

# Opportunities for Inverse Analysis in Dynamic Tensile Testing

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## ABSTRACT

Dynamic tensile testing using Kolsky Bar methods are used to assess crashworthiness of new structural materials needed for lightweight automotive design. High speed Digital Image Correlation (DIC) measurements routinely show that the strain experienced by dynamic tensile specimens deviates markedly from what is expected from the original strain wave analysis techniques used in tensile Kolsky bar metrology. Deviations can be manifest either by different average strain values over the gage section, or by departures from strain uniformity, or both. The former can be attributed to plastic yielding in the specimen outside the gage section, while the latter concerns specimen geometry and material hardening effects. These issues are sometimes difficult to eliminate through simple modifications of the sample or the test design. Finally, it is of interest to make use of the data beyond necking, where the strain state departs significantly from ideal conditions. These metrology issues lend themselves to solution by inverse methods, where full field strain measurements and global load measurement data are available. In this paper we describe typical measurement data and explore methods to identify the constitutive response from dynamic tensile tests.

**KEY WORDS:** High Strain Rate, Advanced High Strength Steels, Digital Image Correlation, Finite Element Analysis, Virtual Fields Method

## INTRODUCTION

High strain rate tensile testing of Advanced High Strength Steels (AHSS) are of interest to automotive manufacturers who need to assess the crashworthiness of these new materials that are being developed to make lighter, more fuel-efficient vehicles. Crashworthiness, in a relative sense, is estimated from dynamic strength and ductility obtained from stress-strain curves measured to failure. Crash performance is analyzed in a more absolute sense by large finite element simulations of full-scale vehicle impact problems, which relies on full stress-strain curves over a range of strain rates, to define the material response up to, but not including, the fracture point. Modeling fracture requires many additional mechanical tests that are not discussed here [1].

In the past several years it has become apparent that traditionally-designed dynamic tensile tests, performed using Kolsky Bar methods can produce flawed results. High speed Digital Image Correlation (DIC) measurements show that the strain experienced in specimens can deviate markedly from what is indicated using traditional Kolsky bar data analysis methods [2]. These deviations can be manifest in a variety of ways. Typically the strain measured by DIC methods in the gage section is substantially less than indicated by Kolsky bar data analysis, owing to some amount of plastic strain occurring outside the nominal gage section. In addition, strain non-uniformities arise when small gage length specimens are used to achieve high overall strain rates, rapid force equilibration and to promote fracture. The ASTM E8 standard for static tensile testing calls for a minimum ratio of gage length to gage width of 8, whereas ISO 26203-1:2010, a standard for dynamic testing, allows for ratios as low as 2. For some materials, particularly high strength ones that have limited strain hardening capacity such as most AHSSs, highly non-uniform strain distributions can result [3]. One way to combat this is to customize the specimen geometry for each individual material. However, this is a daunting prospect and makes it difficult to compare different materials on an equivalent strain rate basis. An alternative is to use so-called inverse methods, which may be employed to provide more accurate estimates of material behavior from non-ideal experiments. In addition, opportunities exist to use inverse methods to extract material information from portions of a Kolsky bar test that traditionally are ignored: the ringup portion of the test, where the forces and strain rates are varying, and the post-necking behavior where tri-axial stresses develop that cannot be analyzed using load-deflection measurements unless approximate Bridgeman-type correction methods are applied [4].

The literature is rich and growing fast on inverse method techniques and applications. Review papers on inverse methods have described different methods considered and benefits and drawbacks of the various approaches [5]. One significant

distinction we note is whether or not finite element analysis (FEA) is performed. The Virtual Fields Method (VFM) is a powerful technique that can identify material parameters directly from DIC strain field data without performing expensive FEA. Instead, parameters can be obtained simply by solving a small linear system of equations for elasticity problems, or by minimizing a cost function for plasticity problems [6]. More recently, the dynamic VFM (dVFM) has been developed that uses acceleration field information derived from DIC displacement field measurements to estimate internal forces [7]. Avoiding expensive FEA computations greatly improves speed and reduces the cost of the analysis, and many practical applications have shown remarkable accuracy when the method is expertly applied. The VFM method is limited, however, to two dimensional problems because the DIC measurements upon which it relies can measure only surface strain fields [6]. FEA-based methods avoid this problem, in that results can be obtained without three dimensional data. Another benefit of using FEA-based inverse methods is that one can use the same numerical approximation method to identify material behavior that is used to solve problems of practical significance. This ability minimizes errors that can arise when numerical approximation methods are mismatched.

In this paper we explore opportunities to apply either VFM or FEA-based inverse methods to better leverage experimental measurements from Kolsky bar strain gage signals and high speed DIC displacement field data to experimental results that may be flawed due to specimen geometry or to portions of the experiments that have not traditionally been used to enhance our understanding of material behavior.

## RESULTS

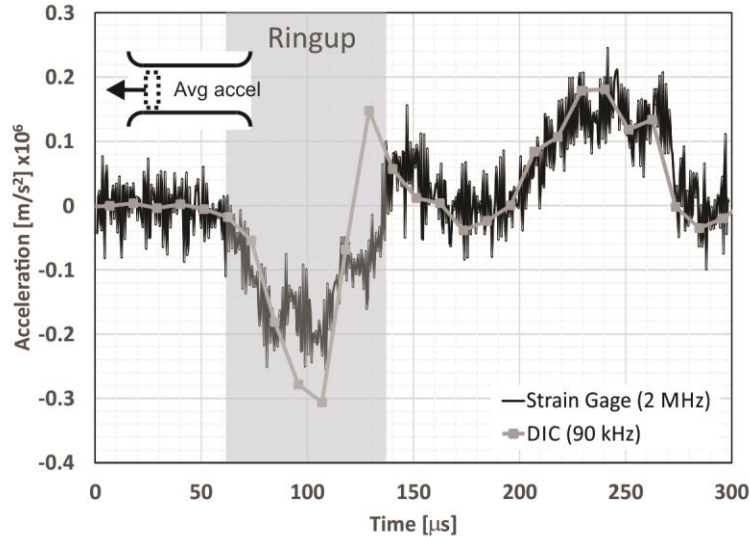
Three aspects of dynamic tensile testing that may lend themselves to inverse methods are considered here. First, an initial “ringup” period exists in the Kolsky bar test where force equilibrium has yet to develop and the data are therefore considered unreliable and ignored. Second, depending on the test geometry and material hardening characteristics, the strain in the gage section can be non-uniform after force equilibrium is established due to end effects. Third, after the onset of localized necking through to failure when uni-axial analysis becomes inappropriate. We note that other localization phenomena exist that may provide opportunities for inverse analysis, such as strain localization due to Portevin-Le Chatelier banding, are not considered here. The three portions of the dynamic tensile test are now considered in order.

### Ringup

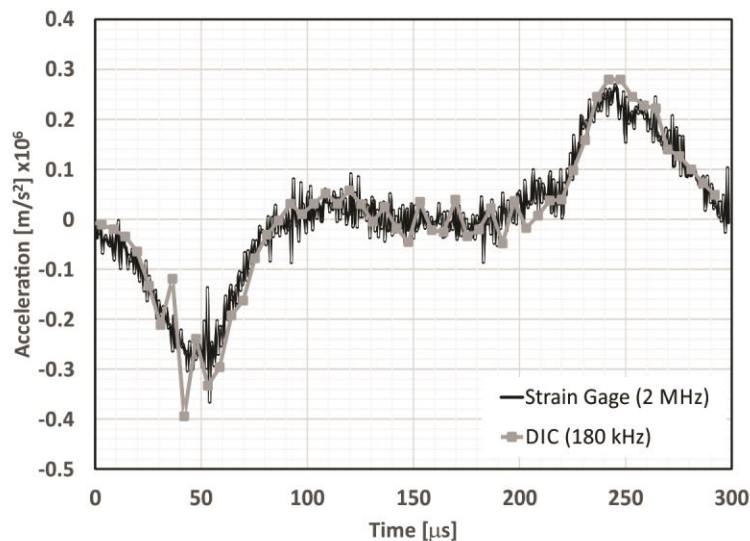
The ordinary way to treat data during the ringup portion of a Kolsky bar test is to ignore it, on the reasonable grounds that the stress and strain state is highly non-uniform and the strain rate is varying rapidly. These are, however, the exact conditions that are exploited by the dVFM [7], which depends on the presence of significant accelerations within the test piece that can be measured with DIC to provide dynamic force information. Typical high speed digital cameras used for dynamic material measurements, such as the ones used in this study, operate at about 100 000 frames per second (100 kHz) and can measure displacement and strain fields with adequate spatial resolution for a tensile Kolsky Bar test. We also note that force and displacement information provided by Kolsky bar strain gage data must be acquired at 200 kHz or more to be fully resolved [8]. Thus typical high speed cameras may not acquire DIC information fast enough to capture the real acceleration fields to make proper use of the dVFM method. Although there are more advanced cameras that can meet or exceed strain gage measurement rates, they can be quite expensive and may also be limited in terms of the total number of frames that can be captured.

In Figure 1 we compare the acceleration obtained from a three dimensional (3D) surface DIC measurement using typical high speed camera equipment and resolution (90 000 fps, 128x288 pixels, 1  $\mu$ s exposure, 15 pixel DIC subset with 3 pixel offset) with the acceleration predicted from an analysis of the reflected strain pulse from the bar end, which is recorded at 2 MHz using a high speed oscilloscope. The sample in this test is a QP-980 high strength steel, and the gage section measures 2.9 mm wide by 7 mm long by 1.0 mm thick. The DIC acceleration is averaged over a slice of the sample taken perpendicular to the load axis, located nearest the incident bar where the acceleration is highest during ringup. Further, the accelerations obtained from both DIC field information and strain gage data are smoothed by central differencing. As this figure shows, the DIC acceleration field data differ from the strain gage result nearest the acceleration period during ringup. Some of this difference may be real and caused by slippage between the sample and the grip, which is typical but undesirable in Kolsky bar testing. To compare accelerations between DIC and strain gage data at higher recording rates without grip effects, Figure 2 plots DIC measurements obtained directly on the end of the incident bar, this time at 180 kHz, against the strain gage signal. With grip slippage eliminated there is better agreement between DIC and strain gage results, but we still see larger noise in the DIC data near the peak accelerations compared to the strain gage result. Clearly, obtaining acceleration information from DIC displacement field measurements is quite prone to noise because the data must be differentiated twice, amplifying any displacement noise. Noise levels are quite significant in the 2 MHz strain gage signals that have only been differentiated a single time. Thus even with higher speed cameras for DIC measurements, noise in the derived acceleration data during the ringup portion of a Kolsky bar experiment presents a challenge to obtaining accurate dVFM results. As

discussed by its inventors [7], dVFM may be better suited to experiments that are intentionally designed to provide large, smoothly changing accelerations. Turning to FEA-based inverse techniques, the question arises whether data obtained during ringup is useful for identification purposes since in principle the optimized FEA solution would be able to capture this portion of the experiment. Displacement data could be used to guide the identification process so double differentiation is avoided. However, because more constitutive parameters are sensitized during this portion of the test (strain rate sensitivity, for example), the identification process becomes more difficult.



**Fig. 1.** Comparison of the acceleration of the incident side of a steel tension specimen obtained from DIC measurements at 90,000 frames/s with the acceleration of the incident bar obtained from the reflected strain pulse recorded at 2 MHz. Accelerations are determined by central differencing.

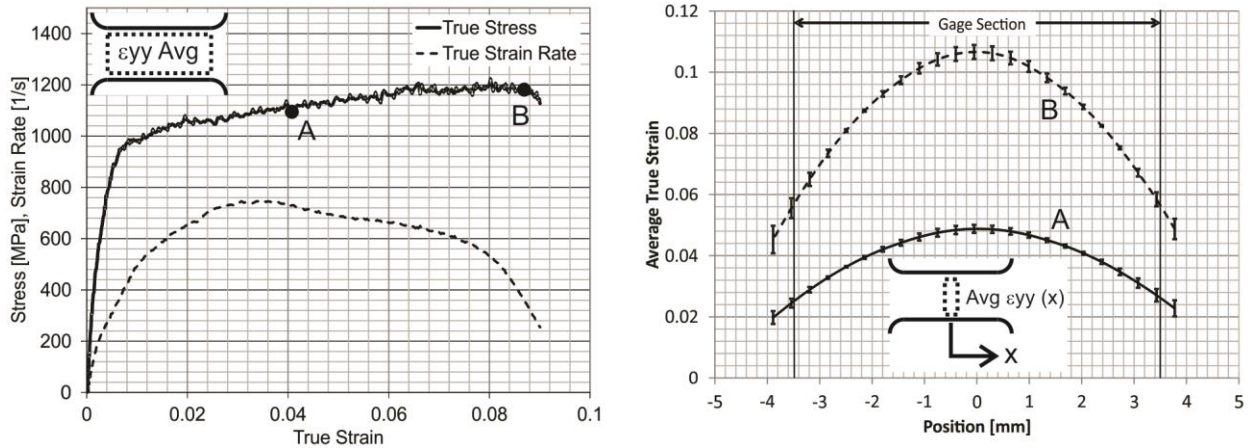


**Fig. 2.** Comparison of the acceleration of the free end of the incident bar with no sample obtained from DIC measurements at 180,000 frames/s with strain gage results obtained at 2 MHz using central differencing.

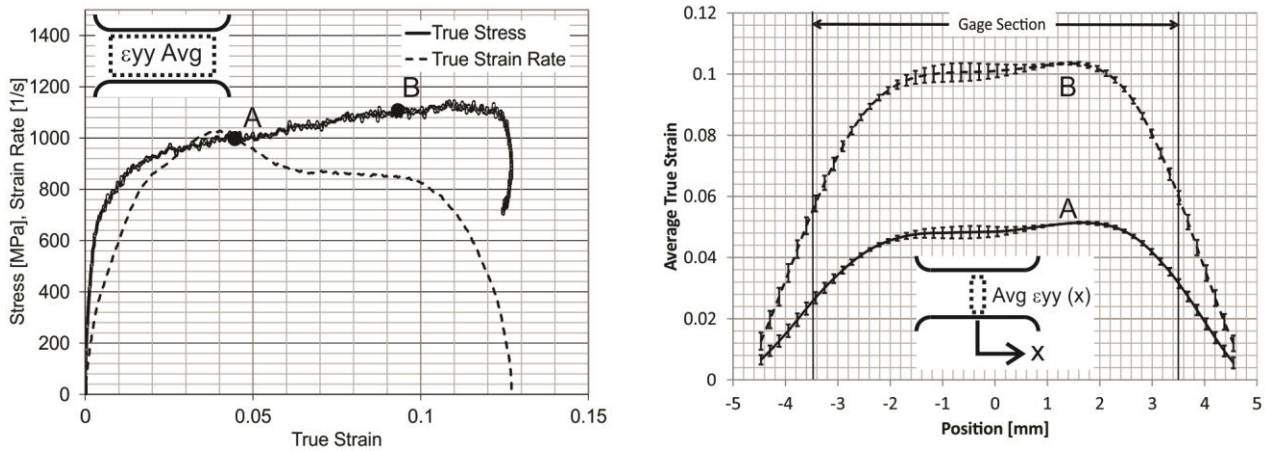
### Equilibrium Deformation

In Fig. 3 shows dynamic tensile test results for a second-generation AHSS using a gage length of 7 mm and a length-to-width ratio of 1.4. An estimate of the dynamic true-stress, true strain response of this material shows limited strain hardening in the material. High speed 3D DIC measurements using the same settings as the previous data (90 000 fps, 128x288 pixels, 1 μs exposure, 15 pixel DIC subset with 3 pixel offset) show highly peaked true strain profiles across the cross section of the specimen for global average true strains exceeding a few percent. For a material with very little strain hardening such as the QP-980 material studied here, achieving uniform strain along the gage section is very difficult. In the limit of a zero strain

hardening material, one can demonstrate with finite element analysis that localized necking begins almost immediately because the material is incapable of diffusing a neck by hardening. In Fig. 4 the same steel is tested with the same gage length (7 mm) but a reduced thickness such that the gage length-to-width ratio is now 2.4. Clearly the strain profile is more uniform, but the strain falls off significantly at the edges even within the gage section. We note that the strain rate is higher in the second test because of reduced cross sectional area of the specimen lowers the transmitted load which, in turn, increases the overall strain and strain rate in the test. Thus even seemingly minor changes to the specimen geometry can impact the test conditions significantly.



**Fig. 3.** Left: Dynamic average stress-strain behavior of an AHSS with a gage length-to-width ratio of 1.4. Right: True strain distribution along the load axis at two average true strain values (labeled A and B), from DIC data averaged across planes perpendicular to the load axis.



**Fig. 4.** Left: Dynamic average stress-strain behavior of an AHSS with a gage length-to-width ratio of 2.4. Right: True strain distribution along the load axis at two average true strain values (labeled A and B), from DIC data averaged across planes perpendicular to the load axis.

Equilibrium deformation can be analyzed by the ordinary (non-dynamic) VFM because the strain fields are quite non-uniform, the equilibrated forces acting to cause the strain field are available from strain gage data, and the specimen is assumed to be in a state of plane stress up to the point of localized necking. Again, depending on the assumptions made and the complexity of the plasticity model, plastic parameters may need to be identified by minimizing a cost function rather than solving a set of algebraic equations. Additionally, the dynamic nature of the test introduces an additional complication that is unique to dynamic plasticity experiments: adiabatic heating of the specimen due to rapidly accumulating plastic strain. In this case an additional step is needed to evaluate the temperature field and its effect on the virtual work estimate which affects the resulting identified material parameter values. Accounting for adiabatic heating has been recognized as a significant challenge by the developers of the VFM [6]. An iterative updating scheme, combined with an assumption, based on measurement data, regarding the fraction of plastic work converted to heat, might be used to account for adiabatic heating

effects in this case. FEA-based inverse methods can also be applied to this portion of the test, albeit at considerably greater complication and computational expense, and with no obvious advantage over the VFM other than providing consistent numerical approximations between the identification problem and the application problem.

### **Necking**

Once localized necking begins, ordinarily no more information can be obtained from a dynamic tensile test. Acceleration levels are much lower during necking than at the onset of the test except when fracture occurs. In addition, the stress state within the developing neck transitions from plane strain to tri-axial, which by itself precludes the use of the VFM because one can no longer accurately estimate virtual work quantities where stresses vary through the thickness of the sample. Further, because the gradients in strain become large in the neck and, in general, high speed cameras have limited spatial resolution, the strain or displacement field resolution is too limited to accurately capture these gradients. This problem also affects the quality of FEA-based identification techniques but it does not preclude their use to estimate material parameters in the neck region up until the fracture point.

### **CONCLUSIONS**

Measuring the dynamic tensile stress-strain curves of advanced high strength automotive sheet steels presents opportunities for inverse analysis to better leverage experimental data obtained when imperfect test conditions arise due specimen geometry effects that are exacerbated by the relatively low hardening rates in these steels. Three portions of the dynamic test were examined regarding the potential of various inverse techniques to improve the value of the test. The dVFM may be very useful to analyze material behavior during ringup, but DIC data must be obtained at very high sampling rates for accurate results in this kind of experiment. After ringup and before necking, ordinary (non-dynamic) VFM can be applied and should perform well if adiabatic heating effects can be adequately accounted for. Once localized necking begins, finite element based inverse methods must be used because the tri-axial stress state that develops in the neck precludes use the VFM. More costly FEA-based identification methods can be applied to all three portions of the test, although the accuracy of the identified parameters can be limited by the quality of the approximations needed in the absence actual of three dimensional data. FEA-based methods can also be selected to minimize approximation errors between the identification problem and the application problems. It is also noteworthy that the present paper deals only with dynamic tests designed using the traditional approach that seeks a uni-axial strain state and rapid development of force equilibrium. As pointed out in [7], there may be significant benefits to designing material tests very differently in order to maximize the utility of the dVFM method in making use of internal accelerations within the test sample that are intentionally avoided by the traditional materials testing approach.

### **ACKNOWLEDGMENT / DISCLAIMER**

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### **References**

- [1] Bao, Y., and Wierzbicki, T. (2004). On Fracture Locus in the Equivalent Strain and Stress Triaxiality Space. *International Journal of Mechanical Sciences*, 46, 81-98.
- [2] Gilat, A., Schmidt, T., and Walker, A. (2009). Full Field Strain Measurement in Compression and Tensile Split Hopkinson Bar Experiments. *Experimental Mechanics*, 49, 291-302.
- [3] Mates, S., and Abu-Farha, F. (2015). Dynamic Tensile Behavior of a Quenched and Partitioned High Strength Steel using a Kolsky Bar. *Proceedings of the Society for Experimental Mechanics 2015 Annual Meeting*. Costa Mesa, CA: Society for Experimental Mechanics.
- [4] Bridgman, P. (1952). *Studies in Large Plastic Flow and Fracture*. New York: McGraw-Hill.
- [5] Avril, S., et al., (2008). Overview of Identification Methods of Mechanical Parameters Based on Full-field Measurements. *Experimental Mechanics*, 48, 381-402.
- [6] Pierron, F., and Grédiac, M. (2012). *The Virtual Fields Method*. New York: Springer.

- [7] Pierron, F., Zhu, H., and Siviour, C. (2014). Beyond Hopkinson's Bar. *Philosophical Transactions of the Royal Society A*, 372 (2023), 24.
- [8] Chen, W., and Song, B. (2010). *Split Hopkinson (Kolsky) Bar*. New York: Springer.