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Forces and dynamics of optically levitated polystyrene particles in air using electrostatic modulation

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ABSTRACT

We demonstrate the simultaneous measurement of optical trap stiffness and quadrant-cell photodetector (QPD) calibration of optically trapped polystyrene particle in air. The analysis is based on the transient response of particles, confined to an optical trap, subject to a pulsed electrostatic field generated by parallel indium tin oxide (ITO) coated substrates. The resonant natural frequency and damping were directly estimated by fitting the analytical solution of the transient response of an underdamped harmonic oscillator to the measured particle displacement from its equilibrium position. Because, the particle size was estimated independently with video microscopy, this approach allowed us to measure the optical force without ignoring the effects of inertia and temperature changes from absorption.

Keywords: Optical trapping, Levitation, Transient response, Electrostatic force, Force measurement

1. INTRODUCTION

Radiation pressure from a tightly focused beam of light across a dielectric boundary (e.g. arising from the presence of a microscopic particle in the focal volume) can be decomposed into attractive (gradient) and repulsive (scattering) components.[1] A stable optical trap is formed when the net gradient force is stronger than perturbing forces, such as scattering and thermal forces, causing a particle to be trapped in the region of maximum intensity (trap center). This phenomenon has been successfully used to manipulate many objects of interest, for example liquid aerosol,[2] metallic particles,[3] biological samples,[4] etc.[5] Since the first demonstration of manipulating microscopic objects,[6] optical tweezers (OT) have served as a primary tool for measuring forces on the order of *pico*newtons, useful when probing physical systems.[7,8]

Under the assumption that the trapping force scales linearly with particle displacement, the forces acting on a particle in an optical trap are easily obtained by multiplying the induced displacement with trap stiffness. Accurate force measurements, however, require calibration of both the positional detector and optical trap stiffness. The trap stiffness has been measured by a variety of methods including the drag force method,[9] from the thermal fluctuations of a particle in equilibrium, or using a power spectrum approach.[10] The simultaneous calibration of QPD and trap stiffness has been demonstrated previously for highly damped systems,[11,12] but their application to underdamped systems (e.g. trapping in air) is less developed. Moreover, a careful revision of previously developed techniques and theories is needed to estimate trap stiffness in this regime.[13]

In this paper, we propose new calibration methods to measure QPD sensitivity and trap stiffness for an underdamped optical trapping system in air or vacuum. The method relies on measuring the transient response of a particle in a stable optical trap to an applied electrostatic force, produced by two parallel conducting plates. The simultaneous measurement of the particle motion with a QPD and video microscopy allowed us to simultaneously measure detector sensitivity and optical trapping stiffness. Here, we describe the details of CCD scaling, particle diameter measurement, and analysis of transient response of the particle in an optical trap.

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2. THEORY

When a system is exposed to an external disturbance, it converges to a new steady state *via* a transient response (natural response). We applied an external electrostatic force to displace a particle in an optical trap from equilibrium. The transient response of the particle to this external force, as it converged to a new steady state, was then analyzed to obtain the natural resonant frequency and damping. Under the assumption that an optical trap is described by a harmonic oscillator, for small (linear) oscillations in one dimension, the motion of a particle in the trap is described by[14]

$$\ddot{x}(t)+\beta\dot{x}(t)+\omega_o^2 x(t)=\Lambda\xi(t) \tag{1}$$

where $\omega_o\left(=\sqrt{k/m}\right)$ is natural frequency, with stiffness k and particle mass m, $\beta\left(=\gamma/m\right)$ is the viscous damping, $\gamma=6\pi R\eta$ is the coefficient of friction, η is the dynamic viscosity of fluid with temperature T. The Brownian stochastic force is given by $\Lambda\xi(t)$, where $\Lambda=\sqrt{2k_BT\beta/m}$, with Boltzmann constant, k_B, and $\xi(t)$ is a random process with mean $\langle\xi(t)\rangle=0$ and autocorrelation $\langle\xi(t)\xi(t')\rangle=\delta(t-t')$ for all t and t'. Due to the random nature of the Langevin force ($\langle\xi(t)\rangle=0$), the time averaged trajectories are simply approximated as a motion of the underdamped harmonic oscillator without external force $\ddot{x}(t)+\beta\dot{x}(t)+\omega_o^2x(t)\cong 0$. Therefore, the time-averaged motion of particle under an external force is simply the transient response (step-response) of the damped oscillator described by

$$x(t)=X_o\left(1-\frac{e^{-\varsigma\omega_o t}}{\sqrt{1-\varsigma^2}}\sin(\omega_o\sqrt{1-\varsigma^2}t+\phi)\right) \tag{2}$$

where the $\phi\equiv\tan^{-1}\left(\sqrt{1-\varsigma^2}/\varsigma\right)$, and the damping ratio $\varsigma=\beta/\beta_{critical}$ is used to represent the motion relative to critically damped ($\beta_{critical}=2\omega_o$) response (i.e. the fastest response without overshoot).[15] Based on the ideal step response in equation 2, we introduce an additional phase (Φ_{add}) and QPD offset (X_{off}) to acquire best fitting results as shown in equation 3.

$$x(t)=X_o\left(1-\frac{e^{-\varsigma\omega_o t}}{\sqrt{1-\varsigma^2}}\sin(\omega_o\sqrt{1-\varsigma^2}t+\phi+\Phi_{add})\right)+X_{off} \tag{3}$$

3. EXPERIMENTAL SETUP

Figure1 shows a schematic illustration of the optical tweezers and electrostatic forcing apparatus (two parallel conducting plates) developed on a commercial inverted optical microscope (Eclipse TE2000, Nikon Instruments Inc.). An optical trap was formed using a diode pumped neodymium yttrium vanadate ($Nd{:}YVO_4$) laser and electro-optic modulator (EOM) to control the optical power delivered to a near-infrared (NIR) corrected long-working distance (LWD) objective lens (Mitutoyo, 20X, NA = 0.4, WD = 20 mm). Once the particle was trapped, a quadrant-cell photodetector (QPD) was used to measure the trajectory of the particle with a sampling frequency of 20 kHz. The particle motion (back scattered visible light from halogen lamp) was also simultaneously recorded using video microscopy (a CCD camera calibrated with Ronchi-ruler). This allowed the QPD voltage signal to be mapped to the displacement induced by an electrostatic force under a uniform electric field generated from two parallel indium tin oxide (ITO) surfaces. The ITO coated cover glass was attached on the two sides of the rectangular sample enclosure (width × height × length = 15 mm × 10 mm × 15mm), 15 mm apart and facing each other along the x-axis. Since the polystyrene (PS) particles are charged during the optical trap loading using a ring-type piezoelectric transducer,[4] we were able to apply a controlled, time-dependent electrostatic force on the trapped particle under an electric field up to 40 kV/m .

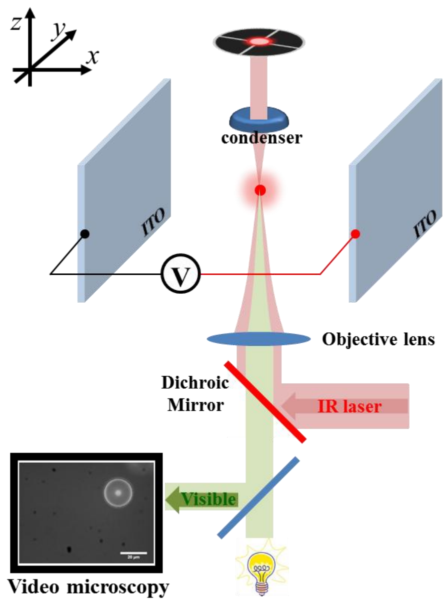

Figure 1 Schematic of the experimental setup: parallel indium tin oxide (ITO) plates were used to apply a known external force to an optically trapped particle along the *x*-axis. The transient response to the disturbance were simultaneously recorded with a quadrant cell photodetector (QPD) and using video microscopy to identify system parameters such as trap stiffness, damping, and steady state displacement.

4. RESULTS

As a first step, the CCD camera used for video microscopy was calibrated with a Ronchi grating composed of chrome lines, with a uniform pitch of 10 μm, on a transparent substrate as shown in **figure2 (a)**. Line edges on the grating were found using Sobel image edge detection, available in the image processing toolbox in MATLAB, which returned the location of the line edges.[16] The image was further processed to estimate the separation between neighboring lines (**figure 2 (b)**), and an average pixel size of 0.158 μm .

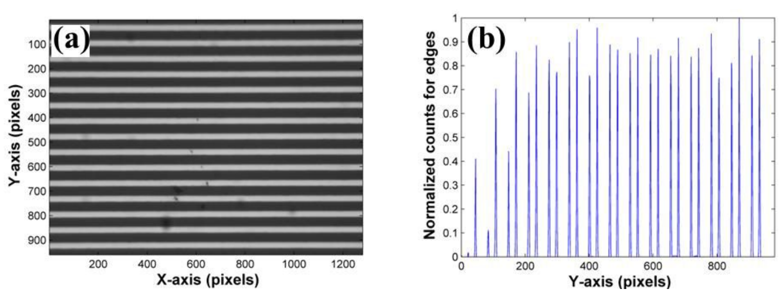

Figure 2 Video microscopy calibration using a Ronchi grating: (a) original image for 10 μm pitch Ronchi grating, (b) The average separation between chrome grating lines is measured from the peak to peak distance for each line of the grating.

Once the scale of CCD image was established, the size of particle was calculated using the automatic particle analysis function embedded in ImageJ. First, a proper threshold value has to be set to distinguish the objects of interest (i.e., particle) from the background image. Based on the selected threshold value, the image is transformed into a binary format with pixels with values lower than the threshold set to black (zeros), while those with intensity higher than threshold set to white (ones). Note that for high contrast images, several automatic algorithms are available to determine a threshold value with minimal user input.[17] The particle area, assuming an ideal sphere, calculated by the image processing software, together with the material density provided from the manufacturer was then used to calculate the particle mass. In our experiment, the PS particle diameter was averaged with three different manual threshold values for three different images captured at slightly varied focus to minimize potential artifacts from the image analysis. The particle diameter was then estimated to be 22.12 μm ± 0.58 μm. The particle mass of 5.950 ng ± 0.58 ng, calculated as described previously was then used to convert the angular resonant frequency (ω_o) into trap stiffness k (= $m\,\omega_o^2$).

The steady state displacement was recorded with video microscopy using slow electrostatic modulation and measuring the maximum displacement of the particle with the CCD. The particle trajectories were extracted using an automated particle tracking plug-in embedded in ImageJ, which returned the location of the particle center in each video frame. By linking neighboring frames, we constructed the particle trajectory as a function of time.[18]

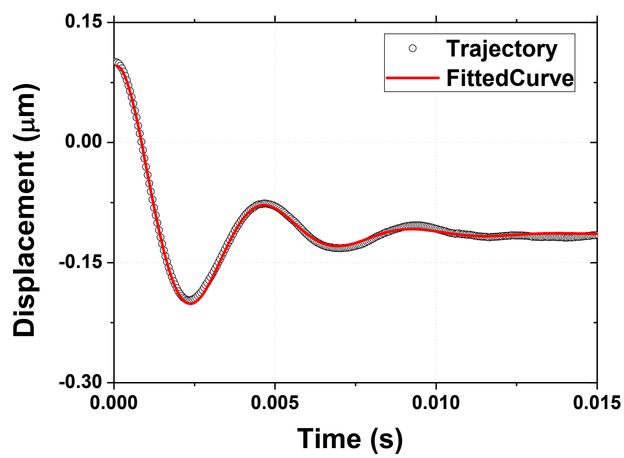

Figure 3 The transient response of trapped 22.12 μm PS particle is well fitted to ideal step response of underdamped harmonic oscillator.

In order to model the transient response analysis, we use the QPD to measure the ringing resulting from a step change in the electrostatic field. We averaged 50 trajectories to minimize the effects of random thermal fluctuations. **Figure 3** show one such averaged trajectory of a trapped particle with diameter = 22.12 μm (*black markers*). The transient response from equation 3 was then fit to the data (**Figure 3**, *red*) using a least squares algorithm for a 15ms trajectory (after the particle converged to a steady-state). From the fit, we obtained the natural resonant frequency ($\omega_o/2\pi$) of 224.2 Hz, damping ratio ς of 0.2699, and induced displacement from the QPD in volts. Note that the QPD signal near the end of the trajectory (approximately 15ms) represents the steady-state displacement at this excitation voltage, allowing calibration of the QPD. Finally, we were able to convert this measured QPD displacement into a particle displacement of 0.2168 μm. The comparison of the QPD position and particle trajectory extracted from video microscopy are summarized in **Table 1**. This technique can be easily extended to obtain the full calibration over the linear range of the QPD, and potentially measure the optical trapping potential beyond the linear region close to the trap center.[19]

Table 1 Fitting results directly acquired from the non-linear least square fit of particle trajectory displayed in figure 3.

Parameters [Units]	X_o [μm]	X_{off} [μm]	ω_o [rad/s]	ς [pure]
Values (95% CB*)	- 0.2168 ± 0.0016	0.1025 ± 0.0015	1427.4 ± 6.0	0.2699 ± 0.0032

* CB represent confident bound of fitting

5. SUMMARY

For the simultaneous calibration of QPD and trap stiffness measurement, we applied an external electrostatic force to the trapped particle and analyzed the transient response of the particle as it reached a steady state. Analysis of the transient response allowed us to estimate the natural frequency and damping of the optical trap. The technique simultaneously allowed the calibration of particle position detected with a QPD using the displacement measured by video microscopy. Our experimental demonstration can be extended to complete system characterization including stiffness, optical force, net charge on the particle and viscous damping. Furthermore, temperature can also be estimated by leveraging the thermal dependence of Stokes damping on gas dynamic viscosity.[20] In future work, we will design additional measurements over a wider range of motion to extend this technique beyond the linear approximation.

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