Laser Refractometer as a Transfer Standard of the Pascal

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***Abstract*—We have developed a new low pressure sensor which is based on the measurement of (nitrogen) gas refractivity inside a Fabry–Perot (FP) cavity. We compare pressure determinations via this laser refractometer to that of well-established ultrasonic manometers throughout the range 100 Pa to 100 000 Pa. The refractometer demonstrates 10^{-6} precision for $p > 50$ kPa;—as good or better than the manometer—we argue that a laser refractometer represents a state-of-the-art transfer standard of the pascal. We also claim the refractometer has an accuracy of $U(p_{\mathrm{FP}}) = [(16\,\mathrm{mPa})^2 + (11.9\times10^{-6}\cdot p)^2]^{1/2}$, as realized through the properties of nitrogen gas.**

***Index Terms*—Fabry–Perot, length measurement, pressure measurement, refractive index, uncertainty.**

I. Introduction

During the past several years we have been developing a low pressure sensor that utilizes a laser refractometer and the ideal gas relation $p \propto (n-1)k_{\mathrm{B}}T$, where the pressure of a gas can be determined by a measure of the gas refractivity $n-1$ and thermal energy $k_{\mathrm{B}}T$ [1]. This approach is a departure from the traditional U-tube manometer where pressure $p = hg\rho$ comes from a measure of the liquid-column height h, with gravity g and the fluid density ρ being well-known [2]. Our chief motivation for this effort is ecological (that is, to move away from the toxin mercury), but we also endeavor to overcome the technical drawbacks of manometers, among which are slowness, size, sensitivity to vibration, and limited range. The metrology behind our technique is interferometry (and laser wavelength), which is used to measure the change in optical length of a Fabry–Perot (FP) cavity going from vacuum to pressure at a level of 3×10^{-10}. Our apparatus is small (about 30 cm^3), fast and precise (1 mPa for 1 s averaging), and can hold this precision across more than five decades of pressure.

II. Method and Results

Our refractometer shown in Fig. 1 consists of two separate FP cavities built out of one piece of low thermal expansion glass; a reference cavity is permanently held at vacuum and a measurement cavity is filled with gas; the cavities have a moderate finesse of 960. The pressure of the gas in the FP cavity is measured as

$$p_{\mathrm{FP}} = \frac{1}{c_1 - d_{\mathrm{m}} - d_{\mathrm{r}}}\left(\frac{\Delta f}{\nu}\right) - \frac{c_2 - c_1 d_{\mathrm{m}}}{(c_1 - d_{\mathrm{m}} - d_{\mathrm{r}})^3}\left(\frac{\Delta f}{\nu}\right)^2 + \frac{2(c_2 - c_1 d_{\mathrm{m}})^2 - c_3(c_1 - d_{\mathrm{m}} - d_{\mathrm{r}})}{(c_1 - d_{\mathrm{m}} - d_{\mathrm{r}})^5}\left(\frac{\Delta f}{\nu}\right)^3, \quad (1)$$

where $\frac{\Delta f}{\nu}$ is the effective fractional change in cavity resonance, d_{m} and d_{r} are (compressibility) distortion terms for the measurement and reference cavities respectively. The proportionality constants

$$\begin{aligned} c_1 &= \frac{3}{2k_{\mathrm{B}}T}A_R \\ c_2 &= \frac{3}{8(k_{\mathrm{B}}T)^2}\left(A_R^2 - 4A_R B_p + 4B_R\right) \\ c_3 &= \frac{3}{16(k_{\mathrm{B}}T)^3}\left(5A_R^3 - 4A_R^2 B_p + 16A_R B_p^2 + 4A_R B_R - 16B_p B_R - 8A_R C_p + 8C_R\right), \end{aligned} \quad (2)$$

are defined by the refractivity virial coefficients (A_R, B_R, and C_R), the density virial coefficients (A_p, B_p, and C_p), and the Boltzmann constant k_{B} and thermodynamic temperature T. The proportionality constants in (1) are fixed properties of the gas species which fills the cavity, and it is the terms $\frac{\Delta f}{\nu}$, d_{m}, and d_{r} that are specific to each FP cavity; these terms need to be characterized before a gas pressure can successfully be determined with a refractometer. The effective fractional change in cavity resonance $\frac{\Delta f}{\nu}$ is what is actually measured for a given change in pressure: it is an rf beat frequency between two HeNe lasers; one locked to the resonance of a reference cavity held at vacuum, and one locked to the resonance of a measurement cavity which is filled with gas; a complete definition of $\frac{\Delta f}{\nu}$ is given in Ref. [1] The distortion term d_{r} is determined by monitoring how the resonance frequency of the reference cavity changes as the exterior of the refractometer is

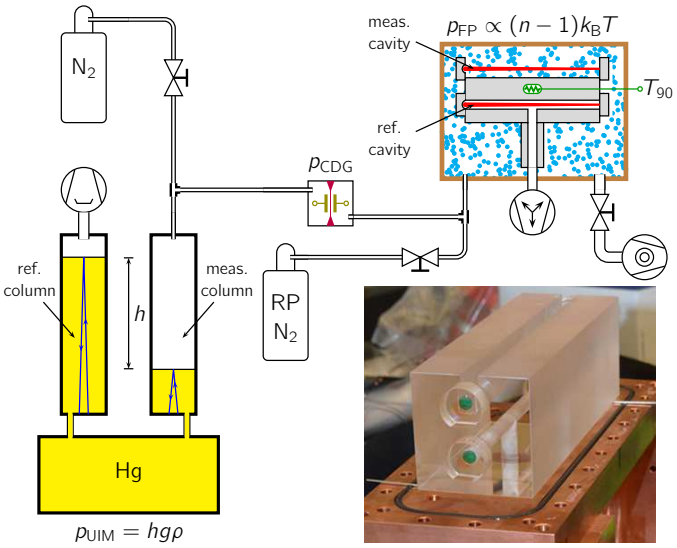

Fig. 1. Plumbing to compare a refractometer to a manometer; inset photograph of refractometer. RP, research purity; CDG, capacitance diaphragm gauge.

TABLE I
EXPANDED UNCERTAINTY FOR PRESSURE MEASURED BY A LASER REFRACTOMETER AT $p = 100\,\mathrm{kPa}$.

parameter	contribution to relative $U(p_{\mathrm{FP}}) \times 10^6$	notes
$A_R = 4.44612(4)\,\mathrm{cm}^3/\mathrm{mol}$	9.0	[i]
$B_R = 0.9(2)\,\mathrm{cm}^6/\mathrm{mol}^2$	1.8	[4]
$C_R = -95(10)\,\mathrm{cm}^9/\mathrm{mol}^3$	0.01	[4]
$B_p = -4.02(15)\,\mathrm{cm}^3/\mathrm{mol}$	6.2	[5]
$C_p = 1434(200)\,\mathrm{cm}^6/\mathrm{mol}^2$	1.0	[5]
$T = 302.919(1)\,\mathrm{K}$	3.3	[ii]
$k_{\mathrm{B}} = 1.3806488(13) \times 10^{-23}\,\mathrm{JK}^{-1}$	1.8	[7]
$\frac{\Delta f}{\nu} = 2.649422(2) \times 10^{-4}$	0.5	[iii,iv]
$d_r = 1.092(2) \times 10^{-6}$	0.4	[iv]
$d_m = 9.83(5) \times 10^{-7}$	1.1	[iv]
gas impurity	0.7	[v]
compression hysteresis	1.1	[vi]
nonlinear length change	15 mPa	[1]
lock offsets	4 mPa	[vii]
anomalous distortion	1.2 mPa	[viii]
outgassing	1.3 mPa	[ix]
intercavity length drift	0.5 mPa	[ix]
overall uncertainty ($k = 2$)	$\left[(16\,\mathrm{mPa})^2 + (11.9 \cdot p)^2\right]^{1/2}$	

[i] Based on the most accurate measurement of nitrogen refractivity [1]—A_R is limited by how accurate the pascal can be realized.
[ii] Measured with an SPRT and includes $U(T - T_{90})$ [6].
[iii] Includes errors in the estimate of cavity length, mirror and diffraction phase shifts, and vacuum-wavelength.
[iv] These terms are specific to one of our laser refractometers. In principle, the terms are correlated with uncertainty already expressed in A_R, and their contribution to $U(p_{\mathrm{FP}})$ is smaller than what is stated.
[v] Worst-case is 0.0001 % CO_2 in 99.9999 % N_2.
[vi] Our FP cavity is made of ULE, which has notably low hysteresis.
[vii] Caused by residual amplitude modulation.
[viii] For temperature changes of 1 mK or less.
[ix] For measurements completed within 0.5 h after a fill.

brought to pressure; the change in resonance is measured by beating the cavity resonance against a known laser frequency reference, in our case an iodine-stabilized laser. Finally, the distortion term d_{m} is determined via helium correction: we fill the measurement cavity with helium of known pressure and temperature, and calculate the theoretical refractivity; the error between the calculated refractivity and what the refractometer measures is attributed to d_{m} [3].

In the top part of Tab. I we list expanded uncertainties for all parameters in (1) and (2), and show the contribution of each parameter to the relative expanded uncertainty for a pressure determination by the refractometer at 100 kPa. It is worth noting that the chief contributor to $U(p_{\mathrm{FP}})$—A_R, the first refractivity virial coefficient—comes from a measurement of nitrogen refractivity at $p = 100.0000(6)\,\mathrm{kPa}$, $T = 302.919(1)\,\mathrm{K}$ and $\lambda_{\mathrm{vac}} = 632.9908(2)\,\mathrm{nm}$; thus, $U(p_{\mathrm{FP}})$ at this particular pressure is entirely independent of other virial coefficients. Furthermore, since we operate at the same temperature and vacuum-wavelength, a certain cancellation of errors occurs at other pressures, leading to a complicated relationship between the uncertainty of the final result and the uncertainty of the parameters in Tab. I. Also, knowledge of A_R is limited by how well nitrogen gas pressure can be measured with a manometer: if the pascal can be realized more accurately than current

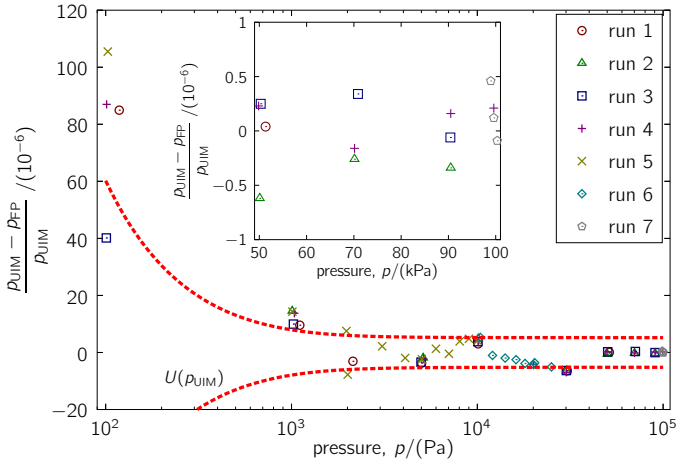

Fig. 2. Disagreement in pressure as measured by a laser refractometer (p_{FP}) and ultrasonic manometer (p_{UIM}); manometer uncertainty $U(p_{\mathrm{UIM}}) = [(6\,\mathrm{mPa})^2 + (5.2 \times 10^{-6} \cdot p)^2]^{1/2}$ also shown.

means, the more accurate measurements of A_R would correspondingly reduce $U(p_{\mathrm{FP}})$. In addition to the uncertainties in the parameters of (1) and (2), there are experimental limitations, as listed in the bottom part of Tab. I. These limitations end up dominating $U(p_{\mathrm{FP}})$ at lower pressures because they are responsible for an offset term in the refractometer (a pressure independent error).

In Fig. 2 we show pressure measurements using the laser refractometer as compared to NIST's ultrasonic mercury manometer, one of the world's most accurate realizations of the pascal. For pressures above 50 kPa we see 1×10^{-6} repeatability, with performance degrading at lower pressures—this poorer performance is caused by the offset term in $U(p_{\mathrm{FP}})$, but uncertainty from mercury vapor in p_{UIM} is non-negligible. Notably, some bands of pressure—1 kPa, 10 kPa, and 30 kPa—are outside the expanded uncertainty of the manometer $U(p_{\mathrm{UIM}})$. At present it is not clear what to attribute these outliers to, but we are in the process of building a second laser refractometer as a cross-check, and our next tests will compare two independent laser refractometers to ultrasonic (oil and mercury) manometers.

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