Microwave Radiometry of Blackbody Radiation

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***Abstract*—We outline the theoretical formulation of radiometry of the free-space radiation emitted by a blackbody target. Simulation shows a much smaller drop of radiation intensity of a Lambertian source than that of an incoherent source in the near-field region, indicative of a powerful influence produced by the coherence property of the blackbody source. Further, the coupling of the radiation to a radiometer is formulated by the plane-wave scattering theory of the radiation field.**

***Index Terms*—Calibration of remote sensing, coherence propagation, electromagnetic radiation, microwave blackbody, plane-wave expansion, thermal noise.**

I. Introduction

Microwave radiometry has become more and more crucial in remote-sensing instruments due to its significant role in weather forecasting and climate studies. Nearly all the environmental parameters observed by such systems are originally represented by temperature, which is directly extracted from total-power radiometer measurements. The measurement accuracy relies on the radiometric calibration, often including the observation of two known thermal noise sources. One typical configuration consists of a man-made blackbody target in conjunction with the naturally accessible cosmic background. The radiation from both sources are independently collected through the front-end antenna of the radiometer to complete a calibration.

On one hand, the radiation arising from the cosmic background is fairly well known to us. On the other hand, the artificial blackbody usually possesses imperfect properties and measurements of its radiation almost always take place in the near-field (NF) region. As a consequence, precise knowledge of the blackbody NF radiation has a preponderant impact on calibration accuracy. In this paper, we furnish a theoretical model of radiometric measurements of blackbody radiation at close range. The radiation emanating from the blackbody is modeled by coherence-propagation theory and the power coupled to the antenna is based on the plane-wave scattering-matrix theory.

II. Plane-Wave Expansion of Primary Thermal Source

In general, blackbody sources can be compartmentalized into two categories, namely a primary source and a secondary source. Throughout this report, we concentrate on the primary source, which often embodies a thermal object with ideal emission characteristics. Handling the secondary source is actually more straightforward and can be simply inferred from the approaches in what follows.

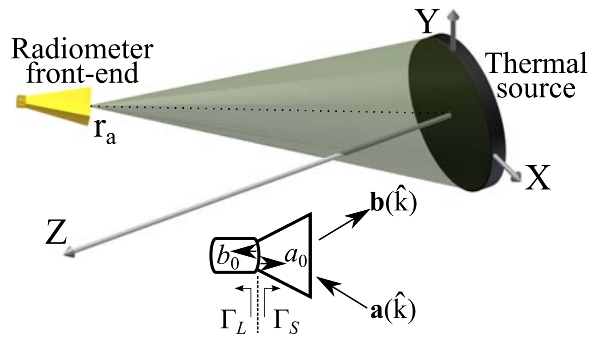

Fig. 1. Illustration of a radiometer collecting radiation from a blackbody and some notations for the plane-wave scattering matrix representing an antenna.

We consider that a radiometric receiver detects the radiation from a planar source located at the plane $z = 0$, as shown in Fig. 1. Although portrayed in a circular shape, the source can have any arbitrary form. We first attempt to obtain the "cumulative spectra" $\mathbf{U}$ of prescribed currents $\mathbf{j}$ in vacuum [1]. It is well known that the vector potential $\mathbf{A}$ relates to $\mathbf{j}$ by

$$\mathbf{A}(\mathbf{r}) = \mu_0 \int_{\mathscr{A}} \mathbf{j}(\mathbf{r}') \frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|} d^2r', \tag{1}$$

where μ_0 is the permeability of free space, $\mathbf{r}$ and $\mathbf{r}'$ symbolize the field and the source point, respectively. The integration is carried over the area $\mathscr{A}$ occupied by the blackbody. With the utility of the following identity

$$\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{ik|\mathbf{r}-\mathbf{r}'|} = \frac{1}{2\pi} \int \exp\left(i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')\right) \frac{d^2K}{k\gamma}, \tag{2}$$

the vector potential $\mathbf{A}$ can be represented in terms of the cumulative spectra as

$$\mathbf{A}(\mathbf{r}) = \frac{i}{2\pi\omega} \int \mathbf{U}(\hat{\mathbf{k}}) \exp(i\mathbf{k}\cdot\mathbf{r}) \frac{d^2K}{k\gamma}, \tag{3}$$

where $\mathbf{U}$ is given by

$$\mathbf{U}(\hat{\mathbf{k}}) = \frac{\mu_0 k\omega}{4\pi} \int_{\mathscr{A}} \mathbf{j}(\mathbf{r}') \exp(-i\mathbf{k}\cdot\mathbf{r}') d^2r'. \tag{4}$$

Here, the wave vector $\mathbf{k}$ is composed of $(\alpha\hat{\mathbf{x}} + \beta\hat{\mathbf{y}} + \gamma\hat{\mathbf{z}})$ with its transverse component $\mathbf{K} = \alpha\hat{\mathbf{x}} + \beta\hat{\mathbf{y}}$ and its unit vector $\hat{\mathbf{k}} = \mathbf{k}/k$. After acquiring the expression of $\mathbf{U}(\hat{\mathbf{k}})$, we next obtain the plane-wave spectrum of the blackbody radiation from the relation between the electric field $\mathbf{E}(\mathbf{r})$ and the vector potential $\mathbf{A}(\mathbf{r})$

$$\mathbf{E}(\mathbf{r}) = i\omega \left(\mathbf{A}(\mathbf{r}) + \frac{1}{k^2} \nabla\nabla\cdot\mathbf{A}(\mathbf{r}) \right). \tag{5}$$

In view of (3) and (5), the plane-wave spectrum $\mathbf{a}(\hat{\mathbf{k}})$ can be conveniently expressed as

$$\mathbf{a}(\hat{\mathbf{k}}) = (\hat{\mathbf{k}}\hat{\mathbf{k}} - \mathbb{I}) \cdot \mathbf{U}(\hat{\mathbf{k}}), \tag{6}$$

where $\mathbb{I}$ is the unit dyad.

III. Coherence Property of Blackbody Radiation Source

In contrast to the customary postulation that blackbody sources are of complete spatial incoherence, the correlation distance is on the order of the radiation wavelength for any blackbody source with its far-field (FF) radiation following the Lambertian cosine law. In light of its coherence property, the correlation tensor of the blackbody source at any pair of points ($\mathbf{r}'_1$ and $\mathbf{r}'_2$) can be factorized into two terms under the quasi-homogeneous condition [2]:

$$\mathbb{W}_{\mathbf{j}}(\mathbf{r}'_1, \mathbf{r}'_2) = L_{\mathbf{j}}(\tfrac{\mathbf{r}'_1+\mathbf{r}'_2}{2}) C_{\mathbf{j}}(\mathbf{r}'_2 - \mathbf{r}'_1)\mathbb{I}, \tag{7}$$

where $L_{\mathbf{j}}$ is the intensity distribution of $\mathbf{j}$ in the source plane and $C_{\mathbf{j}}$ is the source correlation function.

In spite of a small value, the non-negligible correlation length renders a remarkable influence on the radiation field especially in the NF. In Fig. 2, we show the radiation intensity normalized to its FF value as a function of the separation distance for blackbody targets of various sizes. All the normalized quantities approach 1 asymptotically and the target with a smaller footprint appears relatively brighter in the NF. Furthermore, the inset plot clearly indicates a pronounced difference between a completely incoherent source and a partially coherent source (the blackbody).

IV. Antenna Coupling of Radiation

Without losing generality, an antenna can be characterized by the receiving function $\mathbf{s}(\hat{\mathbf{k}})$. Referring to Fig. 1, the emergent wave amplitude b_0 at the antenna feed is

$$b_0 = \Gamma_S a_0 + \int \mathbf{s}'(\hat{\mathbf{k}}) \cdot \mathbf{a}(\hat{\mathbf{k}}) \frac{d^2K}{k\gamma}. \tag{8}$$

This is equivalent to a source with a reflection coefficient Γ_S excited by a generated wave b_G (the second term on the RHS of (8)). Here, the receiving function is primed to distinguish its translated position and rotated orientation $(\mathbf{r_a}; \mathbb{R}\cdot\hat{\mathbf{x}}, \mathbb{R}\cdot\hat{\mathbf{y}}, \mathbb{R}\cdot\hat{\mathbf{z}})$ from the nominal $(\mathbf{O}; \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$. $\mathbf{s}'(\hat{\mathbf{k}})$ relates to $\mathbf{s}(\hat{\mathbf{k}})$ through

$$\mathbf{s}'(\hat{\mathbf{k}}) = \mathbb{R} \cdot \mathbf{s}(\mathbb{R}^{-1} \cdot \hat{\mathbf{k}}) \exp(i\mathbf{k} \cdot \mathbf{r}_a). \tag{9}$$

$\mathbb{R}$ is the transformation matrix pertaining to the Euler angle.

From the radiometric standpoint, the power received by the radiometer can be found as

$$P_{rec} = \frac{1}{2k^2 Z_0} \frac{(1 - |\Gamma_L|^2)|b_G|^2}{|1 - \Gamma_L \Gamma_S|^2}, \tag{10}$$

where $Z_0 \approx 377\ \Omega$ is the wave impedance in vacuum, Γ_L is the reflection coefficient looking into the radiometer from the antenna feed. With (4), (6), and (8) at our disposal, the essential part of the received power can now be quantified by the ensemble average:

$$\langle b_G^* b_G \rangle = \pi^2 k^2 \mu_0^2 \omega^2 \int \frac{d^2K_1}{k\gamma_1^*} \int \frac{d^2K_2}{k\gamma_2} \tag{11}$$
$$\mathbf{s}'^*(\hat{\mathbf{k}}_1) \cdot (\hat{\mathbf{k}}_1^* \hat{\mathbf{k}}_1^* - \mathbb{I}) \cdot \widetilde{\mathbb{W}}_{\mathbf{j}}(-\hat{\mathbf{k}}_1, \hat{\mathbf{k}}_2) \cdot (\hat{\mathbf{k}}_2\hat{\mathbf{k}}_2 - \mathbb{I}) \cdot \mathbf{s}'(\hat{\mathbf{k}}_2),$$

where $\widetilde{\mathbb{W}}_{\mathbf{j}}$ is the Fourier transform of $\mathbb{W}_{\mathbf{j}}$ in (7). So long as the antenna receiving function (radiation pattern) is available to us from simulation or measurements, the received power of the radiometer can be predicted by numerical integration from the closed form of (11). Although the computational cost may seem unduly high, a variety of numerical techniques can be applied to make the calculation feasible in some specialized yet commonly encountered circumstances.

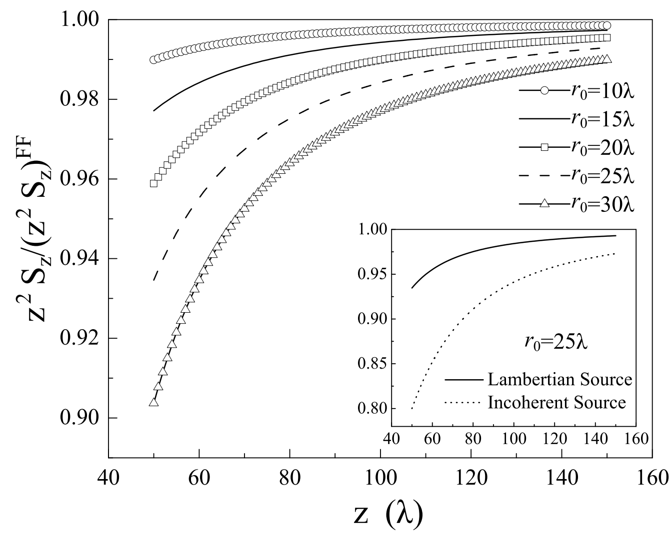

Fig. 2. Normalized radiant intensity $(z^2 \cdot S_z)$ as a function of the separation distance between the blackbody and the on-axis observation point ($\mathbf{r}_a = z\hat{\mathbf{z}}$) for a target radius of 10λ, 15λ, 20λ, 25λ, and 30λ. The inset shows the comparison between the incoherent source and the *Lambertian* source (partially coherent) with a circular profile $r_0 = 25\lambda$.

V. Conclusion

We have established the formulation of blackbody-radiation radiometry in a rigorous way by using coherence-propagation theory and plane-wave scattering theory. The power received by a radiometer is closely tied to the coherence function of the source current in the blackbody through the plane-wave spectrum of the radiation field. The correlation distance at a sub-wavelength scale produces a profound effect on the radiation emerging from the blackbody source. Simulation results including the NF coupling through antennas will be reported at the conference.

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