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A STATISTICAL APPROACH TO ESTIMATING A 95 % CONFIDENCE LOWER LIMIT FOR THE DESIGN CREEP RUPTURE TIME VS. STRESS CURVE WHEN THE STRESS ESTIMATE HAS AN ERROR UP TO 2 % (*)

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ABSTRACT

Recent experimental results on creep-fracture damage with minimum time to failure (minTTF) varying as the 9^{th} power of stress, and a theoretical consequence that the coefficient of variation (CV) of minTTF is necessarily 9 times that of the CV of the stress, created a new engineering requirement that the finite element analysis of pressure vessel and piping systems in power generation and chemical plants be more accurate with an allowable error of no more than 2 % when dealing with a leak-before-break scenario. This new requirement becomes more critical, for example, when one finds a small leakage in the vicinity of a hot steam piping weldment next to an elbow. To illustrate the critical nature of this creep and creep-fatigue interaction problem in engineering design and operation decision-making, we present the analysis of a typical steam piping maintenance problem, where 10 experimental data on the creep rupture time vs. stress (83 to 131 MPa) for an API Grade 91 steel at 571.1 C (1060 F) are fitted with a straight line using the linear least squares (LLSQ) method. The LLSQ fit yields not only a two-parameter model, but also an estimate of the 95 % confidence upper and lower limits of the rupture time as basis for a statistical design of creep and creep-fatigue. In addition, we will show that when an error in stress estimate is 2 % or more, the 95 % confidence lower limit for the rupture time will be reduced from the minimum by as much as 40 %.

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1. INTRODUCTION

In reporting engineering observations such as materials property test data involving two variables, the most common practice is to fit the data with a straight line. An example of this is given in Fig. 1, where 25 observations of variable X_1 (pounds of steam used per month) vs. variable X_8 (average atmospheric temperature in degree F) are plotted with a regression line representing a linear, first-order model [1].

The problem with this engineering practice is that no additional quantitative information about the scatter or uncertainty of the data is also reported, even though additional analysis methodology exists to yield, for example, 95 % confidence limits as shown in Fig. 2 [1]. This deficiency in data reporting and data compilation in engineering design and materials property data handbooks made it impossible for engineers to estimate the useful life of a component or system with evidence-based quantification of uncertainty and to conduct a subsequent risk analysis for decision-making.

This incomplete data analysis problem is compounded by a not-so-well-known but highly inconvenient fact that the independent variable X_8 in the regression model is usually accompanied by some error or uncertainty. An example of this appeared in a recent paper by Cohn, Cronin, Faham, Bosko, and Liebl [2], where creep rupture time (variable X_1) vs. stress (variable X_8) data obtained not from a laboratory but from a handbook curve [3] without uncertainty information were used to make maintenance decisions under the assumption that the stress (X_8) estimated from finite element method (FEM)-based analysis is accurate and without error.

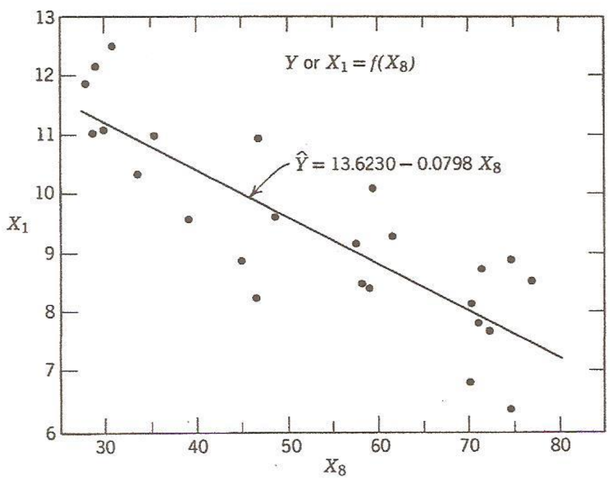

Fig. 1 A linear, first order model of a relationship between X_1 (pounds of steam used per month) and X_8 (average atmospheric temperature in F.) of 25 observations documented by Draper and Smith [1] to illustrate the regression methodology.

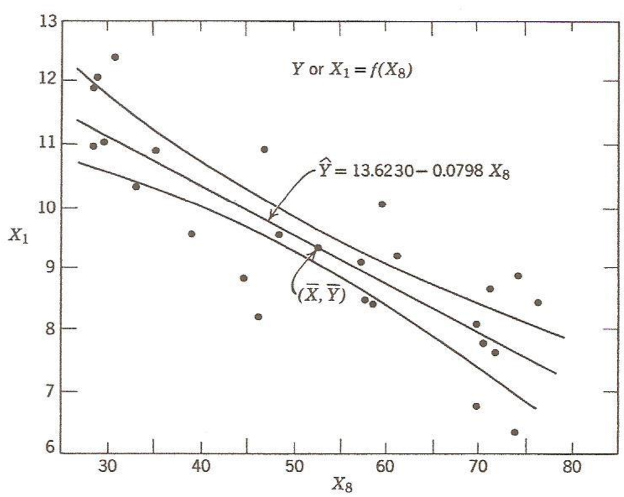

Fig. 2 The same set of data and its regression line as plotted in Fig. 1 now appear with two hyperbolic curves [1].

FEM-based analysis has been applied to estimating stress by engineers since the 1970s [4]. The method is well known to yield approximate solutions that need to be verified and validated before use [5]. As shown in a recent series of papers by Fong, et al. [6 - 10], the accuracy of the estimated stress depends on at least five sources of uncertainty: (1) FEM codes, (2) FEM element type, (3) FEM mesh density, (4) FEM mesh quality such as the mean aspect ratio of elements, and (5) the uncertainties associated with the governing equations, the initial and boundary conditions, the physical and material property parameters, and geometry. More specifically, it was shown in Marcal, et al. [7], Fong, et al. [8, 9] and Rainsberger, et al [10] that different choices of FEM element type, mesh density, mesh quality, and FEM code can yield different estimates of maximum stress in an elastic deformation problem of a pipe-elbow with a surface crack in one of its two girth welds by as much as a factor of two.

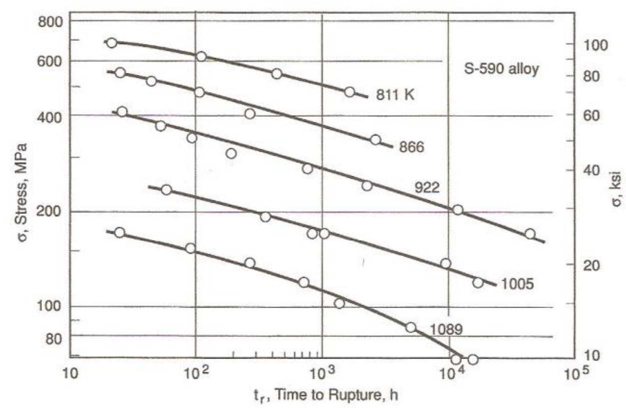

Fig. 3 Stress vs. rupture life curves for S-590, an iron-based heat-resisting alloy, at five temperatures (811 K to 1089 K) as reported by Goodhoff [11] and reproduced in a book by Dowling [12]. This figure is referred to in Section 2.

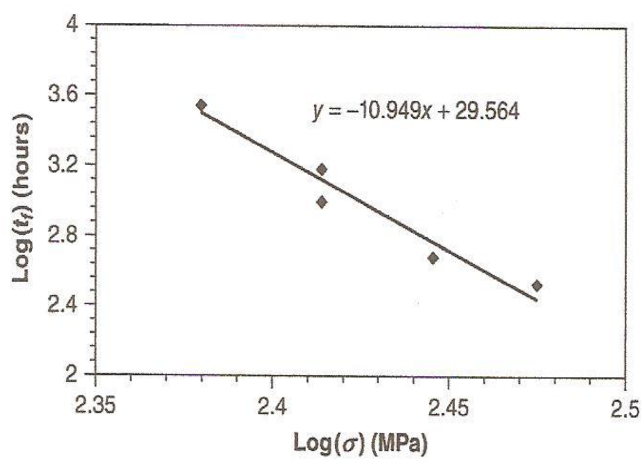

Fig. 4 Uniaxial creep rupture data for 316 stainless steel at 600 C in a log-log plot as reported by Hyde, et al. [13], and reproduced in a book by Hyde, Sun, and Hyde [14]. This figure is referred to in Section 2.

In this paper, we aim to develop a methodology to conduct, through the use of a numerical example, a more complete statistical data analysis of the creep rupture time vs. stress data, assuming a simple power-law model and a 2 % error in stress.

In Section 2, we first present an incomplete analysis of the creep rupture time vs. stress data for two materials, i.e., an iron-based heat-resistant alloy named S-590 at five temperatures [11, 12], and the 316 stainless steel at 600 C [13, 14]. We then present a more complete analysis of the API Grade 91 steel at three temperatures, 550 C [15], 571.1 C [2], and 600 C [15], to show the difference between the two analysis methods.

In Section 3, we present the results of a recent investigation [6 – 10] of the accuracy of a FEM-based estimates of stresses in piping or pipe-elbow with surface crack in one of its girth welds. Our results led us to a conclusion that the assumption of an accurate stress estimate without error in predicting creep rupture time is unwise. We plan to show that even a 2% error in stress may lead to a large over-estimation of rupture life.

In Section 4, we present the methodology and the results of a new analysis of the creep rupture time vs. stress data, where we are able to predict the 95 % confidence lower limit of the creep rupture curve with the effect of a 2 % stress error.

Significance and limitations of this new approach to estimating uncertainty in life prediction due to uncertainty in materials property test data and the FEM-based stress estimates, are presented in Section 5. A discussion, some concluding remarks, and a list of references are given in Sections 6, 7, and 8, respectively.

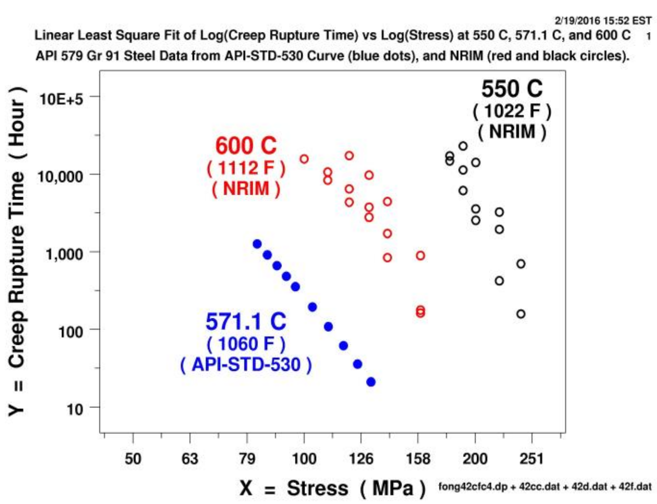

Fig. 5 Creep Rupture Time vs. Stress Data in a log-log plot for API Gr. 91 Steel at 3 temperatures: The 571.1 C (1060 F) data (blue) are from an API minimum design curve [3] as listed in Table 1; the 600 C (1112 F) data (red) are from NRIM [15]; and the 550 C (1022 F) data (black) are also from NRIM [15]. Note that the blue data (571.1 C) have very little scatter because they are derived from an API min. design curve (see Table 1).

2. EXPERIMENTAL DATA AND ANALYSES

As mentioned in Section 1 (Introduction), experimental data in two variables can be analyzed using a linear, first-order model in two ways: (a) without uncertainty information (Fig. 1) and (b) with uncertainty information (Fig. 2). Examples of the uncertainty-free method (a) are given in Figs. 3 and 4, both of which appear in the engineering literature [11 – 14] as recommended design curves. Using this uncertainty-free method (a), Cohn, et al. [2] obtained a regression line of ten data points, as shown in Table 1, from an API STD 530-based recommended table, F31, of creep rupture time vs. stress data for API Gr. 91 steel [3]. In this section, we will analyze the same data with uncertainty and compare the results with two other sets of data given by NRIM [15] for the same material.

Table 1 (after Cohn, et al. [2])
API Gr. 91 Steel Creep Rupture Time vs. Stress at 571.1 C
(minTTF = minimum Time To Failure.)

Stress (ksi)	Stress (MPa)	minTTF (1000 hours)
12	82.74	1266.43
12.5	86.19	914.20
13	89.63	663.75
13.5	93.08	484.60
14	96.53	355.72
15	103.4	194.70
16	110.3	108.70
17	117.2	61.84
18	124.1	35.81
19	131.0	21.10

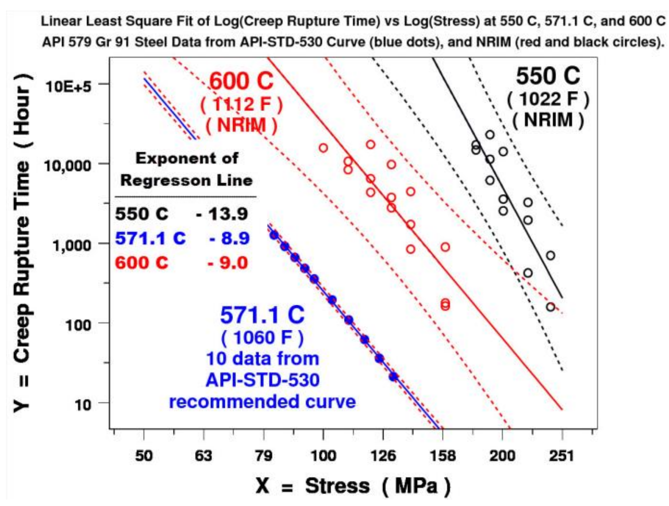

Fig. 6 Linear Least Squares Fit with 95 % Confidence Limits for three sets of Creep Rupture Time vs. Stress Data at 550 C, 571.1 C, and 600 C. The material is API Grade 91 steel. The 550 C and 600 C data are from NRIM [15], and the ten data points for 571.1 C (1060 F) are from an API minimum design curve [3] as listed in Table 1 [2].

A plot of the data in Table 1 is given in Fig. 5 (blue dots) and a regression analysis of those data complete with 95 % confidence limits is given in Fig. 6 (blue dots with red scatter band). It is not surprising that the scatter band is very small, because the data are not from experiments at a testing laboratory, but from engineering literature without uncertainty information [2, 3]. That is similar to the data and regression lines given in Figs. 3 and 4 (see refs. [11, 12, 13, 14]..

Nevertheless, the methodology to compute the 95 % confidence limits for a linear, first-order model [1] exists and is applicable whether the data are from handbooks (see, e.g., Table 1) or experimental values (see, e.g., NRIM [15]).

To show the difference between the results of two analysis methods, one without and a second one with uncertainty quantification, we choose to work with the rupture time data of a single material, i.e., the API Grade 91 steel. In Fig. 5, we present the 571.1 C data of Table 1 (blue dots), and two sets of experimental data from NRIM [15] (represented as red circles for 600 C and black circles for 550 C). We then present in Fig. 6 the 95 % confidence limits for all three sets of data, showing the huge scatter band for the experimental data (600 C and 550 C) and the relatively small band for the handbook data (571.1 C).

A complete exposition of the methodology for computing the 95 % confidence limits is given in a 1966 book by Draper and Smith [1], and has been in the statistical literature way before 1960s. Unfortunately, this elegantly described analysis method with uncertainty quantification is still not well-known to most practicing engineers today, as witnessed by the current prevailing practice of reporting materials property data in handbooks and textbooks without any information on the scatter of the data. For completeness, we provide below the 13 key equations, all quoted directly from Draper and Smith [1], that allow us to compute the uncertainties of three quantities, b_1 , b_0 , and the vector, $\hat{Y}$, in a linear, first-order model:

$$Y = \beta_0 + \beta_1 X + \epsilon, \tag{1}$$

Here, X and Y are two vectors, ($X_1, X_2, \ldots Xi, \ldots X_n$), and ($Y_1, Y_2, \ldots Yi, \ldots Y_n$), and β_0 , β_1 , the two fitting parameters. The vector, ε , represents the error introduced when we fit a set of n data by a linear, first-order model of Eq. (1). Let us introduce two quantities of interest, namely, $\bar{X}$, the average of the X's, and $\bar{Y}$, the average of the Y's.

$$\hat{Y} = b_0 + b_1 X, \tag{2}$$

Here, we introduce a vector quantity, $\hat{Y}$, that is the predicted value of the vector Y using the linear, first-order model and dropping the error vector, ε . We also introduce b_1 and b_0 .

$$b_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} \tag{3}$$

$$b_0 = \bar{Y} - b_1 \bar{X} \tag{4}$$

Eqs. (3) and (4) solve directly for b_1 and b_0 . Eqs. (5), (6) and (7), each containing a new quantity, s , are required to compute the confidence limits of all key quantities of interest, i.e., b_1 , b_0 , and any predicted $\hat{Y}_0$ of $\hat{Y}$ for a given X_0 .

$$b_1 \pm \frac{t(n-2, 1-\frac{1}{2}\alpha)s}{\{\sum (X_i - \bar{X})^2\}^{1/2}} \tag{5}$$

$$b_0 \pm t(n-2, 1-\tfrac{1}{2}\alpha)\left\{\frac{\sum X_i^2}{n \sum (X_i - \bar{X})^2}\right\}^{1/2} s. \tag{6}$$

$$\hat{Y}_0 \pm t(\nu, 0.975)\left\{1 + \frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right\}^{1/2} s, \tag{7}$$

To compute the new quantity, s , we need to understand and solve all of the Eqs. (8) through (13) as listed below:

$$\text{Sum of squares about the mean} = \text{Sum of squares about regression} + \text{Sum of squares due to regression}. \tag{8}$$

$$SS_2 = SS + SS_1 \tag{9}$$

$$SS_2 = \sum Y_i^2 - (\sum Y_i)^2/n \tag{10}$$

$$SS_1 = \frac{\{\sum X_i Y_i - (\sum X_i)(\sum Y_i)/n\}^2}{\{\sum X_i^2 - (\sum X_i)^2/n\}} \tag{11}$$

From Eq. (9),

$$SS = SS_2 - SS_1 \tag{12}$$

$$s^2 = \frac{(SS)}{(n-2)} \tag{13}$$

To solve the above as we did for the hyperbolic curves of the 95 % confidence limits presented in Fig. 6, we wrote a computer code in DATAPLOT [16], which is freely available upon request to the first co-author.

3. UNCERTAINTY IN FEM STRESS ANALYSIS

The large values of the regression line exponents of the three sets of data in Fig. 6 indicate that a small change in creep stress is bound to lead to a large change in rupture life. Since most engineers use the finite element method (FEM) to estimate stress, it is important to ask whether the FEM-based stress estimates are accurate.

As mentioned in Section 1 (Introduction), FEM has been known as an approximation method [4] for a long time. The need to verify and validate FEM-based estimates has also been carefully documented [5]. Largely because of cost and a commitment of time, it is common practice today that FEM-based estimates of stress are delivered to owners and plant operators without uncertainty quantification nor with an appropriate protocol of verification.

Based on a recent series of paper [6 – 10], we show in this section that the assumption of zero error in stress estimate is wrong and unrealistic, and can easily lead to serious consequences in life modeling such as creep rupture time estimation. In Fig. 7 we show a typical problem in powerplants where a surface crack is found in a girth weld of a pipe-elbow of a main steam piping system. Using three types of FEM elements and two different FEM codes [17, 18], we solved the elastic deformation problem of a 900-mm-o.d. (outside diameter), 20-mm-thick, 90-degree-pipe elbow with a 50-mm-long, 10-mm-deep surface crack in one of its two girth welds (Fig. 8). The predicted max. crack tip stress was found to vary from a low of 231.69 MPa to a high of 457.96 MPa (see Appendix A and Ref. [10, 19, 20]. A typical result of the FEM stress analysis with uncertainty quantification is given in Fig. 9 (ABAQUS-Hex-8, 11 mesh densities), and a comparison of the multiple-code-element-type exercise is given in Fig. 10. A ranking of the seven scenarios of FEM runs is given in Appendix A and Ref. [20].

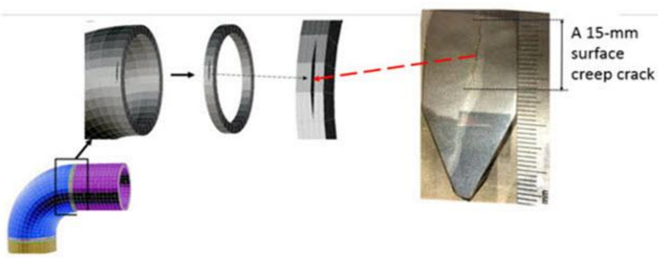

Fig. 7 A typical surface crack in a girth weld of a pipe elbow in the main steam piping system of a power-generation plant [2].

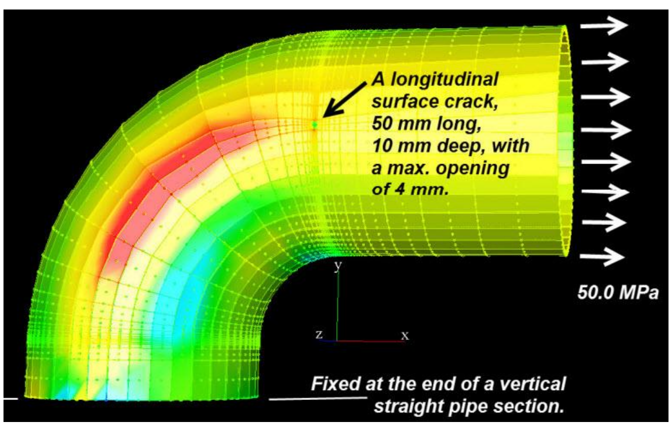

Fig. 8 An FEM solution (MPACT-Hex-27 at 149,706 degrees of freedom) for the elastic deformation of a pipe-elbow with a longitudinal surface crack in one of its two girth welds [8].

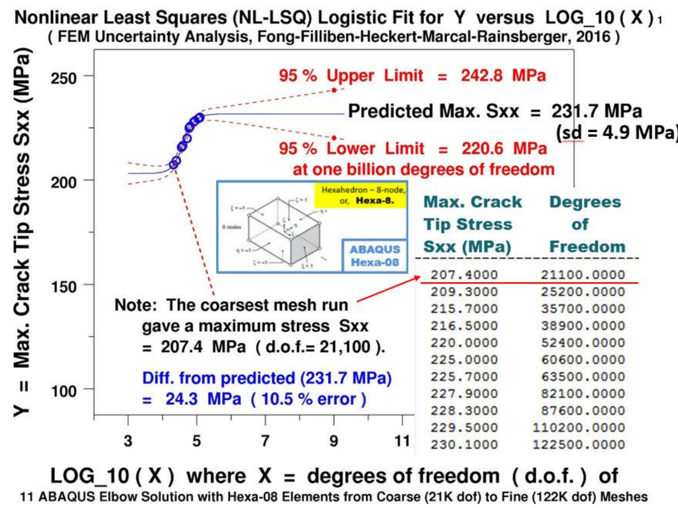

Fig. 9 A Nonlinear Least Squares Logistic Fit of 11 ABAQUS Hexa-08 Solution of an Elbow-Weld-Crack problem with 95 % Confidence Limits of Predicted Max. Stress at 10E+9 d.o.f.

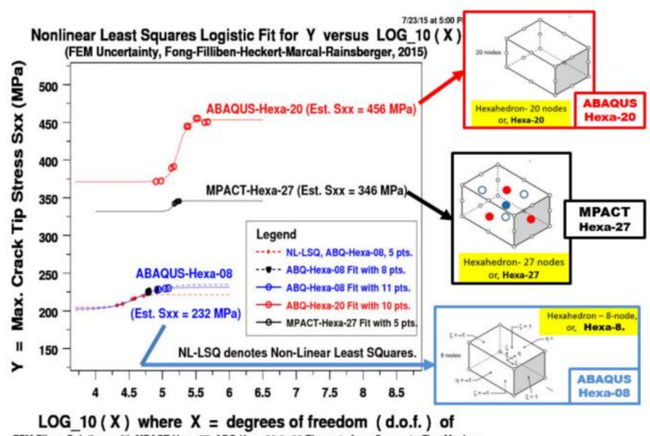

Fig. 10 A comparison of three FEM solutions of a pipe-elbow-with-crack problem using 3 different element types (Hexa-20, Hexa-27, and Hexa-08) and different FEM codes [8].

An examination of the table in Appendix A leads to a rationale to propose at least four new requirements that an engineer to assess the accuracy of an FEM-based stress estimate. The first requirement (Req.), R-1, is to demand that, for a fixed FEM mesh design, a minimum of five mesh densities, or d.o.f. (degrees of freedom), be used not only to check the convergence of a candidate solution, but also to estimate its uncertainty through a nonlinear least squares logistic fit algorithm. As shown by Fong, et al. [8], the reason for requiring a minimum of five mesh densities is that it takes at least five data points to execute a nonlinear least squares fit based on a 4-parameter logistic function.

The 2nd, 3rd, and 4th requirements are to demand that the solution be verified in three ways: Req. R-2 is to make a change in the element type. Req. R-3 is to make a change in the FEM code. Req. R-4 is to add one or more mesh densities to confirm the convergence of the 5-density solution given by R-1.

In the numerical example given in Appendix A, Req. R-2 yields a result that the max. crack tip stress can vary from 457.96 (5 runs, Hex-20), to 246.05 (5 runs, Hex-08), with the larger value differing from the smaller one by a factor of two.

Req. R-3 requires one to change the FEM code. In the numerical example given in Appendix A, we switched from ABAQUS [17] to MPACT [18]. The results were startling, as shown in the following Table 2:

Table 2
A Comparison of FEM Solutions Using Different Codes
Legion: (*) d.o.f. denotes degree of freedom.
(#) Ranking is based on the smaller the better.

	Predicted Max. Crack Tip Stress (MPa)	Predicted Standard Deviation at 10^9 d.o.f. (*)	Coeff. of Var. (C.V.) at 10^9 d.o.f.	Ranking Of Solu. By the C.V. metric (#)
ABAQUS Hex-20 5 runs	457.96	19.70	4.30 %	2
MPACT Hex-27 5 runs	345.47	0.12	0.03 %	1

Not only did the two stress estimates differ by a factor of 1.3, the two measures of uncertainty (the coefficient of variation, or, the C.V.) differed by a factor of over a hundred. In our experience with other problems [8, 10, 20], we found Req. R-3 to be most effective among all four new requirements.

Req. R-4 is also effective even though it is very costly, because it demands more solutions at a finer mesh or larger degrees of freedom than the previous five. Nevertheless, in the absence of conducting a physical experiment to validate a numerical solution such as the FEM, Req. R-4 assures us that the nonlinear least squares logistic fit algorithm did provide consistently an extrapolated solution that converges as the mesh density (number of elements per unit volume) or the degree of freedom approaches infinity.

Finally, a word of caution on ranking a large collection of candidate FEM solutions as a protocol for verification. As discussed by Rainsberger, et al. [10] in their recent paper on a super-parametric method of assessing the accuracy of a FEM-based solution, a key attribute to accuracy is the mesh quality. It was shown in that paper [10] that a change of mesh quality such as the mean aspect ratio of the elements, the stress estimates could be different by as much as a factor of five (see Appendix B for a table of comparison of 6 FEM solutions of a pipe-crack problem where the mean aspect ratio of two different sets of mesh design differs from each other by a factor of 16).

We conclude this section by observing that all FEM-based stress estimates are approximate by design, and, possess uncertainties by nature, that need to be quantified as an integral part of the computational effort. The extra cost of computing the uncertainties and verifying the stress estimates is justified on account of the serious consequences down the line when a decision needs to be made to repair or not to repair a piping system in service.

4. CONFIDENCE LIMITS ON RUPTURE TIME

The availability of a methodology to compute the 95 % confidence limits of a linear, first-order model of a creep rupture time vs. stress relationship, and the fact that all FEM-based stress estimates contain uncertainties not often explicitly reported, provided us a rationale to develop a new approach to estimating the 95 % confidence limits of a typical creep rupture time vs. stress regression line when the stress error is assumed to be small but not less than 2 %.

For convenience, we choose to describe the new approach via a numerical example, where the creep rupture time vs. stress data are given in Table 1 for an API 579 Grade 91 steel at 571.1 C (1060 F). A linear least squares fit of those ten data gives an estimate of the y-intercept, A and the exponent, C , as shown below in Table 3:

Table 3
Typical Output File of a Least Squares Fit

```
             Linear Least Squares Fit

Sample Size:                               10
Model: Y =(A + C*X)
No Replication Case:

------------------------------------------------------------------------
                                   Residual *
 Iteration    Convergence          Standard *      Parameter
    Number        Measure         Deviation *      Estimates
------------------------------------------------------------------------
         1  0.1000000E-01  0.1065179E+01 *    0.1000000E+01  0.1000000E+01
         2  0.5000000E-02  0.1147863E+00 *    0.1710857E+02 -0.7363802E+01
         3  0.2500000E-02  0.2010166E-01 *    0.2007786E+02 -0.8840575E+01
         4  0.1250000E-02  0.1944376E-01 *    0.2021643E+02 -0.8909494E+01

-------------------------------------------------------------------
                                                  Approximate
       Final Parameter Estimates            Standard Deviation   t-Value
-------------------------------------------------------------------
  1  A                             20.2181              0.1890  106.9852
  2  C                             -8.9103              0.0940  -94.8028

Residual Standard Deviation:               0.0194
Residual Degrees of Freedom:               8
```

In Fig. 6, the straight line in blue with a very narrow band of confidence limits in red is a log-log plot of the data in Table 1. The same data when plotted in natural scales appear in Fig. 11 as blue circles with regression line in blue and the confidence limits again in red. We choose to work with two cases to study the effect of a 2 % stress error: Case 1. Stress = 101.4 MPa. Case 2. Stress = 69.9 MPa. An enlarged view of the Case 1 data is given in Fig. 12. For Case 1, the min. time to failure (minTTF) is 220.9 hours, and the 95 % lower limit is at 198.2, a drop of 22.7 hours in creep life.

Let us consider a 2% error in stress estimate for Case 1. In Fig. 12, we note that a 2 % stress error causes a drop of 35.3 hours in minTTF, and a further drop in the 95 % confidence limit. In Fig. 13, we show a plot of the same data with a red band of confidence limits for the creep and a blue band for the combined effect of creep and 2 % stress error when stress = 101.4 MPa. In Fig. 14, we show a similar plot for Case 2 when stress = 69.6 MPa. In both cases, the total extra loss of life due to a 2 % stress error and the 95 % confidence limit is 40 %.

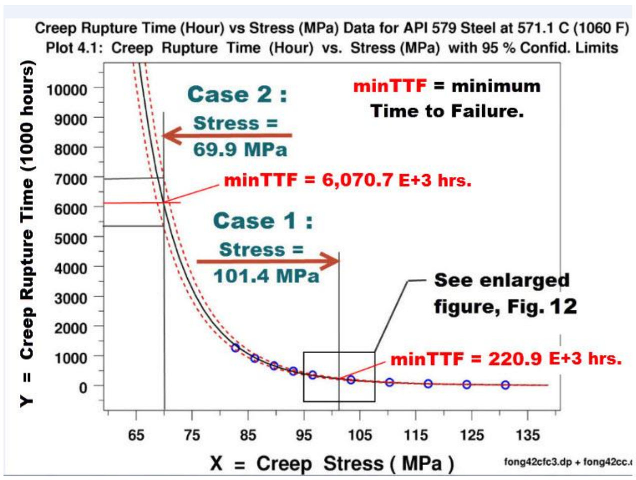

Fig. 11 Creep Rupture Time vs. Stress Data with a Power-law Fit based on a linear log-log model for API Grade 91 steel at 571.1 C (1060 F), and 95 % Confidence Limits for two stress Cases: (1) Stress = 101.4 MPa. (2) Stress = 69.9 MPa.

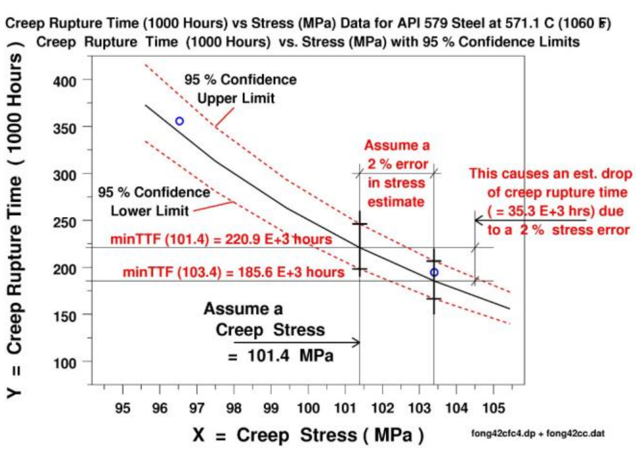

Fig. 12 An enlarged view of the creep rupture time vs. stress curve at 571.1 C for Case 1 (Stress = 101.4 MPa) investigation of the effect of a 2 % error in stress estimate on rupture time.

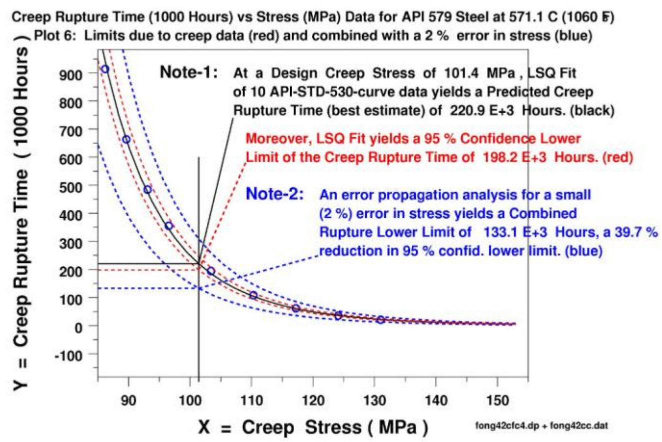

Fig. 13 A 95 % Confidence Lower Limit Approach to a Case 1 (Stress = 101.4 MPa) investigation of the effect of a 2 % error in stress estimate on Creep Rupture Time for API Grade 91 steel at 571.1 C. The combined uncertainty due to creep and 2% stress error causes a 39.7 % drop in Creep Rupture Time.

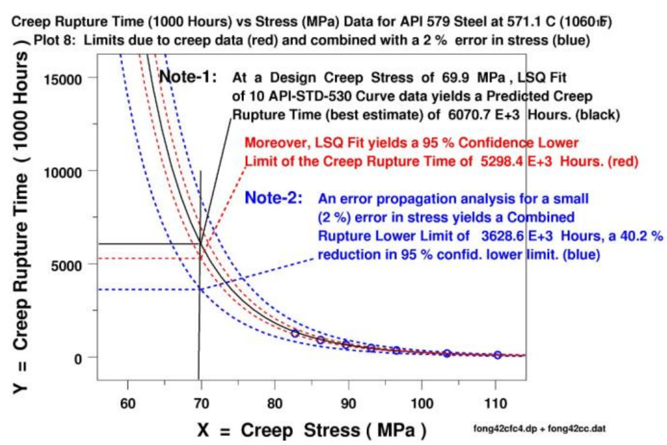

Fig. 14 A 95 % Confidence Lower Limit Approach to a Case 2 (Stress = 69.9 MPa) investigation of the effect of a 2 % error in stress estimate on Creep Rupture Time for API Grade 91 steel at 571.1 C. The combined uncertainty due to creep and 2% stress error causes a 40.2 % drop in Creep Rupture Time.

5. SIGNIFICANCE AND LIMITATIONS OF THE NEW APPROACH TO CREEP DESIGN

The statistical 95 % confidence limits approach to analyzing creep rupture time vs. stress data with an assumption of a 2 % error in FEM-based stress estimate is significant in at least two ways:

(1) The new approach allows a product designer or a maintenance engineer to better understand the true value of those laboratory-generated test data that always have a quantifiable uncertainty. That uncertainty, if and when reported and evaluated, allows a decision-maker to compute an evidence-based estimate of creep rupture time, also with uncertainty, of a full-size component or structure.

(2) The new approach provides a decision maker a rationale and four requirements to communicate with an FEM analyst, whose stress estimates are used in a creep rupture time model based on those laboratory-generated test data. In a nutshell, the four requirements outlined in Sect. 3 allow one to ask the analyst to provide stress estimates not as a deterministic quantity (single-valued), but a statistical one (with uncertainty).

The approach outlined in this paper is not without limitations. First and foremost, the assumption of a linear, first-order model for a set of creep rupture time vs. stress data, or a straight-line fit of a log-log plot of the data, is plausible within the range of the test variables, but not necessarily valid outside the range of stress being tested.

Secondly, the add-on complication of a small stress error is computationally rigorous but physically over-conservative, because it invokes the 95 % confidence limit tool twice.

Nevertheless, on balance, the new approach gives a decision maker a path to a more rational use of creep test data.

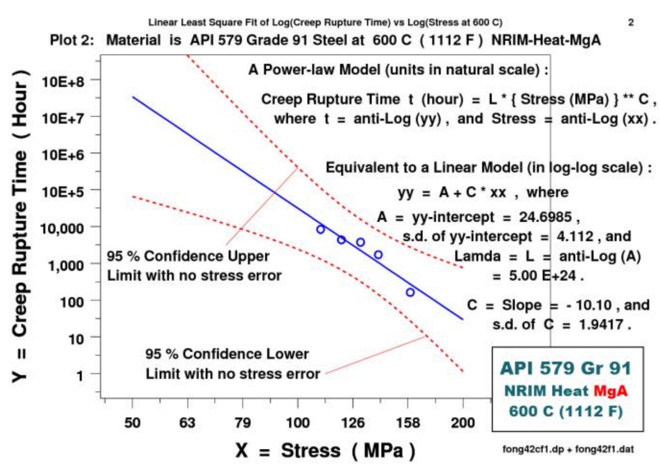

Fig. 15 Linear Least Squares Fit with 95 % Confidence Limits for a set of heat-specific Creep Rupture Time vs. Stress Data at 600 C. The material is heat MgA of the API Grade 91 steel, and the data were reported by NRIM [15]. Note that the exponent of the fit is 10.1, which is considerably larger than that of the same steel at 571.1 C given by API-STD-530 curve.

It is interesting to inquire that, had the data of Table 1 (API 579 Grade 91 steel at 571.1 C) not originate from a handbook [2, 3], but from a laboratory such as those reported by NRIM [15], will a 2 % stress error lead to a similar drop, such as 40 %, in creep rupture time? We can answer this by reviewing Fig. 6, where both the 571.1 C (Table 1) and the 600 C (NRIM) data have comparable exponents (-8.9 vs. -9.0), but the scatter band of the laboratory 600 C data is much wider.

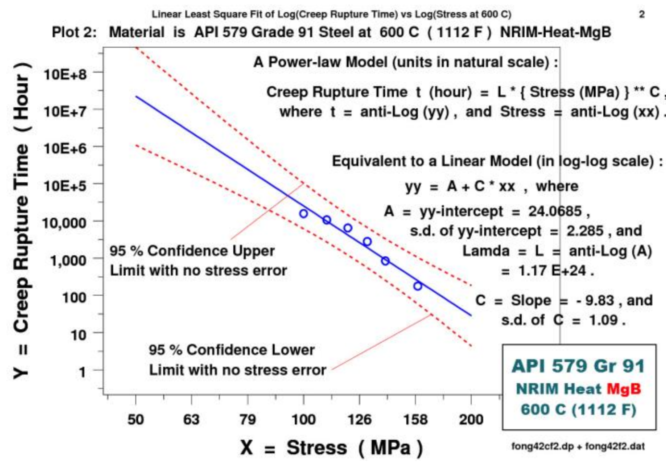

Fig. 16 Linear Least Squares Fit with 95 % Confidence Limits for a set of heat-specific Creep Rupture Time vs. Stress Data at 600 C. The material is heat MgB of the API Grade 91 steel, and the data were reported by NRIM [15]. Note that the exponent of the fit is 9.8, which is considerably larger than that of the same steel at 571.1 C given by API-STD-530 curve.

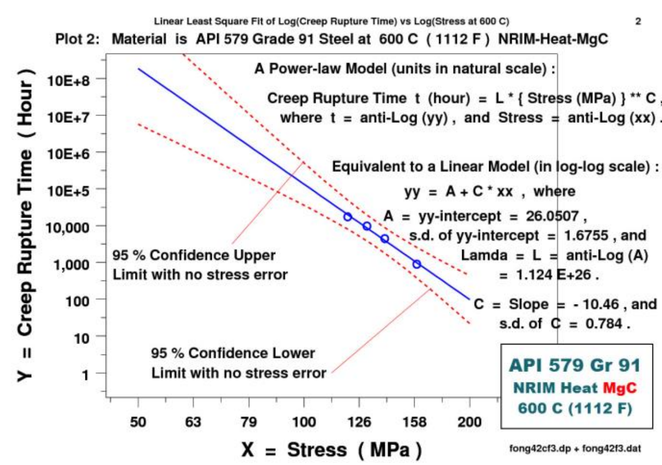

Fig. 17 Linear Least Squares Fit with 95 % Confidence Limits for a set of heat-specific Creep Rupture Time vs. Stress Data at 600 C. The material is heat MgC of the API Grade 91 steel, and the data were reported by NRIM [15]. Note that the exponent of the fit is 10.5, which is considerably larger than that of the same steel at 571.1 C given by API-STD-530 curve.

Recognizing the fact that the 600 C NRIM laboratory data came from three heats, MgA, MgB, and MgC, we present in Figs. 15-17 plots of a regression line with scatter bands for each of those three heats. We observe that the absolute values of the exponents for the three heats vary from a low of 9.8 (heat MgB) to a high of 10.5 (heat MgC), the scatter bands of all three heats are much wider than that of the 571.1 C data as shown in Fig. 6. Since the broader the scatter band, the higher is the uncertainty, so we can logically deduce that a 2 % stress error for a laboratory-generated test data on creep rupture time vs. stress will lead to a more than 40 % total drop in the 95 % confidence limit of rupture time.

6. DISCUSSION

It is important to note that the handbook-generated data of Table 1 (after Cohn, et al. [2, 3]), used in this paper as a vehicle to present a new approach to analyzing creep rupture time vs. stress (CRT-S) data, was strictly intended as an example to highlight a key feature of the CRT-S data, namely, its high value of exponent (close to -9) indicating a high sensitivity to stress.

As a matter of fact, the data of Table 1 for the API Grade 91 steel at 571.1 C were from a minimum curve [2, 3], and not an average one. In either case, however, the uncertainty information was not available, leading to an unrealistic result of a very narrow scatter band as shown in Fig. 6.

It is worth noting that our choice of making the data of Table 1 as an example to illustrate the methodology of our new analysis approach was appropriate, because it clearly showed the pitfall of quoting handbook data without uncertainty characterization to estimate creep rupture time with uncertainty for decision making.

A key result of this paper is to confirm the observation stated by Cohn, et al. [2] that the value of the exponent associated with their data (Table 1 for 571.1 C) is as high as those found for each of the three heats of the same steel at 600 C (Figs. 15 – 17) based on the NRIM data [15].

This result also confirms the observation made by Cohn, et al [2] that the relative ranking of girth weld applied stresses is extremely important in determining the few lead-the-fleet group of girth welds subject to creep rupture failure. As a result of this observation, Cohn, et al. [2] noted that from a practical point of view, examining the top 3 to 6 high stress ranked locations (approximately the girth welds subject to the top 15 % multiaxial stress), is a better strategy than selecting locations by alternative methods such as (1) picking shop welds versus field welds, (2) picking high traffic areas, (3) picking all fitting weldments, or (4) examination of one-third of the total population of girth welds every 5 years during a scheduled major outage (and then in the 15th year starting from the first set).

It is also useful to note that the example data set of Table 1 (API Grade 91 steel at 571.1 C) are for a base metal. Since girth welds typically fail in the heat-affected-zone (HAZ) of a weld metal, which is usually weaker than the base metal, the use of a CRT-S curve for a base metal to estimate the rupture time of a weld metal region is not appropriate. The resulting estimate of rupture time for a weld region using the properties of a base metal is not conservative, i.e., the estimate will be too optimistic.

Regarding the example problem of the elastic deformation of a pipe-elbow-crack configuration fixed at the left base and loaded at the right end of the pipe (Fig. 8), it is important to note that while the example serves the purpose of illustrating the existence of uncertainty in an FEM stress analysis exercise, it by no means tells the whole story of the nature of piping stress uncertainties in a typical power-generating plant.

For example, as described by Cohn, et al. [2] in Fig. 10 of their paper, there exists uncertainty in the estimate of the redistribution time when the elastic piping stresses redistribute to inelastic stresses through wall and axial to the pipe.

Secondly, there exists uncertainty in time-dependent variations in the external loads, such as malfunctioning spring supports. Such uncertainty can be accounted for by evaluating the current piping configuration (hot and cold piping system walkdowns) with topped-out and bottomed-out supports. Experience told us that after 100,000 operating hours there are usually unexpected piping displacements because of malfunctioning supports. The malfunctioning supports may increase the loads and stresses by more than 20 %.

7. CONCLUDING REMARKS

A statistical methodology to analyze creep rupture time vs. stress (CRT-S) test data with an assumption of a 2 % error (equivalent to a 1 % standard deviation and a 95 % lower confidence limit) in finite element method-based stress estimates has been presented via a numerical example based on handbook-generated values of the CRT-S data.

In two case studies involving different creep stress values, namely, Case 1 (stress = 101.4 MPa), and Case 2 (stress = 69.9 MPa), it was found that the combined effect of creep and a 2 % stress error causes a 40 % drop in the 95 % lower limit of the rupture life.

If the CRT-S data are not from a handbook, but are laboratory-generated, we found that the scatter band is likely to be wider and the drop in the 95 % lower limit of the rupture life to be higher than 40 %.

The huge impact of a small error in stress estimate (such as 2 %) on the 95 % lower limit of the rupture life (such as 40 %) produces four new engineering requirements that the estimate of stress by a computer-assisted method such as FEM be properly verified and validated with an error estimate of no more than 2 %.

8. ACKNOWLEDGMENT

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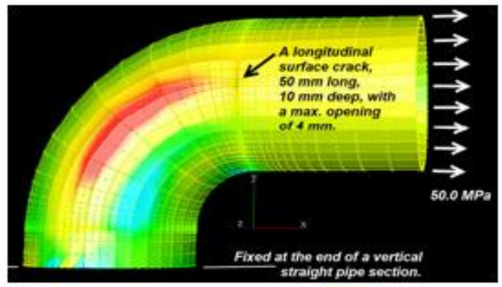
APPENDIX A

A COMPARISON OF MAX. STRESS ESTIMATES FROM 7 FEM SOLUTIONS OF A PIPE-ELBOW-WITH-CRACK PROBLEM WITH UNCERTAINTY ESTIMATION BASED ON A NONLINEAR LEAST SQUARES LOGISTIC FIT [8, 20]

A 900-mm (36-in) o.d. Pipe 90-deg. Elbow with a surface crack in one of its two welds

Est. Max. Crack Tip Stress ***Sxx (MPa)*** *at* ***1 billion*** *(10^9)* ***degrees of freedom*** *using a Nonlinear Least Squares Logistic Fit of 5 or more FEM solutions of the same mesh design at increasing mesh densities*



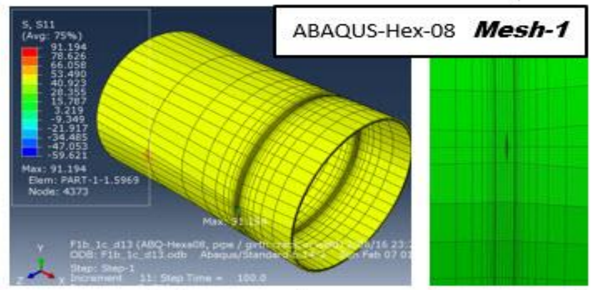
FEM Code-Element Type *No. of Runs* (Best Estimated Solution)	95 % Lower Limit at 10^9 d.o.f. (MPa)	Predicted Max. Crack Tip Stress at 10^9 d.o.f. (MPa)	95 % Upper Limit at 10^9 d.o.f. (MPa)	Stand. Dev. (S.D.) at 10^9 d.o.f. (MPa)	Coeff. of Variation (C.V.) at 10^9 d.o.f. (%)	Ranking of Solutions by C.V. (least being the best)
ABQ-Hex20 ***7 runs*** (455.20)	407.32	**457.96**	508.60	**19.70**	**4.30 %**	**6**
ABQ-Hex20 ***9 runs*** (455.50)	413.80	**454.23**	494.67	**17.10**	**3.76 %**	**5**
ABQ-Hex20 ***10 runs*** (455.50)	418.74	**453.17**	487.61	**14.93**	**3.29 %**	**4**
MPACT-Hex27 ***5 runs*** (345.48)	345.10	**345.47**	345.85	**0.12**	**0.03 %** ***(lowest)***	**1**
ABQ-Hex08 ***5 runs*** (220.00)	203.02	**246.05**	289.09	**13.52**	**5.49 %**	**7**
ABQ-Hex08 ***9 runs*** (228.30)	215.78	**233.37**	250.96	**7.44**	**3.19 %**	**3**
ABQ-Hex08 ***11 runs*** (230.10)	220.56	**231.69**	242.82	**4.92**	**2.12 %**	**2**

APPENDIX B

A COMPARISON OF MAXIMUM STRESS ESTIMATES FROM 6 FEM SOLUTIONS OF A PIPE-CRACK PROBLEM WITH UNCERTAINTY QUANTIFICATION BASED ON A NONLINEAR LEAST SQUARES LOGISTIC FIT [8, 19, 20]

A 900-mm (36-in) o.d., 20-mm thick Steel Pipe with a 50-mm long surface crack in its weld

Est. Max. Crack Tip Stress ***Sxx (MPa)*** *at* ***1 billion*** *(10^9)* ***degrees of freedom*** *using a Nonlinear Least Squares Logistic Fit of 5 or more FEM solutions of the same mesh design at increasing mesh densities*



FEM Code-Element Type ***Mesh Design*** (Best Est. Max. Stress in MPa; No. of runs)	Mean Aspect Ratio (MAR)	95 % Lower Limit at 10^9 d.o.f. (MPa)	Predicted Max. Crack Tip Stress at 10^9 d.o.f. (MPa)	95 % Upper Limit at 10^9 d.o.f. (MPa)		Stand. Dev. (S.D.) at 10^9 d.o.f. (MPa)	Coeff. of Variation (C.V.) at 10^9 d.o.f. (%)	Ranking of Solutions by C.V. (least being the best)
ABQ-Tet-04 ***Mesh-1*** (139.5; 7 runs)	32	321.0	**700.9**	1080.9		**148.0**	**21.1 %**	**5**
ABQ-Hex-08 ***Mesh-1*** (99.7; 7 runs)	32	85.7	**99.2**	112.6		**5.2**	**5.2 %**	**2**
ABQ-Hex-20R ***Mesh-1*** (174.3; 6 runs)	32	95.2	**174.1**	252.9		**28.4**	**16.3 %**	**4**
ABQ-Tet-04 ***Mesh-2*** (135.8; 6 runs)	2	28.6	**130.6**	232.7		**36.8**	**28.2 %**	**6**
ABQ-Hex-08 ***Mesh-2*** (148.5; 5 runs)	2	145.4	**148.5**	151.6		**1.0**	**0.7 %** ***(lowest)***	**1**
ABQ-Hex-20R ***Mesh-2*** (294.8; 10 runs)	2	219.7	**307.8**	396.0		**38.2**	**12.4 %**	**3**

DISCLAIMER

Certain commercial equipment, instruments, materials, or computer software is identified in this paper in order to specify the experimental or computational procedure adequately. Such identification is not intended to imply recommendation or endorsement by the U.S. National Institute of Standards and Technology, nor is it intended to imply that the materials, equipment, or software identified are necessarily the best available for the purpose.