**RU177**

**Wind Load Factors for Use in the Wind Tunnel Procedure**

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**ABSTRACT**

A 2004 Skidmore Owings and Merrill report (in Simiu E. (2011) *Design of Buildings for Wind,* Appendix 5, Wiley, Hoboken, NJ) notes that the ASCE 7 Standard (American Society of Civil Engineers (2002) ASCE 7-02, Reston, Va) is incomplete insofar as it provides no guidance on wind load factors appropriate for use with the Standard’s wind tunnel procedure. The purpose of this paper is to contribute to such guidance. Based on a classical definition of wind load factors as functions of uncertainties in the micrometeorological, wind climatological, aerodynamics and structural dynamics elements that determine wind loads, the paper presents a simple, straightforward approach that allows practitioners to use appropriate wind load factors applicable when those uncertainties are either the same as or different from those assumed in the development of the ASCE 7 Standard. Illustrations of the approach are presented for a variety of cases of practical interest. In estimating design wind loads, the various uncertainties should not be accounted for in isolation, for example by specifying peak pressure coefficients with percentage points higher than those corresponding to their expected values. Rather, to achieve risk-consistent designs, the uncertainties should be accounted for collectively, in terms of their joint effect on the design wind loading. The design wind effect is equal to the estimated expectation of the peak wind effect times a load factor that, in most cases, is not significantly different from the load factor explicitly or implicitly specified in the ASCE 7 Standard. Notably, the load factor is not affected significantly by errors associated with interpolations required in typical Database Assisted Design applications. However, if the available wind speed records are several times shorter than, say, 20 to 30 years, the wind load factors increase by amounts of the order of 15 %.

**KEYWORDS**

Aerodynamics; load factors; micrometeorology; safety; structural reliability; uncertainties; wind climatology; wind engineering.

**INTRODUCTION**

“Wind engineering is an emerging technology and there is no consensus on certain aspects of current practice…. Unfortunately, the use of ASCE 7 with wind tunnel-produced loadings is not straightforward” (Skidmore Owings and Merrill (SOM) 2005). Referring to reports developed by two laboratories on the World Trade Center (WTC) towers, SOM (2005) notes: “Neither wind tunnel report gives guidance on how to use the provided forces with ASCE-7 load factors.”

This paper is specifically addressed to structural designers. Its purpose is to contribute to and stimulate discussions on the development of such guidance. It considers the wind load factor as a function of uncertainties in the micrometeorological, climatological, aerodynamics, and structural dynamics components of the expression for the wind effects. The paper is an outcome of the NIST recommendation, following the Federal Building and Fire Investigation of the WTC Disaster, that “nationally accepted performance standards be developed for … estimating wind loads and their effects on … buildings for use in design, based on wind tunnel testing data and directional wind speed data” (Status of NIST’s Recommendations, 2011)

(<http://www.nist.gov/el/disasterstudies/wtc/upload/WTCRecommendationsStatusTable.pdf>, August 8, 2011). The approach used in this work is applicable to the wind load factor as specified explicitly in earlier versions of the ASCE 7 Standard. However, given the straightforward relation between that wind load factor and its implicit counterpart specified in the ASCE 7-10 Standard via an increase in the mean recurrence interval (MRI) of the wind speed, it is applicable to that counterpart as well, as is indicated subsequently.

Guidance on the specification of wind load factors is especially needed by practitioners and code developers in instances where at least one of the uncertainties considered in the development of wind load factors differs significantly from its counterpart assumed in the development of the ASCE-7 Standard. To estimate the dependence of the wind load factor upon those uncertainties, it is necessary to consider the latter within the context of the *total* uncertainty in the wind loading, rather than individually. This can be done by adapting to the task at hand a simple, approximate reliability-based approach proposed by Ellingwood et al. (1980). The purpose of this paper is to illustrate the application of such an approach to cases in which, in addition to measurement errors, one or several uncertainties affecting the design wind loading are due to (1) the wind speed record’s relatively short duration, (2) the pressure record’s relatively short duration, which may be imposed, for example, by the high cost of aerodynamic testing in large-scale facilities, (3) improved terrain exposure factor estimates achieved in wind tunnel tests, (4) interpolations between or among data contained in aerodynamic databases used in database-assisted design, which cover a necessarily limited number of building models, (5) lack of information on the orientation of the building being designed, and/or (6) imperfect knowledge of the parameters of the dynamic response of flexible structures.

**DEFINITION OF DESIGN PEAK WIND EFFECTS AND TYPICAL UNCERTAINTIES**

The peak wind effect is a random variable: it varies from realization to realization. The following expressions hold for the expectation and coefficient of variation (COV, i.e., the ratio between the standard deviation and the expectation) of the peak wind effect (e.g., pressure, force, moment, deflection, acceleration) with an *N-*year MRI:

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| --- | --- |
|  | (1) |

The results in Eq. 1 and 2 are obtained by considering the definitions of the expectation, variance, and the COV, and neglecting higher order terms in the Taylor series expansion for a product of variables, where the product is expanded around its expectation. In Eq. 1 the factor *a* is assumed to be a deterministic constant. The aerodynamic coefficient depends upon the area being considered, which can be as small as a roof tile or as large as an entire building. Once this dependence is taken into consideration, for rigid structures the gust response factor *G* is unity, and COV(*G*) = 0. *V*(*N*) is the wind speed with an *N-*year MRI, estimated from samples of largest wind speeds regardless of direction; *θm* is the direction for which the product   
*G* (*θ*) *Cp,pk* (*θ*) is largest; *E*z is a terrain exposure factor assumed for simplicity to be independent of direction; *z* denotes height above the surface; and *Kd*  is a wind directionality reduction factor that takes into account the fact that the direction *θm* and the directions of the largest directional wind speeds typically do not coincide. Equation 2 is valid if the variables of concern are uncorrelated, which may be assumed to be the case for the variables just defined.

The design peak wind effect with a 50-year MRI may be defined as

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|  | (3) |

where is the expectation, and is the coefficient of variation, of the peak wind effect *ppk* with a 50*-*year MRI. For codification purposes the factor *k* has been determined by calibration with respect to past practice and consensus among expert practitioners; the value *k ≈* 2 appears to be reasonable (Ellingwood et al. 1980, pp. 6-7) and is adopted herein for illustrative purposes. The quantity

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|  | (4) |

is the factor by which the estimated expected peak wind effect with MRI *N* = 50 years must be multiplied to yield the design peak wind effect. Therefore

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|  | (5) |

Ellingwood et al. (1980) suggested COV(*Ez*)≈ 0.16, COV(*Kd*) ≈0, and COV(*G*) = 0.11 for flexible structures. For rigid structures with specified areas COV(*G*) = 0, as noted earlier. It may further be assumed that, typically, COV(*Cp,pk*) ≈ (0.112 + 0.102)1/2 = 0.15, where 0.11 is the assumed contribution to the uncertainty due to measurement errors and 0.10 is the assumed contribution due to sampling errors (i.e., to the limited sample size). For typical conditions it is reasonable to assume COV[*V*(*N =* 50 yrs)] ≈ 0.10.These measures of uncertainty are approximately consistent with those used in the development of the ASCE 7-10 Standard. For rigid structures they result in a wind load factor *γ* ≈ 1.6. This factor may be used in Eq. 5 in conjunction with the estimated expected peak wind effect in cases where the uncertainties affecting the wind loading do not differ significantly from those underlying the ASCE 7-10 provisions. However, measures of uncertainty that differ from those just listed may be used in applications, as appropriate. The simple procedure presented in this paper would then result in wind load factors *γ* (*N* = 50 years) that may differ from 1.6. Also, at the request of the authority having jurisdiction, the owner, the insurer, or other stakeholders, special structures may warrant higher safety levels, in which case in Eqs. 4 and 5 a factor *k* > 2 and a mean recurrence interval *N >* 50 years should be used*.*

The ASCE 7-10 no longer uses Eq. 5. Instead, with a view to simplifying the Standard, it does away with the wind load factor *γ.* However, to maintain the same safety level for , it requires that

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|  | (6) |

where *N*1 is specified so that

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|  | (7) |

Equation 7 is typically satisfied if *N*1 ≈ 700 years. A 700-yr MRI, rather than a 50-yr MRI, is therefore specified in ASCE 7-10 Standard for buildings designed in accordance with earlier ASCE versions of the Standard (i.e., using Eq. 5). For certain types of structure, MRIs different from 50 years and 700 years, respectively, are used in the ASCE 7-10 Standard.

Alternative formats and/or parameter values that may be used for the definition of peak effects may yield somewhat different numerical values for the wind load factors being estimated. However, the ratios between those values and the values corresponding to conditions assumed in the development of the ASCE 7 Standard will provide a useful indication of the approximate effect of any of the six deviations from those conditions listed at the end of the Introduction.

**EFFECT OF WIND SPEED RECORD LENGTH**

Assume that for a region of interest the length of the record of the largest yearly wind speeds is 6 years, rather than the more typical 30 years. Assume further that the coefficient of variation corresponding to typical conditions, COV(*V*) ≈ 0.10, is due to two contributions, one due to measurement and modeling errors, and the other due to sampling errors, each of the contributions being characterized by a coefficient of variation equal to 0.07, that is, COV(*V*) = (0.072 + 0.072)1/2 ≈ 0.10. Since the standard deviation of the sampling error is approximately proportional to the reciprocal of the square root of the sample size (see, e.g., Gumbel 2004), for a region where the length of the wind speed record is only 6 years, rather than 30 years, the coefficient of variation characterizing the sampling errors may be assumed to be approximately times larger, so that COV(*V*) ≈ [0.072 + (0.07 × )2]1/2 = 0.17. Instead of *γ* ≈ 1.6, the estimated wind load factor of a rigid structure should then be

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|  | (8) |

The ratio between the wind load factors based on the 6-year record and on the 30-year record is approximately 1.15.

**EFFECT OF PRESSURE RECORD LENGTH**

This section considers time series of pressures. However, its results are applicable to other types of time series, e.g., time series of internal forces.

*Estimated Expected Value of the Peak Pressures.* Let the *Cp* (*θ*, *t*) record have length *T* and bedivided into a number *n* of subintervals (”epochs” of length *T/n*, or “trials”)*.* The peak value of *Cp* (*θ*) in any one epoch *i* (*i* = 1, 2, ..., *n*) (i.e., over any one subinterval of length *T/n*), denoted by , forms a sample ofsize *n* of data assumed to be independent, identically distributed, and best fitted by a Type I Extreme Value (EV I) cumulative distribution function

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|  | (9) |

that is, is the probability that the variate is not exceeded during any one epoch of length *T/n*. The probability that the variate is not exceeded during the 1st epoch, *and* the 2nd epoch, …, *and* the *r*th epoch, is

Inversion of Eq. 10 yields

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|  | (11) |

Equation 11 shows that *Fr* is an EV I cumulative distribution function with location parameter equal to *μ + σ* ln *r* and scale parameter *σ*.

Consider the relations between the expectation and standard deviation of variates with an EV I distribution, and the location and scale parameters of that distribution (see Appendix or, e.g., Simiu and Scanlan 1986, p. 607):

Expectation = location parameter + 0.5772 × scale parameter

Standard deviation = × scale parameter.

where 0.5772 is the Euler - Mascheroni constant. It follows from the first of these relations that the expectation of the variate over *r* epochs, denoted by, corresponds to a probability *F* such that

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|  | (12) |

For Eq. 12 to be satisfied it follows from Eq. 11 that *–* ln(*–* ln *Fr*) = 0.5772, that is,

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|  | (13) |

Equation 13 may be interpreted as follows. Given a large number of realizations, in 57 % of the cases the observed peak will be lower, and in 43 % of the cases it will be larger than the expected value. The parameters *μ* and *σ* can be estimated from the sample of data ,(*i* = 1, 2, …, *n*) by using, for example, the Best Linear Unbiased Estimator (BLUE, Lieblein 1974) or the method of moments (see, e.g., Simiu and Scanlan 1996).

In applications, design peak pressures are commonly estimated by substituting in Eq. 11 estimated values for the “true” values of the parameters *μ* and *σ*, and assuming the probability *Fr* = 0.78 or 0.8 (as specified in ISO 2009, p. 22), rather than *Fr* = 0.5704. Assuming that the EV Type I distribution (Eq. 11) is an appropriate model, the use of the probability *Fr* = 0.8 rather than *Fr =* 0.5704 would be an instance of double counting, by increasing in Eq. 1 the pressure (or force) coefficient above its expected value, while also accounting in Eq. 2 for the deviation of the pressure from its expected value.

It could be argued that the use of the 0.78 or 0.8 value of *Fr* is consistent with storm durations in excess of one hour (e.g., three hours). Note, however, that if a storm duration longer than one hour were assumed, the expected peak corresponding to it should be estimated directly by using in Eq. 10 or Eq. 11 a value of *r* consistent with that duration. Also, the assumption that storm durations are longer than one hour would clearly violate accepted design practice, which follows the convention of a storm duration of one hour (see, e.g., ASCE 7-10, Eq. 26.9-11; ASCE 7-10 Commentary, Fig. C26.5-1; and Mooneghi, Irwin and Chowdhury 2015).

To the probability *Fr* = 0.8 there corresponds a value

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|  | (14) |

such that, if the EV I distribution were correct, in 80 % of the cases the observed peak will be lower, and in 20 % of the cases it will be larger than the expected value plus 0.7 times the standard deviation.

*Sampling Errors.* The variance of the estimate of the peak value in Eq. 11, obtained by substituting method of moments estimators for *μ* and *σ*, is approximated by the expression

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|  | (15) |
|  | (16) |

Equations 15 and 16 are derived in the Appendix (Eq. 33).

*Numerical Example.* Consider a *T =* 90-s long record of pressures on the roof of a model with geometric scale 1:8 and velocity scale 1:2. The length of the prototype counterpart of the record is obtained from the condition

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|  | (17) |

Let *n* = 16. The prototype length of each subinterval is then *Tprot* /16 = 360/16 = 22.5 s. The estimated values of the location and scale parameters of the EVI distribution of the peaks of the *n =* 16 intervals of length *Tprot* = 22.5 s were found by the BLUE estimator to be *μ* = 4.414 and *σ* = 0.536. It is assumed that *Fr* = 0.5704. From Eq. 12, the corresponding estimated expectation of *Cp,pk* (*θ, T/n*)during the prototype 16-epoch interval is

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|  | (18) |

Alternatively, the method of moments may be used, in conjunction with the assumption *Fr* = 0.5704. The sample mean and the sample standard deviation for the 16 peaks *Cp,pk i*(*θ, T/n*) (*i* = 1, 2,…., 16) were found to be |E[*Cp pk* (*θ, T/n*)]| = 4.72 and SD[*Cp,pk* (*θ, T/n*)] = 0.75, respectively, yielding the estimates *σ* = (61/2/*π*) × 0.75 = 0.585 and *μ* = 4.72 − 0.5772 × 0.58 = 4.39. Therefore, the estimated expectation of *Cp,pk* (*θ, T/*16, *r* = 16)during the prototype 16-epoch interval (i.e., a 360 s prototype time) is

From Eqs. 15 and 16, the standard deviation of the sampling error in the estimation of is

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|  | (20) |

to which there corresponds a coefficient of variation 0.555/6.35 = 0.09. The observed peak of the pressure coefficient record is 6.33. Had more than one record been available, each of the respective peaks would of course have been different.

The number *r* of epochs for which the peak value of is required for design purposes is assumed to correspond to a 3600 s prototype length of record, rather than 360 s, that is, to *r* =160 epochs. Using method of moments estimates of the location and scale parameters, the estimated expectation of during the prototype 160-epoch interval is

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|  | (21) |

For *Fr* = 0.5704 (corresponding to the estimated expectation of the peak), the standard deviation of the sampling error in the estimation of is

|  |  |
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|  | (22) |

to which there corresponds a coefficient of variation 0.91/7.70 = 0.12.

The wind load factor may be adjusted to account for the larger variability in the estimated pressure coefficient *Cp,pk*(*θ,T/*16*,r=*160). In this example, using the estimated coefficients of variation shown following Eq. 5, but replacing COV[*Cp****,pk*** (*θ,*1 hr)] = [0.112 + 0.102]1/2 by, say, [0.112 + 0.122]1/2 = 0.16, does not result in a significant change in the estimated coefficient of variation of the peak pressure and therefore in the estimated wind load factor *γ*. However, this may not be the case for pressure records exhibiting very high peaks.   
 These results are consistent with the estimation of peak pressure coefficients of prototype time series of the order of one hour from measured time series to which there correspond prototype lengths of the order of a few minutes. However, for a given situation, it would be prudent to repeat these calculations to be certain that remains true. Indeed, doubling the estimate of the EV I scale parameter doubles the standard error of our estimate of the peak.

In this section it was assumed that bias errors in the estimation of the peak effect are negligible. For tests conducted at Reynolds numbers much lower than the prototype Reynolds number, this assumption may not be valid; an example are peak pressures at roof corners under wind skewed with respect to the sides of the roof (see, e.g., Long 2004, Simiu 2011, p. 178). In such cases corrections for bias are required.

**EFFECT OF INTERPOLATION ERROR IN DATABASE-ASSISTED DESIGN (DAD)**Aerodynamic pressure data used for DAD do not cover all possible model dimensions and roof slopes. For this reason, interpolations based on existing models are typically required in the design process. Calculations reported in Main and Fritz (2006) have shown that such interpolations entail errors which, depending upon the number of models in the database, can have COVs as large as 0.1, say. Accounting for this COV in the expression for the load factor used in this example yields

rather than 1.59; that is, the increase in the estimated value of the wind load factor in this example is 2 %.

**EFFECT OF REDUCING UNCERTAINTY IN THE TERRAIN EXPOSURE FACTOR**

Ad-hoc wind tunnel testing that reproduces to scale the built environment of the structure being designed has the advantage of reducing the uncertainty in the terrain exposure factor, from COV(*Ez*)= 0.16 to 0.08 or even, for perfect wind tunnel simulations, to 0. This results in a reduction of the estimated wind load factor from LF ≈ 1.59 to LF ≈ 1 + 2(0.082 + 0.152 + 4 × 0.102)1/2  = 1.52 or 1.50, respectively, that is, by approximately 4 % or 6 %, respectively. In reality wind tunnel simulations are not perfect. In principle, the factor *Ez* subsumes the effects of the deviations of the wind tunnel flow simulations from the target atmospheric boundary layer models. However, because in practice such deviations can be significant, as shown for example by the results of Fritz et al. (2008), the errors they produce in the estimation of the structural response warrant future detailed research.

**RIGID BUILDINGS WITH UNKNOWN ORIENTATION**

The ASCE 7-10 Standard specifies for buildings the value , which was found in Habte et al. (2015) to be reasonably appropriate for non-hurricane regions. For and the coefficients of variation listed in the previous section, it follows from Eqs. 2 and 4 that COV[*ppk* (*N* = 50 yrs)] ≈ 0.302 and *γ* ≈ 1.604.

Calculations reported by Habte et al. (2015) showed, however, that rather than being vanishingly small, COV(*Kd*) values corresponding to *Kd* = 0.85 are of the order of 0.10. This means that *Kd* can vary fairly significantly as a function of building orientation, which is assumed in ASCE 7-10 to be unknown and therefore has a contribution to the overall measure of uncertainty in the wind effect, COV[*p*(*N*)]. With COV (*Kd*) ≈0.1, it follows from Eq. 2 that COV[*p*(*N* = 50 yrs)] ≈ 0.312, and Eq. 4 now yields *γ* ≈ 1.62. Accounting for the variability of *Kd* thus results in an increase of the design load *ppk* (*N =* 50 yrs) by (1.62 – 1.60)/1.60 = 1 %. This shows that the neglect of the variability of *Kd* is acceptable in this case.

The conclusion drawn from this example is that a non-negligible uncertainty (with COV = 0.10) in the magnitude of the wind directionality reduction factor, which could in theory contribute to the uncertainty in the wind loading, can have a negligible effect on the estimated design wind loading. This is the case because the relative weight of the uncertainty with respect to that factor is small relative to the *total* uncertainty in the design wind load.

**FLEXIBLE BUILDINGS**

The difference between load factors for rigid and flexible buildings is the fact that the latter experience dynamic effects embodied in the gust response factor, *G.* The factor *G* depends upon the type of structure and may vary from member to member. For typical situations the assumption COV(*G*) = 0.11 may be used (see Vickery 1970). The factor may be calculated for any wind effect on the structure being designed by using procedures outlined, e.g., in Yeo and Simiu (2011). In special cases an estimate of the variability of the gust response factor may be performed by considering assumed variabilities of the natural frequencies of vibration and of the damping ratios. As shown in Gabbai and Simiu (2014), the application of a procedure developed therein led to the conclusion that, for buildings of up to 300 m height, the variability of the dynamic parameters (i.e., the natural frequencies and damping ratios) may cause the magnitude of the requisite wind load factor to increase by approximately 5 % or less. That procedure accounted for structural responses to wind from 16 azimuth directions, and can lead to far more accurate results than the use of responses to wind from just the two orthogonal directions parallel to the building’s principal axes. Note that the terms “along-wind response” and “across-wind response” as traditionally used have been rendered obsolete by the availability of publicly available software for directional analyses capable of calculating the “along-wind” and “across-wind” responses associated not only with the wind directions parallel to the building’s principal axes, but also with wind directions skewed with respect to the principal axes (Yeo and Simiu 2011).

**CONCLUSIONS**

Based on a classical definition of wind load factors as functions of uncertainties in the micrometeorological, wind climatological, aerodynamics and structural dynamics elements that determine wind loads, the paper presents a simple, straightforward approach to the development and use of wind load factors. Load factors developed by that approach can be applied to wind tunnel estimates of peak wind effects when those uncertainties are either approximately the same as or different from those assumed in the development of the ASCE 7 Standard. Illustrations of the approach are presented for a variety of cases of practical interest, including cases in which: (i) the wind speed record is relatively short; (ii) the pressure time histories are relatively short; (iii) the uncertainties in the estimate of the terrain roughness are smaller or larger than their typical counterparts; (iv) the building orientation is unknown; (v) the pressures or the wind effects are obtained from a database by interpolation; and/or (vi) dynamic effects are estimated on the basis of uncertain dynamic properties of the structure. In estimating design wind loads, the uncertainties should not be accounted for in isolation, for example by specifying peak pressure coefficients with percentage points higher than those corresponding to their expected values. Rather, to achieve risk-consistent designs, the uncertainties should be accounted for collectively, in terms of their joint effect on the wind loading. The design wind effect is equal to the estimated expectation of the peak wind effect times a load factor *γ*. For uncertainties considered to be typical, *γ* is approximately the same as the load factor explicitly or implicitly specified in the ASCE 7 Standard. However, if the length of the wind speed record is shorter than the typical 20- to 30-year length by a factor of, say, five, an increase of the order of 15 % in the value of the typical wind load factor would be necessary. For other uncertainties that differ from those assumed in the ASCE 7 Standard, including uncertainties in the parameters associated with the dynamic behavior of structures with heights of up to approximately 300 m, the changes in the estimated values of the load factor are typically modest – of the order of 5 % or less. In particular, this was found to be true for the interesting case of uncertainties associated with interpolations among wind effects that correspond to data available in aerodynamic databases. Finally, it is recommended that typical uncertainties on which current practice is based be the object of renewed scrutiny aimed at achieving professional consensus. This is, in particular, the case for uncertainties associated with directional effects, dynamic effects, and wind tunnel testing, on which research is currently being performed by the authors with a view to improving upon simplified assumptions considered in this paper.

**APPENDIX – STANDARD DEVIATION OF SAMPLING ERROR IN THE METHOD OF MOMENTS ESTIMATION OF**

This appendix has two goals: 1) recapitulate material on the estimation of extreme value (EV) Type I distribution parameters by the method of moments (MOM) and on the estimation of using the MOM estimators; and 2) derive a standard error for that estimation.

# *MOM Estimators*

Recall the form of the EV type I cumulative distribution function

(23)

with expected value (mean) and variance given, respectively, by

(24)

and

(25)

Setting the first two sample moments equal to the appropriate theoretical quantities and solving leads to

(26)

and (27)

where is the sample average, is the sample standard deviation, and is the Euler-Mascheroni constant. *Estimator*

Substituting and into Equation (12) from the main text provides an estimator of  
:

(28a, b)

# *Standard Error of*

From the previous equation,

(29a, b)

Since the variance of the sample mean is the variance of a single observation divided by the number of observations, we have

(30)

With the use of two tools: 1) an expression for the variance of in terms of the first four central moments and 2) an approximate expression for the variance of based on Taylor’s formula, it can be found that

(31)

With two more tools: 1) an expression for the covariance between and based on the first three central moments and 2) an approximate expression for the covariance between and based on the multivariate version of Taylor’s formula, it can be found that

(32)

where is Apéry’s constant. Putting everything together gives

|  |  |
| --- | --- |
|  | (33) |

To estimate , and thus , substitute for .

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