$$N_{b,T} = \chi_T A f_{\gamma T} \tag{4}$$

with

$$\chi_T = \frac{1}{\varphi_T + \sqrt{\varphi_T^2 - \overline{\lambda}_T^2}}$$
(5)

$$\varphi_T = \frac{1}{2} [1 + \alpha \overline{\lambda}_T + \overline{\lambda}_T^2]$$
(6)

where  $\alpha = 0.65\sqrt{235/f_{y20}}$ ,  $\overline{\lambda}_T = \sqrt{Af_{yT}/P_{ET}}$ . *A* is the steel cross section area and  $P_{ET}$  is Euler bucking load at elevated temperature. By solving  $P_T = N_{b,T}$ , we obtain the column buckling temperature. Here  $P_T$  is the column service load under fire condition.

#### **ANSI/AISC design approach**

The 2005 and 2010 editions of the ANSI/AISC-360 Appendix 4 [4] specify to use the Eurocode 3 temperature-dependent mechanical properties for design of steel

<sup>&</sup>lt;sup>1</sup> The Eurocode 3 yield strength plotted here is determined at the 0.2 % offset for comparison purposes, while the high-temperature yield strength in the Eurocode 3 is defined at 2% strain.

members at elevated temperatures. According to the 2005 edition, the critical buckling stress,  $F_{cr}(T)$ , for steel column for fire conditions can be computed using the standard design equations (i.e., in Chapter E of the ANSI/AISC-360), as expressed in Eqs [7] through [9], with the temperature-dependent values of elastic modulus, E(T), and yield strength,  $F_y(T)$ . On the other hand, the 2010 edition prescribes Eq [10] to compute flexural buckling strength of columns at elevated temperatures. The Eq [10] is valid only when Eurocode 3 mechanical properties are considered for design. Both versions of the equations use the effective column slenderness ratio, KL/r, which is independent of temperatures, to compute the temperature-dependent elastic buckling stress  $F_e(T)$  (given in Eq [9]).

$$F_{cr}(T) = \left[0.658^{\frac{F_y(T)}{F_e(T)}}\right] \cdot F_y(T) \quad \text{for } F_e(T) \ge 0.44F_y(T) \tag{7}$$

$$F_{cr}(T) = 0.877 \cdot F_e(T) \quad \text{for } F_e(T) < 0.44F_y(T)$$
 (8)

$$F_{e}(T) = \frac{\pi^{2} E(T)}{\left(\frac{KL}{r}\right)^{2}}$$
<sup>(9)</sup>

$$F_{cr}(T) = F_{y}(T) \cdot 0.42^{(F_{y}(T)/F_{e}(T))^{0.5}}$$
(10)

#### **FE** approach

#### COLUMN MODEL

The three-dimensional shell element, SHELL181, implemented in ANSYS 14.0.0 [10] was used since this element is suitable for analyzing thin to moderately thick shell structures. The column cross sections were discretized into twenty elements based on mesh optimization study. The shape of initial column crookedness was defined as the first mode obtained from elastic buckling analysis. The initial deflection amplitude at mid-height, if not specified, was taken as L/1000. Neither the effect of residual stress due to cooling of the hot-rolled shape nor the thermal gradient from fire was modeled explicitly. The buckling temperature of columns was computed from the point at which the force equilibrium could not be achieved.

#### **RESTRAINED BEAM MODEL**

Figure 2 shows a FE structural model for a restrained steel I-shaped beam. The steel beam was modeled using SHELL181, and the restraints at the beam ends were modeled using spring-damper element COMBIN14. As shown at the right corner in Figure 2, an axial spring and a rotational spring located at mid-height of the beam end section were used to provide axial and rotational restraints, respectively. This approach can be used to model various end conditions.



Figure 2. FE model of a restrained steel I beam.

## **TEST DATA**

## **Steel columns**

The five column data sets, which were selected from Zhang et al. [11], were used for FE simulations and design calculations, Table II shows a total of twenty individual column specimens along with the reported failure temperatures ( $T_{b,meas}$ ) and other test parameters, such as the ambient temperature yield strength ( $f_{y20}$ ), column length (L),slenderness ratio ( $\lambda = L/r$ , where r is the radius of gyration), the applied axial load( $P_T$ ), the boundary conditions (ends, where P-P is pinned-pinned; F-F is fixedfixed; and P-R is pinned-rotationally restrained), and the initial eccentricity (e).

| Data     | Test | Shape        | $f_{y20}$ | L    | λ   | $P_T$ | e    | Ends | $T_{b,mea}$ |
|----------|------|--------------|-----------|------|-----|-------|------|------|-------------|
|          |      |              | (MPa      | (mm) |     | (kN)  | (m   |      | (°C)        |
| Ali [12] | Ali1 | UC152×152×23 | 320       | 1800 | 47  | 186   | 0    | P-P  | 701         |
|          | Ali2 | UC152×152×23 | 320       | 1800 | 47  | 373   | 0    | P-P  | 626         |
|          | Ali3 | UC152×152×23 | 320       | 1800 | 47  | 559   | 0    | P-P  | 557         |
|          | Ali4 | UB178×102×19 | 320       | 1800 | 75  | 202   | 0    | P-P  | 629         |
|          | Ali5 | UB178×102×19 | 320       | 1801 | 75  | 303   | 0    | P-P  | 539         |
|          | Ali6 | UB178×102×19 | 320       | 1802 | 75  | 101   | 0    | P-P  | 644         |
|          | Ali7 | UB127×76×13  | 320       | 1803 | 97  | 50    | 0    | P-P  | 717         |
|          | Ali8 | UB127×76×13  | 320       | 1804 | 97  | 101   | 0    | P-P  | 658         |
|          | Ali9 | UB127×76×13  | 320       | 1805 | 97  | 151   | 0    | P-P  | 567         |
| Choe[13] | 1    | W8×35        | 413       | 3500 | 67. | 1134  | 0    | P-P  | 500         |
|          | 2    | W8×35        | 413       | 3500 | 67. | 800   | 0    | P-P  | 600         |
|          | 3    | W14×53       | 406       | 3450 | 70. | 1435  | 0    | P-P  | 500         |
|          | 4    | W14×53       | 406       | 3450 | 70. | 1070  | 0    | P-P  | 600         |
|          | Liel | W10×60       | 300       | 3810 | 34  | 1760  | 0    | F-F  | 565         |
| Lie [2]  | Lie2 | W10×49       | 300       | 3810 | 34  | 1424  | 0    | F-F  | 586         |
|          | Lie3 | W10×49       | 300       | 3810 | 34  | 1424  | 0    | F-F  | 584         |
|          | RS45 | UC152×152×37 | 326       | 1500 | 38  | 708.5 | 1.74 | P-P  | 647         |
| Tan [14] | RS55 | UB203×133×25 | 357       | 1500 | 47  | 444.3 | 3.19 | P-P  | 571         |
|          | RS81 | UB152×89×16  | 312       | 1500 | 70  | 260.6 | 2.38 | P-P  | 499         |
|          | RS97 | UB127×76×13  | 320       | 1500 | 83  | 134   | 4.08 | P-P  | 606         |

\*Note: Ali, Lie, and Tan - transient tests; Choe - steady state tests.

#### **Restrained steel beams**

Two tests were considered to evaluate the response of restrained beams in fire. Test on specimen 1 in Li and Guo [15] and test on "FUR15" in Liu et al.[16] were considered. In [15], the tested beam had a cross section H250×250×8×12 and a clear span length of 4500 mm. Two concentrated loads were applied symmetrically on the restrained beam by two jacks. The space between these two point loads was 1500 mm. The load ratio of the restrained beam was 0.7. The axial stiffness provided by the restrained frame was  $k_a$ =39.54 kN/mm and the rotational stiffness was  $k_r$ =1.09×10<sup>8</sup> Nm/rad. In [16], the tested beam had a cross section 178×102×19UB and a clear span length of 2000 mm. Two symmetrical concentrated loads were applied. The space between these two point loads was 800 mm. The load ratio of the restrained beam was 0.5. End-plate beam-to-column connections were used. The axial stiffness provided was  $k_a$ =8 kN/mm and the rotational stiffness was  $k_r$ =1.4×10<sup>5</sup> Nm/rad.

## RESULTS

#### **Buckling temperatures**

Figure 3 shows comparisons among the predicted and measured values for column buckling temperature by using different material models. Table III shows the statistics of the ratios of the difference among the analytical results and measured data for different material models. The mean and standard deviation (Std) are presented in the table. For FE approach, all three models give acceptable predictions, and NIST and TT Lie models give better prediction than the EC3 model. For Eurocode 3 approach, all three models give under-predictions, and NIST and EC3 models give better prediction than the TT Lie model.



Figure 3. Column buckling temperatures predicted using FEM (a) and Eurocode 3 method (b).

| <b>Statistics</b>        | NIST   | EC3    | TT Lie |  |
|--------------------------|--------|--------|--------|--|
| FEM: mean                | 0.060  | 0.110  | 0.024  |  |
| FEM: standard Std        | 0.075  | 0.091  | 0.084  |  |
| Eurocode 3: mean         | -0.066 | -0.096 | -0.156 |  |
| Eurocode 3: standard Std | 0.091  | 0.068  | 0.113  |  |

TABLE III. STATISTICS OF THE RATIOS OF THE DIFFERENCE AMONG THE ANALYTICAL RESULTS AND MEASURED DATA\*.

\*Note: the ratio is defined as  $(T_{b,pred} - T_{b,meas})/T_{b,meas}$ .

Figure 4 shows that the current ANSI/AISC 360-10 Appendix 4 equation conservatively estimate the buckling temperature of the tested column specimens with difference of 20% on average (Figure 4a). When the standard column equation in the Chapter E of ANSI/AISC 360 was used, both the EC 3 and the NIST models accurately predict the buckling temperature of the tested column specimen with the difference less than 5% on average (Figure 4b).



Figure 4. Column buckling temperature predicted using ANSI/AISC-360 (a) Appendix 4 (b) Chapter E.

## **Response of restrained beam**

Figure 5 show the FE predicted results for the restrained steel beam. All three material models give good prediction of the response of the restrained beams.

## CONCLUSIONS

A comparative study of three high temperature steel constitutive models for structural fire analyses was presented. All investigated material models give acceptable prediction of the buckling temperature of steel columns. For the FE approach, using NIST and TTLie models give better prediction than the EC3 model; and for the Eurocode analytical approach, NIST and EC3 models give better prediction than the TTLie model. All three models give good prediction of the response of restrained steel beams subjected to fire.

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Figure 5. FE results for restrained force and mid-span deflection for restrained steel beam.

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