Estimation of waveform state levels and uncertainties using the histogram and shorth methods

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***Abstract* — State level calculations are fundamental to extract parameters of a time-domain waveform. Here we compare state level calculations using two different estimators, the histogram mode method and the shorth method. Our results show that both methods yield accurate results. The conclusions are applicable to any waveform ranging from slow geological to ultrafast optical events.**

***Index Terms* — documentary standards, histogram, mode, shorth, state level, uncertainty, waveform.**

I. INTRODUCTION

The International Electrotechnical Commission (IEC) and the Institute of Electrical and Electronics Engineers (IEEE) publish documentary standards [1,2] for waveform parameters that are used to describe the characteristics of a waveform. These standards also contain suggested methods of computing the values of these waveform parameters. A documentary standard for the computation of waveform parameter uncertainty was recently started [3] to complement the existing waveform parameter standards.

The purpose of this manuscript is to support these standards by comparing two methods of computing state level (*level*(s), where “s” refers to a state) and its uncertainty. *level*(s) is common to almost all of the other important waveform parameters. Accordingly, the measurement uncertainty of *level*(s) affects the measurement uncertainty of all those other waveform parameters. We chose two different estimators for *level*(s), the histogram mode and the shorth for our study. The mode is the bin from a histogram of waveform values that has the largest number of occurrences of waveform values. The *level*(s) is equal to the middle of the range of the mode bin. The shorth is a specific nondecreasing sequence of waveform values comprising a certain fraction of the waveform values in the state. The *level*(s) is computed as the average value of this shorth sequence. While the histogram mode is one of the most commonly used estimators in industry, the shorth estimator is considered as an alternative to the mode [4].

II. WAVEFORM CONSTRUCTION

Our analysis is based on simulated waveforms, which have been constructed as follows. First, we use a waveform containing 10 000 elements (samples) that are equally spaced in time and based on the convolution of an impulse with a Butterworth or a Chebyshev filter. The advantage of using these functions is that they approach a fixed value equal to zero at later times. This waveform type was replicated 100 times. Next, we added multiplicative and additive noise to each of the 100 waveforms. The additive noise was determined independently for each waveform sample (with mean equal to zero and a standard deviation ranging between 1% and 10% of the waveform maximum). The multiplicative noise was determined independently for each of the 100 waveforms (with mean equal to one and a standard deviation ranging between 1% and 10%). The 100 noisy waveforms and their average were taken for the state level analysis of multiple and single waveforms, respectively. This process was repeated 4000 times, setting the filter properties (order, ripples, -3dB point) and the amplitude of multiplicative and additive noise using Monte-Carlo methods. Figure 1 shows an example of one waveform type including noise.

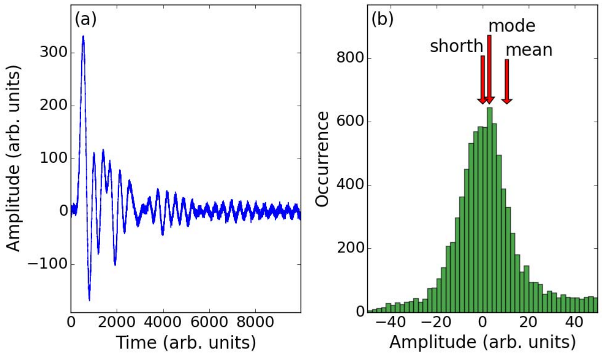

Fig. 1: (a) One of the 4000 different waveform types, which were analyzed, including noise. (b) Histogram of the waveform shown in (a). The histogram mode, the mean and the shorth derived from the waveform data are indicated by arrows.

III. BEST ESTIMATE AND UNCERTAINTY CALCULATIONS

The formulas shown here are taken from [3], as are the descriptions for the derivations of the formulas and their variables.

A. Mode of single waveform

The single waveform provides one mode value that gives $level(s)$. The histogram that yielded $level(s)$ comprised 500 bins ($N_{bin,0} = 500$). The combined uncertainty is given by

$$u_{mode,s} = \sqrt{\sigma_n^2 + u_{\text{bin}}^2 + u_0^2}\,, \tag{1}$$

where σ_n is the standard deviation of the waveform values that are expected to represent noise. The second uncertainty component of (1) accounts for the finite bin width A_{bin} and is expressed as $u_{bin} = A_{bin}/\sqrt{12}$. The last uncertainty component expresses the variation of the mode versus N_{bin}. To derive this component we varied N_{bin} between $N_{bin,0}/2$ and $2N_{bin,0}$ and calculated the deviation of *level*(s) with respect to the corresponding N_{bin}.

B. Mode of multiple waveforms

When N_w waveforms are available, the best estimate of *level*(s) is obtained from the mean of the N_w mode values and the uncertainty from

$$u_{mode,m} = \sqrt{u_{std,mode}^2 + u_{\text{bin}}^2 + u_0^2}\,, \tag{2}$$

with u_{bin} and u_0 being calculated as detailed in subsection *A* using the averaged waveform. The component $u_{std,mode}$ denotes the standard deviation of the mean of the N_w mode values.

C. Shorth of single *waveform*

The single waveform will provide a single shorth value taken as *level*(s) that is calculated with a shorth fraction $f_{s,0} = 0.5$ [4]. The uncertainty is obtained from

$$u_{shorth,s} = \sqrt{\sigma_n^2 + u_{L_{shorth}}^2}\,, \tag{3}$$

where σ_n is identical to the definition in subsection *A*. The second uncertainty component of (3) accounts for the variation of the state level for different shorth fractions. To derive this component we have varied f_s between 0.3 and 0.7 and calculated the deviation of *level*(s) with respect to the corresponding f_s.

D. Shorth of multiple waveforms

When N_w waveforms are available, we calculate *level*(s) from the shorth of the averaged waveform, **Y,** and the uncertainty from:

$$u_{shorth,m} = \sqrt{u_{cov}^2 + u_{L_{shorth}}^2}\,. \tag{4}$$

The first uncertainty component is given by $u_{cov} = \mathbf{H}_L\,\mathbf{\Sigma}_{\mathrm{Y}}\mathbf{H}_L^{\mathrm{T}}$, with $\mathbf{\Sigma}_{\mathrm{Y}}$ being the covariance matrix of the N_w waveforms. $\mathbf{H}_L$ is a row vector with the same length (N) as **Y**. Its elements are equal to $1/N$ if they belong to the shorth, and equal to zero otherwise. The superscript "T" denotes the transpose. The second uncertainty component is calculated as described in subsection *C* using **Y**.

IV. DISCUSSION AND CONCLUSIONS

The results of our simulations are shown in Fig. 2. The value of *level*(s) and its uncertainty, $u_{level(\mathrm{s})}$, can both be computed using the mode and shorth method to yield results that are realistic. For the waveforms studied here, the shorth method provides more accurate results than does the mode method. Yet, the shorth method for multiple waveforms is computationally expensive, such that the calculation of *level*(s) and $u_{level(\mathrm{s})}$ using the mode might be advantageous in certain situations. The data as shown in Fig. 2 do not distinguish the differences between the mode and shorth results depending on the type of waveform and type and magnitude of noise. Additional calculation methods along with different waveforms will be presented at the conference.

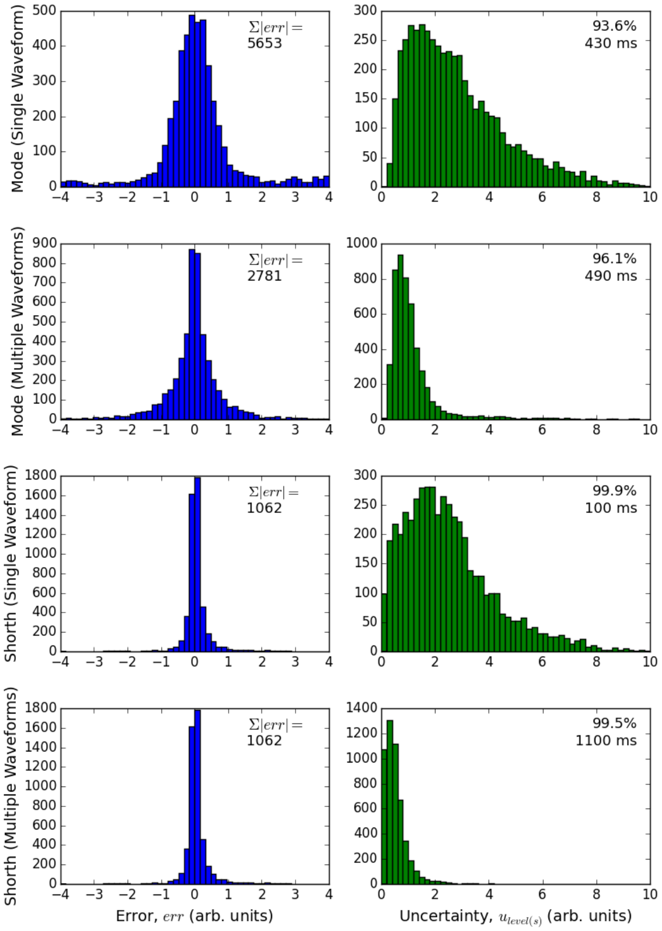

Fig. 2: Results of four different state level calculation methods for 4000 waveform types. Plotted are histograms of the errors in their *level*(s) and their uncertainties per equations (1) to (4). The sum over the absolute errors, $|err|$, is indicated in the upper right corner in the plots in the left column. The percentage shown in the upper right corner of the plots in the right column denotes how often the expanded uncertainty ($k = 2$) exceeds $|err|$ for the 4000 waveform types. The calculation time of each method is also indicated in the right column.

ACKNOWLEDGEMENT

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On behalf of the National Research Council of Canada, I would like to welcome you to Ottawa, Canada, for the 2016 Conference on Precision Electromagnetic Measurements. This year, as we celebrate the 100th anniversary of NRC, we are proudly reminded that we are part of an international scientific community with a long tradition of research on precision measurement science.

It is a very exciting time for metrology as we move towards a revised International System of Units based on fundamental constants. The technical program reflects the increasing use of quantum standards with topics such as emerging graphene quantum Hall standards, improvements in Josephson AC voltage metrology and measurements of Planck's constant using watt balances and silicon spheres, which achieve levels of accuracy and consistency that seemed optimistic only a few years ago and pave the way for the redefinition of the kilogram.

We are pleased to feature five outstanding plenary speakers who will share their knowledge and insights on gravitational measurements, atom interferometry, optical clocks, quantum voltage metrology and the linking of the SI to fundamental constants.

I would like to thank our sponsors and exhibitors, the organizing committee and the 130 members of the technical program committee who made this conference possible. I would also like to thank NIST for funding the SIM travel grant program that provided support for 12 scientists to attend CPEM 2016 and to NCSLI for administering the grant program.

I look forward to engaging with old friends and colleagues, as much as welcoming new faces and minds, during this conference and beyond. I hope that you enjoy the tour of our Measurement Science and Standards laboratories and the diversity of activities that Ottawa has to offer.

Sincerely,

Alan Steele
Canada's Chief Metrologist
General Manager, NRC Measurement Science and Standards