Consensus Value Methods to Compile On-Axis Gain Measurement Results

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Abstract – This paper compares methods for computing consensus values of on-axis gain measurements that have a large range of values and uncertainties. A variety of methods are used to analyze multiple data sets such as unweighted averages, weighted averages and other statistical means. The appropriate method is dependent on characteristics of the data sets such as, the number of data sets, the spread of the data set values and spread of the uncertainty values for each data set. One method computes a weighted mean where the weights are chosen as the inverses of the fractional error in the data values. A second method removes data sets that are determined to be outliers, then computes an unweighted mean. A third computes a simple unweighted mean. The results of this comparison show that the method chosen to compute a consensus value is fairly independent of the data sets.

Keywords: consensus value, weighted mean, outlier removal, unweighted mean.

I. INTRODUCTION

There are a variety of compilation methods that can be used for calculating consensus values, mean or dispersion of data [1] [2]. Often those compiling the data are not certain which method to use. This paper compares three potential methods for computing consensus values. One method uses weighted averages of the data, where the weights are chosen as the inverses of the fractional error in the data values [3]. This weighted mean method applies a larger weight to data values that have smaller uncertainty. It assumes that the measurement uncertainty is determined consistently. A second method for compiling data is an outlier removal method [4]. This method uses an algorithm that calculates a median value and removes data that vary by more than a specified amount from the median. An unweighted averaging technique can then be applied to the remaining data values. The third method computes an unweighted mean value. The data sets used for the comparison are twelve on-axis antenna gain values and accompanying measurement uncertainties.

II. COMPILATION METHODS

Weighted Mean

The weighted mean value compilation method applies weighted averages. The weights are chosen as the inverses of the uncertainty in the data values. Beginning with on-axis gain values, G_i' (dB) and uncertainties, $\Delta G_i'$ (dB), where, i refers to the ith data set.

$$G_i = 10^{G_i'/10},$$

and fractional uncertainty,

$$\Delta G_i / G_i = 1 - 10^{-\Delta G_i'/10}.$$

Next, we use a particular choice and associate the weight w_i with the ith data set,

$$w_i = \frac{G_i}{\alpha \Delta G_i},$$

where,

$$\alpha = max\left(\frac{G_i}{\Delta G_i}\right), \qquad w_i > 0, \qquad \max(w_i) = 1$$

and the effective number of measurements is

$$w = \sum_i w_i$$

Then calculate the overall gain,

$$G_{weighted\ mean} = \frac{1}{w} \sum_i w_i G_i,$$

and the uncertainty,

$$u(G_{weighted\ mean}) = \frac{1}{w} \sqrt{\sum_i w_i (G_i - G_{weighted\ mean})^2}.$$

Finally, compute the value of weighted mean of the on-axis gain values in dB,

$$G'_{weighted\ mean} = 10 \log G_{weighted\ mean},$$

and the uncertainty for the compiled weighted mean,

$$u'(G_{weighted\ mean}) = -10 log\left(1 - \left(\frac{u(G_{weighted\ mean})}{G_{weighted\ mean}}\right)\right).$$

Outlier Removal

The outlier removal compilation method tests for the presence of outliers. First, estimate the standard deviation σ of the underlying distribution. The most common such estimate is just the standard deviation of the sample, or data set in this paper. However, the sample standard deviation is itself quite sensitive to outliers, a more robust estimate can be obtained by using the Median of Absolute Deviations (MAD) [4].

Beginning with,

$$\sigma \approx S(MAD) \equiv 1.596 median_j\{|G_i - G_{med}|\}.$$

Where, G_{med} is the median of the sample $\{G_i\}$, and the factor of 1.596 is a normalization factor that produces the correct estimate of σ for Gaussian error distributions (i.e., in the absence of outliers). A value of G_i which differs from the median by more than 2.5 times S(MAD) is commonly considered an outlier, and the criterion to test each point.

If

$$|G_i - G_{med}| > 2.5 \times S(MAD),$$

the sample G_i is identified as an outlier and is removed from the averaging process.

For this exercise the remaining on-axis gain values are compiled as an unweighted mean, $G_{outlier\ removal}$, defined as,

$$G_{outlier\ removal} = \frac{1}{N'}\sum_{i=1}^{N'} G_i\ ,$$

where G_i are the remaining on-axis gain values and N' is the number of remaining gain values after outlier removal.

The uncertainty after outlier removal, $u(G_{outlier\ removal})$, defined as,

$$u(G_{outlier\ removal}) = \frac{1}{N'}\sqrt{\sum_{i=1}^{N'} (G_i - G_{outlier\ removal})^2}\ ,$$

where G_i are the values of the remaining on-axis gain values and N is the number of remaining gain values after outlier removal.

It is important to note that the gain values and uncertainties are converted to linear values before calculations and then converted back to dB values.

Unweighted Mean

The unweighted mean method is a simple average of all of the on-axis gain values, $G_{unweighted\ mean}$, defined as,

$$G_{unweighted\ mean} = \frac{1}{N}\sum_{i=1}^{N} G_i\ ,$$

where G_i are the on-axis gain values and N is the number of gain values.

Similarly, the uncertainty for the unweighted mean of the on-axis gain values, $u(G_{unweighted\ mean})$, is defined as,

$$u(G_{unweighted\ mean}) = \frac{1}{N}\sqrt{\sum_{i=1}^{N} (G_i - G_{unweighted\ mean})^2},$$

where N is the number of gain values.

Again gain values and uncertainties are converted to linear values before the calculations then converted back to dB values.

III. MEASUREMENT DATA SETS

The data sets are twelve, independently different, measured on-axis antenna gain values and accompanying measurement uncertainties. The measurements were performed at three different frequencies listed as f1, f2 and f3. These gain values and uncertainties are listed in Table I and are shown graphically in Figure 1. For ease in viewing, the values are plotted nominally with respect to the measurement frequency.

TABLE I. ON-AXIS GAIN VALUES AND UNCERTAINTIES*

Data Set	Gain Value (dB)		
	f1	f2	f3
1	23.63 ±0.05	24.43 ±0.05	24.83 ±0.05
2	23.63 ±0.20	24.47 ±0.20	24.83 ±0.20
3	23.65 ±0.21	24.41 ±0.23	24.79 ±0.25
4	23.63 ±0.24	24.45 ±0.24	24.89 ±0.24
5	23.56 ±0.16	24.44 ±0.20	24.99 ±0.26
6	23.61 ±0.09	24.39 ±0.14	24.84 ±0.15
7	23.70 ±0.11	24.47 ±0.11	24.88 ±0.11
8	23.40 ±0.43	24.00 ±0.50	24.20 ±0.50
9	23.89 ±0.35	24.63 ±0.53	25.43 ±0.56
10	23.61 ±0.84	24.14 ±0.84	24.51 ±0.84
11	23.62 ±0.03	24.44 ±0.03	24.86 ±0.03
12	23.66 ±0.07	24.48 ±0.07	24.88 ±0.07

* Uncertainties listed in Table I have a coverage factor of 2.

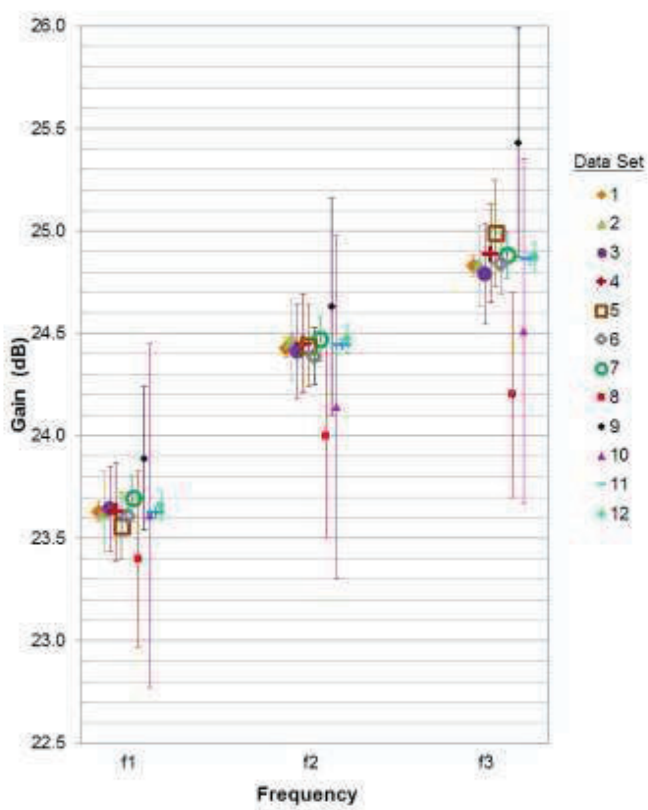

Figure 1. On-axis gain values.

IV. APPLICATION OF COMPILATION METHODS

Weighted Mean

First, the weighted mean compilation method is applied to the on-axis gain values for the three measurement frequencies. The results are listed in Table II.

TABLE II. COMPILED WEIGHTED MEANS AND STANDARD UNCERTAINTIES

f1	f2	f3
23.63 ±0.03	24.44 ±0.04	24.85 ±0.07

Outlier Removal

Next, the outlier removal compilation method is applied to the on-axis gain values for each measurement frequency. The outlier on-axis gain values are defined and listed in *bold red italics* in Table III. After the removal of outliers the unweighted mean is determined for the remaining on-axis gain values for each measurement frequency. These unweighted mean values are listed in *bold blue italics* at the bottom of Table III.

TABLE III. ON-AXIS GAIN VALUES WITH OUTLIERS DEFINED AND CALCULATED UNWEIGHTED MEAN AFTER OUTLIER REMOVAL

Data Set	Gain Value (dB)		
	f1	f2	f3
1	23.63 ±0.05	24.43 ±0.05	24.83 ±0.05
2	23.63 ±0.20	24.47 ±0.20	24.83 ±0.20
3	23.65 ±0.21	24.41 ±0.23	24.79 ±0.25
4	23.63 ±0.24	24.45 ±0.24	24.89 ±0.24
5	23.56 ±0.16	24.44 ±0.20	**24.99 ±0.26**
6	23.61 ±0.09	24.39 ±0.14	24.84 ±0.15
7	23.70 ±0.11	24.47 ±0.11	24.88 ±0.11
8	**23.40 ±0.43**	**24.00 ±0.50**	**24.20 ±0.50**
9	**23.89 ±0.35**	**24.63 ±0.53**	**25.43 ±0.56**
10	23.61 ±0.84	**24.14 ±0.84**	**24.51 ±0.84**
11	23.62 ±0.03	24.44 ±0.03	24.86 ±0.03
12	23.66 ±0.07	24.48 ±0.07	24.88 ±0.07
Unweighted Mean & Standard Uncertainty (After Outlier Removal)	**23.63 ±0.10**	**24.44 ±0.05**	**24.85 ±0.06**

Unweighted Mean

Finally, the unweighted mean compilation method is applied to the on-axis gain values for the three measurement frequencies. The results are listed in Table IV.

TABLE IV. COMPILED UNWEIGHTED MEANS AND STANDARD UNCERTAINTIES

f1	f2	f3
23.63 ±0.03	24.40 ±0.05	24.84 ±0.08

V. RESULTS

The results after applying each compilation method to the twelve data sets at the three measurement frequencies are listed in Table V. These results are also shown graphically along with the on-axis gain values in Figure 2. For ease in viewing, the values are plotted nominally with respect to the measurement frequency.

The results are comparable for the compilation. The compiled results are the same for f1 except for the uncertainties. This is expected due to the close grouping of the on-axis gain values except for the two outlier data sets 8 and 9. The outliers offset each other for the unweighted mean method, are not included in the outlier removal method and due to their large measurement uncertainty have little effect in the weighted mean method. The difference between compiled results is greatest at f2. There are three outlier on-axis gain values for the measurements at f2, data sets 8, 9 and 10. Data sets 5, 8, 9 and 10 are defined as outliers for f3. The compiled on-axis gain and uncertainty results are similar at f3.

TABLE V. RESULTING GAIN CONSENSUS VALUES AND STANDARD UNCERTAINTIES OF COMPILATION METHODS

Compilation Method	f1	f2	f3
Weighted Mean	23.63 ±0.03	24.44 ±0.04	24.85 ±0.07
Outlier Removal	23.63 ±0.10	24.44 ±0.05	24.85 ±0.06
Unweighted Mean	23.63 ±0.03	24.40 ±0.05	24.84 ±0.08

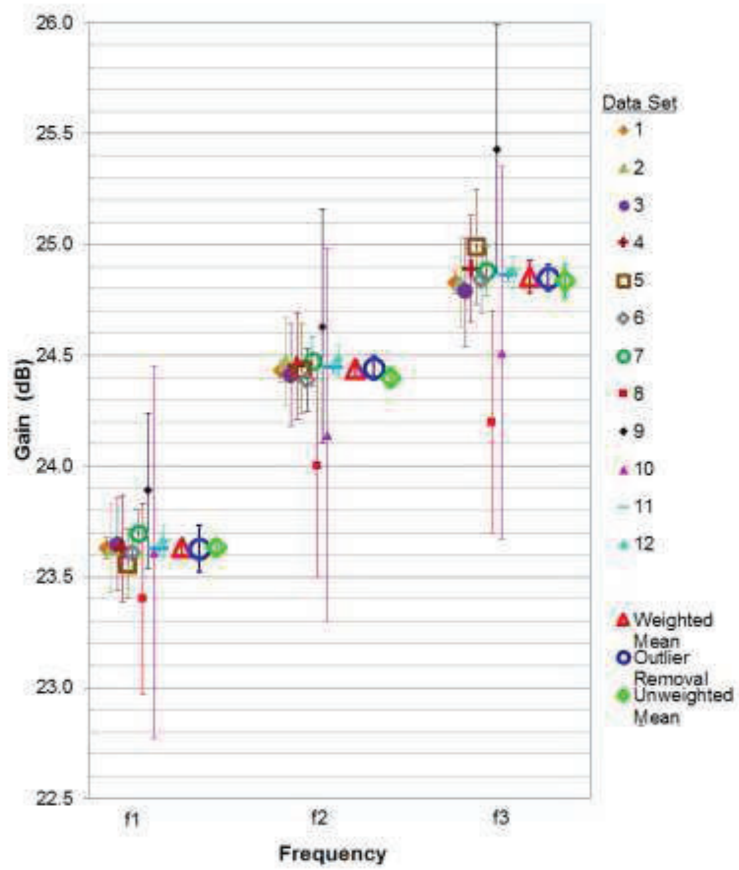

Figure 2. On-axis gain values and the results of the three compilation methods.

VI. CONCLUSION

Three potential compilation methods for computing consensus values were compared using on-axis antenna gain values and associate uncertainties. The compiled gain values compared favorably. The unweighted mean method at f2 differs by the largest amount, but is still within the uncertainty. The uncertainty determined by outlier removal method is larger at f1. The outlier removal method is probably better suited for data sets where the calculated uncertainties are suspect and the comparison can afford to remove data sets. The weighted mean method allows for all data sets to be used in the compilation, which is important for small samples sizes. However, it assumes that the uncertainties are reasonable.

There are a variety of methods used to compile data. One method is not necessarily better, or more correct than another. However, it is important to inform associated parties of the method used and to provide the relevant data for result verification and additional calculation.

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