Josephson-Based Full Digital Bridge for High-Accuracy Impedance Comparisons

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Abstract—This paper describes a Josephson-based impedance bridge capable of comparing any types of impedance over a large bandwidth. The heart of the bridge is a dual AC Josephson Voltage Standard (ACJVS) source which offers unprecedented flexibility in high-precision impedance calibration (i.e., calibration at arbitrary ratios and phase angles) allowing full coverage of the complex plane using a single bridge.

Index Terms—Impedance comparison, ac coaxial bridge, AC Josephson voltage standard, digital bridge.

I. INTRODUCTION

Impedance metrology makes intensive use of ac coaxial bridges for the realization of the capacitance, resistance and inductance scales at kilohertz frequencies. The type and complexity of the bridge depends on the type of the comparison: ratio bridge for the comparison of impedances of the same kind, quadrature bridge for comparing capacitance to resistance, and Maxwell-Wien or resonance bridge for comparing inductance to resistance and capacitance [1]. The common property of these measuring circuits is that once the bridge is balanced, the impedance ratio to be measured is directly given by a voltage ratio. The precise and accurate generation or measurement of this voltage ratio is therefore the cornerstone of all impedance metrology.

Prior to this work, the best voltage ratios were generated using transformers or inductive voltage dividers. However, the main drawback of such devices is that the voltage ratio is set at the fabrication stage of the transformer, by choosing the number of turns of the different windings, and the phase shift between the generated voltages is limited to either 0 or 180 degrees.

Programmable Josephson Voltage Standards (PJVS) can generate stable and precise stepwise approximated ac waveforms and were previously used to generate an accurate voltage ratio. The first two-terminal-pair bridge based on PJVS synthesized voltages was recently demonstrated [2]. This bridge was used to compare impedances of the same type (R - Rand C - C) with an accuracy comparable to transformerbased bridges over a frequency range from 20 Hz to 10 kHz. However, the large harmonic content of the PJVS waveform makes the comparison of impedances of different kinds (R-C, R-L or L-C) more challenging [3] and limits the bandwidth



Fig. 1. Simplified bridge circuit of the JB-FDB. Once the bridge is balanced (i.e. $V_i = 0, i = 1...4$) by adjusting the amplitude and the phase of the bottom ACJVS source as well as the voltages $S_i, i = 1...3$, the complex impedance ratio is equal to the complex voltage ratio: $Z_{\rm bot}/Z_{\rm top} = -V_{\rm bot}/V_{\rm top}$.

to a few kilohertz.

On the other hand, ACJVS are perfect digital-to-analog converters that produce distortion-free waveforms with intrinsically accurate voltage over a bandwidth ranging from a few Hertz to 1 MHz. Combining and synchronizing two such ACJVS systems enables generation of a perfectly calculable voltage ratio with an arbitrary ratio and at any relative phase angle. In this work, these ideal voltage sources are implemented in a fully digital bridge able to compare any impedances with arbitrary ratios and phase shifts over a large bandwidth. In the near future, such bridges will greatly simplify the realization and maintenance of the various impedance scales in many NMIs around the world.

A brief description of the Josephson-Based Full Digital Bridge (JB-FDB) bridge as well as the preliminary results are given in the following sections.

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II. BRIDGE DESCRIPTION

Figure 1 shows a simplified schematic of the JB-FDB developed for high accuracy comparison of two impedance standards Z_{top} and Z_{bot} . The working principle of the JB-FDB is very similar to the digitally assisted bridge (DAB) recently developed at METAS [4] and can be summarized as follows: Once the bridge is balanced, i.e., once $V_i = 0$, i = 1...4, the four-terminal-pair definition of the two impedance standards is realized and the impedance ratio Z_{bot}/Z_{top} is equal to the voltage ratio $-V_{bot}/V_{top}$. The main difference between the JB-FDB and the DAB is that the accurate and stable voltage ratio is generated using two ACJVS instead of a ratio transformer. In other words, the amplitudes and the phase of V_{bot} and V_{top} can each be set to any desired values, thus making the comparison of arbitrary impedances possible using a single bridge.

The two voltage sources required by the JB-FDB are provided by two independent pulse-driven ac Josephson voltage standards operated in separate Dewars of liquid helium. Each ACJVS system can generate a maximum rms voltage output of 1 V [5], [6] and each has an operating current range of 1.4 mA, that is, each can provide ± 0.7 mA of current compliance to a given input. The performance of the JB-FDB method relies on the intrinsic stability, linearity, and tunability of the two ACJVS systems. The relative phase stability of the two ACJVS pulse generators is guaranteed by having the two pulse generators share a 14.4 GHz clock and by triggering the start of both pulse generators using a fast rise-time trigger. The amplitude and phase of the two ACJVS output voltages are adjusted as part of the JB-FDB balancing procedure by recalculating the pulse pattern using a delta-sigma algorithm [7]. Each system can be controlled at the timing resolution of a single pulse, resulting in an approximate relative phase resolution of 70 ps.

III. FIRST RESULTS

The JB-FDB has been used to compare two impedances of 12.906 k Ω over a frequency range from 1 kHz to 20 kHz. The frequency dependence of the real part of the impedance ratio is shown in Fig. 2. For comparison, the same two impedance standards have been measured using the DAB and the result is also represented in Fig. 2.

The uncertainty bars represent the combined (k=1) uncertainties for the measurements made with the DAB while they correspond to the Type A uncertainties only for the measurements made using the JB-FDB. At a few frequencies, the comparison has been repeated a few times during several days, and corrections for the small but finite drift of the resistances have been applied. The residual spread of the results is larger than the Type A uncertainty and indicates that there remain some systematic effects that need to be further investigated.

Nevertheless, the good agreement between the results obtained with the JB-FDB and the DAB clearly shows, for the first time, the functionality of the ACJVS sources when implemented with an impedance bridge. Moreover, the potential accuracy of such a bridge is below 0.05 $\mu\Omega/\Omega$.



Fig. 2. Frequency dependence of real part of the ratio of two impedances of 12.906 k Ω measured at METAS with the DAB and at NIST with the JB-FDB.

IV. CONCLUSION

For the first time, two ACJVS systems have been integrated into a four-terminal-pair bridge, allowing the comparison of any impedances over a broad frequency range.

The first test of the bridge was carried out by comparing two resistances of 12.906 k Ω between 1 kHz and 20 kHz. The measured frequency dependence of the resistance ratios are in good agreement (<0.05 $\mu\Omega/\Omega$) with the frequency dependences measured with a classical analog bridge.

This digital bridge is an ideal tool for comparison of impedances of different kinds (R - C, R - L or L - C) and for comparison of resistances having non-conventional ratios (10 k Ω to 12.906 k Ω , for example) as will be shown at CPEM in July.

ACKNOWLEDGMENT

The authors would like to thank B. Jeckelmann for his continuous support, P. Dresselhaus for the development and design of the ACJVS devices, and H. Bärtschi for his technical skills.

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