Reply to Comments on ‘Dimensionless units in the SI’

2015 Metrologia 52 617

(http://iopscience.iop.org/0026-1394/52/4/617)

IOP Publishing | Bureau International des Poids et Mesures Metrologia

Metrologia **52** (2015) 617–618 doi:10.1088/0026-1394/52/4/617

Reply

Reply to Comments on 'Dimensionless units in the SI'

Peter J Mohr and William D Phillips

National Institute of Standards and Technology, Gaithersburg, MD 20899, USA

E-mail: mohr@nist.gov and wphillips@nist.gov

Received 23 February 2015
Accepted for publication 18 June 2015
Published 3 August 2015

Abstract

We reply to a Comment on our paper 'Dimensionless units in the SI', addressing the objections by clarifying and expanding the arguments of that paper.

1. Introduction

In our paper 'Dimensionless units in the SI', we suggest changes to the International System of Units (SI) which amount to treating so-called dimensionless units, as defined in the current SI, as ordinary units in order to avoid potential inconsistencies and better reflect contemporary usage by the scientific community [1].

Leonard takes issue with changes we suggest and points out that some of them are not consistent with the current SI [2]. We agree that the formulation we suggest is not consistent with the current SI, however this stands to reason as we are proposing that the current SI be modified.

2. Hz versus rad s^{-1}

One of the main problems in the current SI, as pointed out in our paper, is the fact that for periodic phenomena, there are two different units for frequency, which stems from the treatment of angles as dimensionless units. The two units are 'cycles per second' (or Hz) and 'radians per second' which both reduce to 'per second' in the current SI. However, a given physical frequency, whether describing wave motion, mechanical rotation, or some other periodic phenomenon, will have a numerical coefficient that differs by 2π, depending on whether the units are Hz or radians per second. This leads to ambiguity since either unit may be replaced by 'per second' in the current SI. Incidentally, the square brackets in our equation (15) of [1] are unnecessary, as is correctly pointed out by Leonard.

Leonard claims that the inequivalence of 'angular velocity' and 'rotational frequency' negates our equalities expressed in equation (14) and the discussion surrounding it. However, we deliberately do not use the expressions 'angular velocity' and 'rotational frequency' so as to avoid the kind of confusion expressed in his arguments. In fact, both expressions are names for a physical quantity known generally as 'frequency'. It is common, but hardly universal, that when one uses the term 'angular velocity' one intends to express the frequency in radians per second and that when one uses 'rotational frequency' one intends to express the frequency in Hz. But in spite of this, the frequency of the periodic phenomenon is the same physical quantity regardless of the units in which it is expressed. This commonly agreed-upon usage is expressed in our equation (3) of [1]. Furthermore, Leonard has misconstrued our discussion of cycles and radians. In our paper we note that one cycle and 2π radians both *correspond* to one period of a periodic phenomenon. He apparently believes that we meant that a cycle was *equal* to a period, which he finds to be inconsistent on dimensional grounds. Of course we meant no such thing. It should be evident that a 'cycle' is what goes on during the time of one period; it is not that time itself.

Leonard suggests that plane angles should be base quantities with the unit radian [2]. We agree that angles should have units, as discussed in our paper. However in the revised SI, the distinction between base units and derived units will no longer be meaningful, so the designation as base unit for angles is unnecessary. Similarly, we propose in general treating dimensionless units as ordinary units.

To illustrate the conflict between cycles and radians, we consider the following example. In electromagnetism or quantum mechanics the time dependence of a Fourier component of radiation or waves is given by the function $e^{-i\omega t}$, where ω is often called the angular frequency and conventionally is understood to have units radians per second. This means that the phase ωt has the unit radian, which is omitted or ignored in common usage. It is omitted, because the argument of the exponential function, defined by a power series, must be simply a number in order for the terms in the series to have the same unit. Further, the radian unit may be omitted everywhere with no consequence if all angles and phases are expressed in the same units, namely radians, since then there will be no leftover numerical factors or inconsistencies. This makes the radian the natural choice to be the coherent unit for angles in the SI.

On the other hand, one may give the time dependence as $e^{-i2\pi\nu t}$, where ν is the frequency and conventionally is understood to have units cycles per second or Hz. (The equivalence of these names is stated in the SI Brochure, although Leonard appears to question the appropriateness of this equivalence). The factor of 2π in the above exponent is necessary to assure that the function is periodic with period $T = 1/\nu$. In this case it would be cycles that is dropped and if that were done for all angles, there would be no leftover numerical factors and cycles would be the coherent unit for angles in the SI. Leonard argues that cycles is not a unit and should not be included at all, or replaced by 1. This choice of units would be consistent with traditional spectroscopy units, but would require additional factors of 2π in many of the equations of quantum mechanics and electromagnetism. We note that if we accept Leonard's suggestion that the radian be a base unit, the coherent derived unit of frequency must be the radian per second and not the hertz, because the latter would require an additional numerical factor, disallowed for coherent units.

The problem with the current SI is that it attempts to accommodate two systems, one in which the unit of frequency is radians per second and another in which the unit of frequency is cycles per second (Hz). As a consequence, if a calculation is done using SI units throughout, and the outcome is a number for a frequency, the units could be radians per second or Hz. This results in an ambiguity of a factor of 2π. One might argue that by carefully tracing through the calculation the unit could be determined, but that can lead to mistakes. In fact, there are mistakes in the literature that correspond to the incorrect choice being made for the frequency unit. This would also be problematic for computer software that incorporates units and reports results with units.

The solution we propose is to choose radian per second as the coherent unit for frequency in the SI, which has the consequence that the Hz cannot also be a coherent unit.

3. The Avogadro constant

Another dimensionless quantity in the current SI is entities. Here we suggest that the SI follow common usage in which the entity is specified in the unit. This differs from the current approach in the SI where the entity is specified in a descriptive phrase preceding the quantity being considered. In particular, the prescription advocated by Leonard is as stated in section 7.5 of his [3], and represents the *status quo*. We argue that this *status quo* prescription has the disadvantage that one cannot do calculations or conversions with phrases that precede the quantity in question. It should be noted that we recommend that entities may be replaced by specific names such as atoms or whatever is being considered, however it is obviously not necessary for the SI to specify every conceivable entity.

Leonard's discussion of the Avogadro constant is based on an admittedly unconventional definition of his own [2]. We have suggested only a slight modification of the conventional form of that constant, as given in [10] of our paper in [4].

4. Conclusion

In order to make our reply brief, we have not addressed every specific objection made in the Comment by Leonard [2]. In most cases his remarks support the *status quo* of the SI, something we specifically argue against. We stand by all the statements in our paper. We have considered two examples of disagreement and have expanded our discussion of those topics to help clarify our position.

References

[1] Mohr P J and Phillips W D 2015 Dimensionless units in the SI *Metrologia* **52** 40–7
[2] Leonard B P 2015 Comment on 'Dimensionless units in the SI' *Metrologia* **52** 613–6
[3] Thompson A and Taylor B N 2008 Guide for the use of the International System of Units (SI) *NST Special Publication 811* section 7.5.
[4] Mohr P J, Taylor B N and Newell D B 2012 CODATA recommened values of the fundamental physical constants: 2010 *Rev. Mod. Phys.* **84** 1527–605