

NIST's Fully Dynamic Gravimetric Liquid Flowmeter Standard

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Abstract

We describe a new dynamic, gravimetric, liquid flow standard (LFS) that determines flow by measuring the rate of change of the liquid mass accumulating in a collection tank. The LFS is a fully-automated, 15 kg/s system that uses a proportional–integral–derivative (PID) control loop to achieve liquid flows with a stability that is the smaller of 0.001 % or 0.1 kg/s. The expanded uncertainty (corresponding to 95 % confidence level) is 0.021 % for the flow range of the standard; 15 kg/s to 0.22 kg/s. We verify the uncertainty budget by comparing the LFS results with two well-established NIST primary flow standards.

1. Introduction

Dynamic weighing methods are well documented and used by national metrology institutes to calibrate liquid flow meters [1,2]. These methods can achieve low uncertainties while avoiding the expense and complexity of either a flow diverter (generally required by static weighing methods) or the mechanical complexity of provers. Here, we describe the National Institute of Standards and Technology's (NIST's) new 15 kg/s liquid flow standard (LFS). This primary standard uses dynamic weighing to measure flows ranging from 15 kg/s down to 0.22 kg/s with an expanded uncertainty (corresponding to 95 % confidence level) of 0.021 %.

The 15 kg/s LFS is a fully-automated, closed-loop system comprised of three major components: 1) a flow generation and control system, 2) a dynamic weighing system that makes SI (International System of Units) traceable flow measurements, and 3) a test section where customer meters-under-test (MUT) are installed and calibrated. The dynamic weighing system consists of a weigh scale and a collection tank. The scale readings are recorded at approximately 0.2 s intervals during flow accumulation into the collection tank. As shown by Shinder and Moldover the slope of the time-stamped, buoyancy-corrected weight measurements equals the mass flow [3].

The uncertainty of the 15 kg/s LFS is determined using the Monte Carlo method [4]. (See Section 6.) We verify the calculated uncertainty by comparing the 15 kg/s LFS results with two well-characterized NIST flow facilities, the 65 kg/s primary LFS [5] and 2.5 L/s primary piston prover [6]. Figure 1 shows comparison results, which are in good agreement, being less than 0.031 %; Section 5 explains this comparison in detail.

This manuscript describes the 15 kg/s LFS, documents its capabilities as determined by comparisons with well-established flow standards, explains the principle of the dynamic weighing method, and shows the results of the Monte Carlo simulations.

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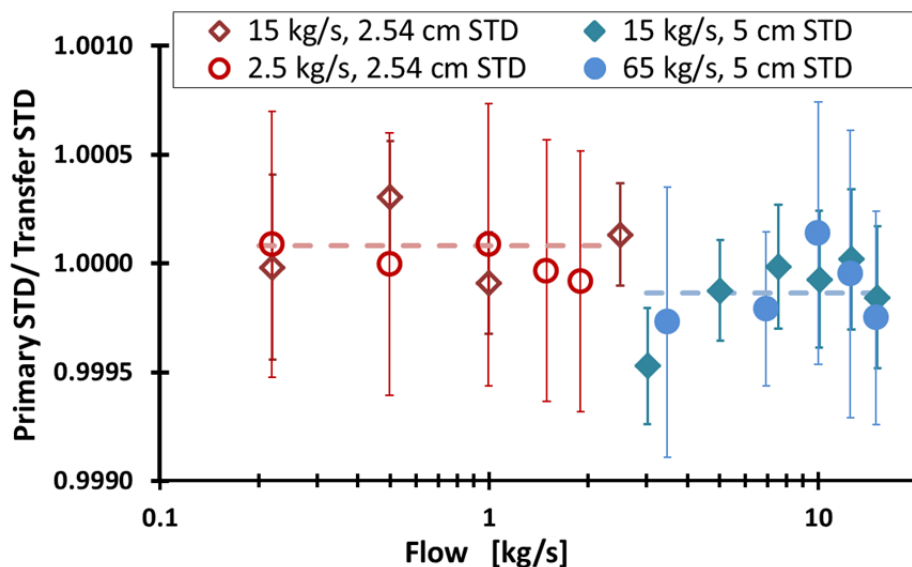


Fig 1. Comparison of new 15 kg/s with NIST's 65 kg/s and 2.5 L/s liquid flow standards using 5 cm transfer standard (solid symbols) and 2.54 cm transfer standard (open symbols). The pink and the blue base lines are the mean reading of the 2.54 cm and the 5 cm transfer standard on the 15 kg/s LFS, respectively. Error bars show the expanded uncertainty in the measurement.

2. Description of the Liquid Flow Standard

The 15 kg/s LFS is a closed-loop liquid flow calibration facility that is automated using the National Instruments LabViewⁱⁱ environment. Custom software was developed to operate mechanical components of the standard and to acquire and store calibration data. Calibrations are done at ambient temperatures with the fluid pressurized to no less than 140 kPa to avoid cavitation. The schematic in Fig. 2 shows the major components of the LFS including: 1) the flow generation and control system consisting of a variable flow pump, reservoir tank, check standard (or reference) flowmeter, butterfly valve, and data acquisition system with digital proportional-integral-derivative (PID) controller; 2) a test section that accommodates a MUT with pipe diameter ranging from 1.25 cm to 5 cm; and 3) the dynamic weighing system comprised of a collection tank and weigh scale. A picture of the 15 kg/s LFS is shown in Fig. 3. Table 1 gives the nominal characteristics of the LFS.

2.1 Flow generation, control, and stabilization

The 15 kg/s LFS generates and maintains steady flow at a desired set point using a pump, a butterfly valve, and reference flow meter. The pump pressurizes and circulates the reservoir tank water throughout the system. The flow and line pressure are stabilized at a desired set point using a PID controller to continuously adjust the pump speed and butterfly valve. When stable, the set point specified in the LabView program will equal the flow indicated by the reference meter. The flow is considered stable if for a time period of 60 seconds, the maximum of the first and second derivative of the flow from the reference meter is less than 0.0001 kg/s^2 and 0.01 kg/s^3 respectively and one of the following conditions are met: 1) the maximum percent error in the flow from the set flow is less than 0.001 % or 2) the maximum flow error from the set flow is less than 0.1 kg/s.

ⁱⁱ Certain commercial equipment, instruments, or materials are identified in this paper to foster understanding. Such identification does not imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

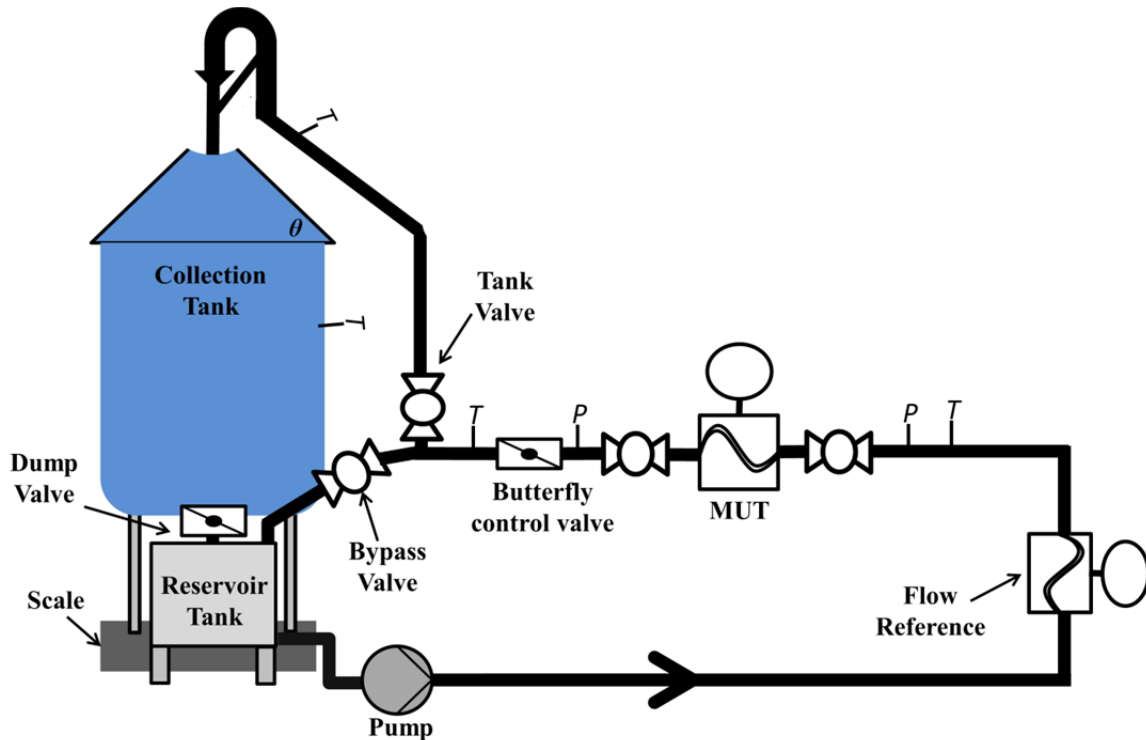


Fig. 2. Schematic of the 15 kg/s LFS. During a calibration, the flow is stabilized after closing the bypass valve and diverting the flow through the collection tank into the reservoir tank.

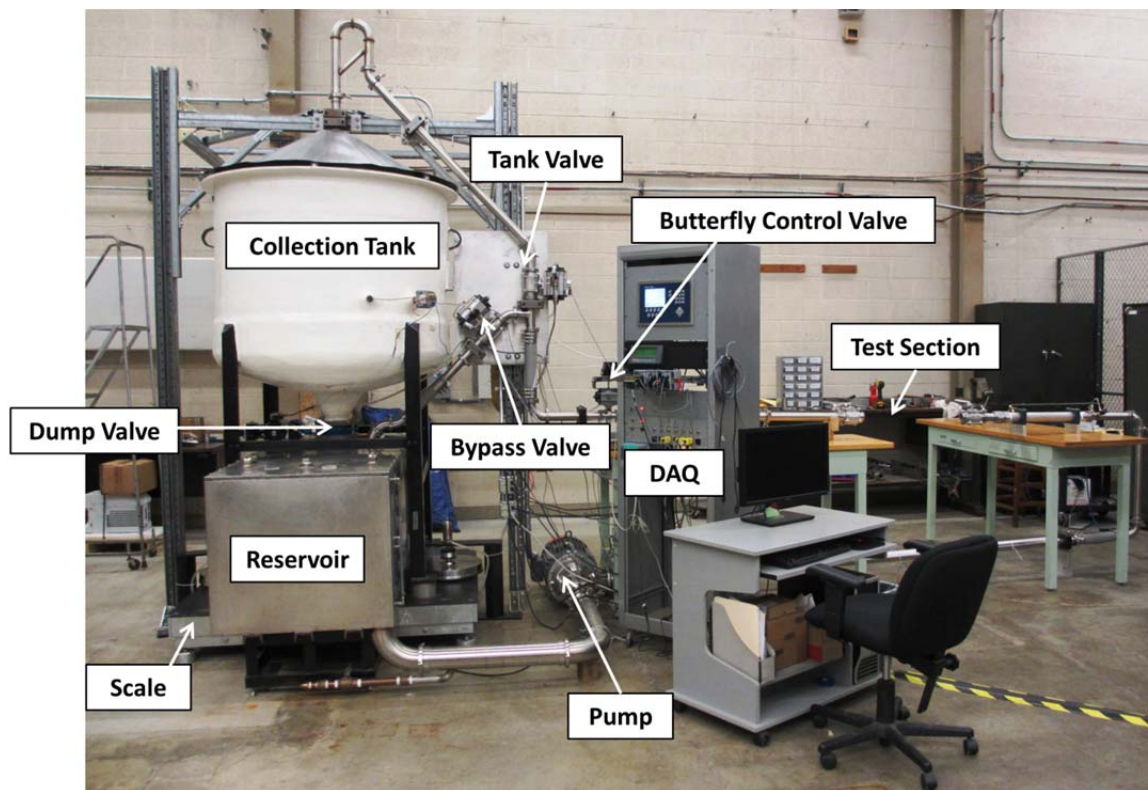


Fig. 3. The 15 kg/s LFS.

Table 1. Nominal Characteristics of the 15 kg/s LFS

Fluid(s)	water(currently), 5 % propylene glycol in water (future)
Flow range [kg/s]	0.22 to 15
Temperature Range ⁱⁱⁱ [°C]	20 to 25
Pressure Range [kPa]	140 to 550
Volume of collection tank [m ³]	0.90
Volume of reservoir tank [m ³]	1.1
Diameter of piping [m]	0.051
Length of test section [m]	3.3
Connecting volume between MUT and collection tank [m ³]	0.01
Collection times	15 s to 3000 s

During the flow stabilization period the system operates in an open loop mode, where the bypass valve is closed and both the tank valve and the dump valve in the collection tank remain open. Flow leaving the reservoir tank is directed through the reference meter, the MUT, and finally through the collection tank before it returns to the reservoir tank. The dump valve closes to start a flow measurement. Flow accumulates in the collection tank for 15 s to 200 s depending on flow. At the lowest flow of 0.22 kg/s the tank can accumulate liquid for up to 3000 s, however, this amount of mass is not necessary to acquire a calibration point and the collection is limited to 200 s. Following the flow measurement interval, the dump valve opens to drain the collection tank in preparation for the next flow measurement. Therefore, the collection tank is constantly either filling or draining during a calibration.

An alternative mode of operation of the 15 kg/s LFS is to use the reference meter as the working standard to calibrate customer meters. This mode also enables research on the effects of unstable flows on meter performance. In this mode, the LFS collection tank is bypassed. The system tank valve is closed and the bypass valve opened to achieve a flow loop.

2.2 Test section

The test section is a straight length of 5 cm diameter stainless steel piping. Flow calibrations are performed with the MUT installed after a minimum of 30 diameters of straight pipe length. We measure temperature and pressure upstream and downstream of the MUT. The fluid temperature is not controlled and increases by as much as 0.5 K due to flow work from the pump during a flow collection. These temperature changes cause the mass of liquid stored in the connecting volume (*i.e.*, volume between the MUT and pipe exit leading to collection tank in Fig. 2) to change between the beginning and ending of a collection. These mass storage effects are accounted for in the mass flow determination and uncertainty analysis of the LFS. The pressure in the test section is controlled and remains essentially constant during a collection, but can vary with flow and the size of the MUT. In general, the pressure increases for larger flow and smaller flow meters. During a calibration we make sure the pressure is sufficiently large to avoid cavitation. During the validation of the 15 kg/s LFS, the pressure in the test section was highest when calibrating a 2.5 cm coriolis meter; it ranged from 140 kPa at a flow of 0.22 kg/s to 445 kPa at a flow of 5 kg/s. When calibrating a 5 cm coriolis meter, the test section pressure was nominally 245 kPa at a flow of 15 kg/s. The maximum pressure is limited to the pump output; 552 kPa.

ⁱⁱⁱ Calibrations are performed with the fluid at the ambient temperature.

2.3 Dynamic weighing system

During a primary flow calibration the bypass remains closed and the flow is continuously either filling or being drained from the collection tank. The tank outlet is a 10 cm hole that is opened and closed by the dump valve (Figs. 2 and 3). The tank drains directly into the reservoir tank, which is suspended above the weigh scale, but not in physical contact with it. The collection tank is covered to prevent water from splashing out of the tank. The inlet plumbing into the collection tank terminates with a *cone* that lies beneath the tank cover. The cone and cover are shown in Fig. 4. The collection tank including the cone and cover are the only parts of the standard in contact with the scale. The top and bottom portions of the cone form an annulus that direct flow into the collection tank. The cone diffuses any horizontal component of momentum with radial symmetry, thereby, minimizing flow-dependent horizontal forces on the weigh scale.

The weigh scale has a 1.2 m x 1.5 m weigh platform with a 1500 kg capacity and 20 g resolution. The data acquisition system continuously reads the scale output that is updated every $0.209718 \text{ s} \pm 0.000001 \text{ s}$, based on the scale internal 24 MHz clock. The scale timing accuracy was verified by multiple long term timing tests performed on the scale. Measurements from the meters and line temperature and pressure sensors are taken every 0.2 s, and from the environmental monitor every 0.46 s. The software uses the scale time as the reference for all other measurements.



Fig. 4. The cover and cone removed from the collection tank. The cover rests on the collection tank without touching the cone that is used to direct the liquid into the tank. The incoming liquid flows between the two plates that make up the cone.

3. Flow Measurement Principle of Dynamic Weighing

Flow determinations are based on the rate of change of the buoyancy corrected mass in the collection tank. The weigh scale reads the sum of three terms: 1) the weight of the fluid, 2) the tare, and 3) the momentum exchange of the fluid at the point of impact in the collection tank:

$$W = m_a g + \dot{p} + \text{tare}, \quad (1)$$

where W is the calibrated weight reported by the scale, g is the acceleration due to gravity, and $m_a = m(1 - \rho_{\text{air}}/\rho_{\text{water}})$ is the apparent mass. The apparent mass equals the true mass multiplied by a buoyancy correction factor where ρ_{air} and ρ_{water} are densities of air and water respectively. The vertical momentum transfer from the falling liquid to the scale equals the mass flow multiplied by the vertical velocity, $\dot{p} = \dot{m}V$. Here \dot{m} is the mass flow we are trying to measure and V is the velocity at time t at the interface where the jet impinges with the water in the collection tank.

Figure 5 shows a control volume inside the collection tank as it is being filled. The tank on the left shows the state of the system at time t_1 , which precedes the tank on the right that shows the state of the system at time t_2 . Based on Eqn. 1 the difference in the scale weights between t_1 and t_2 is

$$W_2 - W_1 = (m_2 - m_1)g(1 - \rho_{\text{air}}/\rho_{\text{water}}) + \dot{m}(V_2 - V_1). \quad (2)$$

A mass balance around the control volume in Fig. 5 results in the following expression

$$m_2 - m_1 = \dot{m}(t_2 - t_1) + m_{\text{jet}}, \quad (3)$$

where the first term on the right hand side is the mass accumulated into the control volume during the collection period, and m_{jet} is the mass of the annular jet falling into the collection tank.

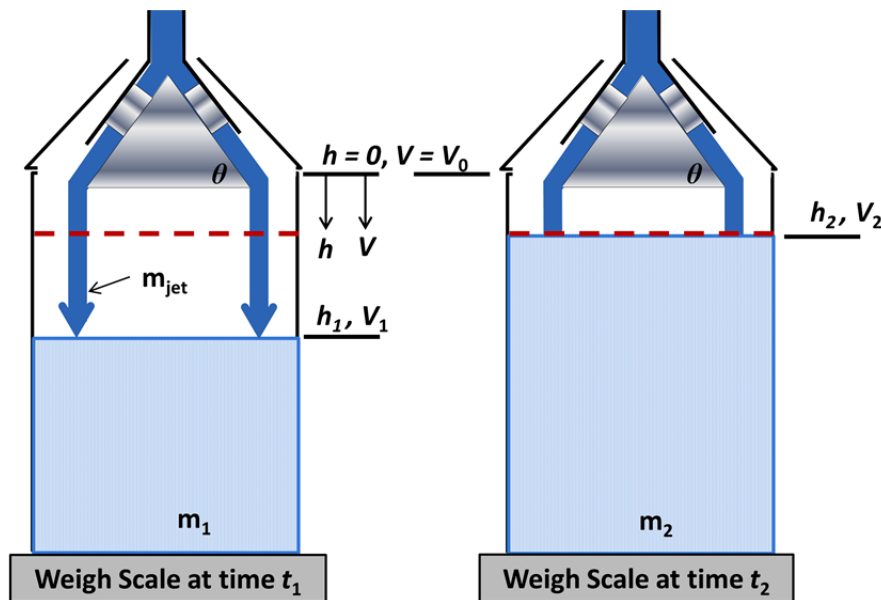


Fig. 5: The red dashed line and the tank boundary makes up the control volume. The tank on the left is the state of the control volume at t_1 and the tank on the right is the state of the control volume at t_2 . Drawing is for illustration purposes only.

Under steady flow conditions the mass of jet is

$$m_{\text{jet}} = \dot{m}t_{h_2-h_1}, \quad (4)$$

where $t_{h_2-h_1}$ is the time it takes the jet to fall from height h_2 to height h_1 . Neglecting air resistance the fall time is determined by the following kinematic equation

$$t_{h_2-h_1} = (V_1 - V_2)/g(1 - \rho_{\text{air}}/\rho_{\text{water}}) \quad (5)$$

where V_1 and V_2 are the respective vertical fluid velocity at h_1 and h_2 . By combining Eqns. 2 through 5 the change in the scale weight between t_1 and t_2 is

$$W_2 - W_1 = \dot{m}(t_2 - t_1)g(1 - \rho_{\text{air}}/\rho_{\text{water}}). \quad (6)$$

Solving Eqn. 6 for mass flow gives

$$\dot{m} = \frac{1}{g(1 - \rho_{\text{air}}/\rho_{\text{water}})} \left(\frac{W_2 - W_1}{t_2 - t_1} \right) = \left(\frac{M_2 - M_1}{t_2 - t_1} \right), \quad (7)$$

where $M = W/g(1 - \rho_{\text{air}}/\rho_{\text{liquid}})$ is the buoyancy corrected, calibrated scale mass readings. For infinitesimally small times between scales readings, Eqn. 7 can be expressed in differential form by

$$\dot{m}_{\text{LFS}} = \frac{dM}{dt}, \quad (8)$$

where the subscript ‘‘LFS’’ has been added to the mass flow to denote that it is determined by the LFS dynamic weigh standard. The simple expression in Eqn. 8 relates the mass flow into the collection tank to the rate of change of the buoyancy corrected, calibrated scale readings. This formulation is valid for steady flows with no mass storage in the collection tank. As discussed by Shinder and Moldover, the model must be extended to apply it to unsteady flows [3]. However, the flow control loop used in the LFS primary standard is able to provide suitably stable flows so that Eqn. 8 is used as the basis for measuring mass flow.

Because the scale response is linear over its operating range as determined by repeated calibrations and the flow is stable, the scale mass versus time is a straight line with the slope of the line equal to the mass flow. Herein, we use linear regression of the time stamped scale readings to determine the slope, and hence the mass flow. The linear regression model for the slope is [4]

$$\dot{m}_{\text{LFS}} = \frac{N \sum_{i=1}^N t_i M_i - \sum_{i=1}^N t_i \sum_{i=1}^N M_i}{N \sum_{i=1}^N t_i^2 - \left(\sum_{i=1}^N t_i \right)^2}, \quad (9)$$

where t_i and M_i are the respective time and mass reported by the scale. We also determine the intercept:

$$c = \frac{\sum_{i=1}^N t_i^2 \sum_{i=1}^N M_i - \sum_{i=1}^N t_i \sum_{i=1}^N M_i t_i}{N \sum_{i=1}^N t_i^2 - \left(\sum_{i=1}^N t_i\right)^2}, \quad (10)$$

which is used for the uncertainty analysis. The masses (M_i) and times (t_i) in Eqns. 9 and 10 are boxcar filtered to attenuate noise in the data [7]. The boxcar averaged times are

$$t_i = \frac{1}{B} \sum_{n=i}^{i+B-1} \tilde{t}_n \quad (11)$$

and the boxcar averaged masses are

$$M_i = \frac{1}{B} \sum_{n=i}^{i+B-1} \tilde{M}_n, \quad (12)$$

where \tilde{t}_n and \tilde{M}_n are the respective unfiltered times and masses, and B is the size of the boxcar filter. The length of the mass (or time) arrays of the boxcar averaged data used to calculate slope is $N = \tilde{N} - B + 1$ where \tilde{N} is the length of the unfiltered data.

3.1 Flow at the MUT

The mass storage in the connecting volume between the MUT and the collection tank must be accounted for to compare the mass flow at the MUT to the standard. The fluid temperature and pressure is measured in the connecting volume at 0.2 s intervals. The measured temperatures and pressures are used to calculate the fluid density. Because the pressure is controlled by the PID, there is no significant change in the pressure over a collection period. Therefore, fluid density change in the connecting volume due to temperature change between the start and end of a collection is the only contributor to connecting volume mass storage (\dot{m}_{CV}). The mass flow at the MUT is given by:

$$\dot{m}_{MUT} = \dot{m}_{LFS} + \dot{m}_{CV} = \dot{m}_{LFS} + \left(\frac{\rho_{final} - \rho_{initial}}{\Delta t}\right) V_{CV}, \quad (13)$$

where $\rho_{initial}$ and ρ_{final} are the respective densities in the connecting volume at the beginning and end of the collection, Δt is the duration of the collection period, and V_{CV} is the size of the connecting volume.

The size of the connecting volume V_{CV} will change with temperature and pressure, however, it is considered constant in Eqn. 13. This is because changes in V_{CV} due to the pressure and temperature being different than when it was measured (*i.e.*, reference conditions) are too small to quantify being that a 10 % uncertainty is given to this volume, Section 6.4. By the same reasoning, any change in V_{CV} during a collection due to temperature change is also negligible.

4. Collecting and Reducing Data

Figure 6 illustrates the reduction technique for the raw data that the 15 kg/s LFS generates. The traces shown are from the calibration of a 5 cm coriolis meter. The top graphs show the mass in the collection tank as it is filled. The starting value is the open loop stable mass in the

tank when the dump valve is initially closed. At the maximum flow of 15 kg/s, generally 50 to 70 data points are collected during a single fill. At the lower flows, the number of data points for a fill can be as large as several thousand; therefore, the total mass collected can be reduced and still give enough data points to determine the slope within the desired uncertainty.

Equation 9 is used to yield the mass flow in the collection tank as shown in the middle row of graphs in Fig. 6. We filter the data to ensure that the slope calculation uses only the portion of data where the flow is steady. The stability criteria for assessing steady flow conditions depends on the following: 1) the value of the second derivative of the mass versus time plot generated from the weigh scale, 2) the standard deviation of flow measurements from the reference meter, and 3) the maximum and minimum values of the flow measurements from the reference meter. The bottom row of graphs in Fig. 6 shows the absolute value of the mass flow rate of change during the collection of mass shown in the top row. Experience has shown that a data collection is sufficiently stable if after filtering the data the uncertainty in the slope calculation is less than 0.009 %.

5. Comparison to NISTs Existing LFSs

The 15 kg/s LFS was validated using two of NIST's existing liquid flow standards. These consisted of the 65 kg/s LFS [5], which is a larger dynamic, gravimetric standard, and the smaller 2.5 L/s LFS [6], a piston prover. Figure 1 shows the comparison of calibration data of a 5 cm and a 2.54 cm coriolis mass flow meter. The larger meter was calibrated on the 15 kg/s LFS and the 65 kg/s LFS. The smaller meter was calibrated on the 15 kg/s LFS and the 2.5 L/s LFS. The error bars for each LFS is calculated by the expanded uncertainty of that standard (*i.e.*, 0.021 % for the 15 kg/s LFS, 0.033 % for the 65 kg/s LFS, and 0.064 % for the 2.5 L/s LFS) root-sum-squared with the standard deviation of the mean of a minimum of three repeated measurements. The 15 kg/s LFS agrees with the existing standards well within their uncertainties.

Figure 7 shows the degree of equivalence (E_n numbers) comparison results, which validate the claimed measurement capabilities of all three of the NIST LFS's as demonstrated by the magnitude of E_n being less than unity. The degree of equivalence is calculated by:

$$E_n = \frac{100 \left(\frac{\dot{m}_{15 \text{ kg/s LFS}}}{\dot{m}_{\text{MUT}, 15 \text{ kg/s}}} / \frac{\dot{m}_{\text{STD}}}{\dot{m}_{\text{MUT}, \text{STD}}} - 1 \right)}{\sqrt{U_{15 \text{ kg/s LFS}}^2 + U_{\text{STD}}^2 + 2\sigma_{\text{TS}}^2}}, \quad (14)$$

where \dot{m} is the mass flow, the subscript STD denotes either the 65 kg/s or the 2.5 L/s LFS, and U is the expanded uncertainty of the standards expressed in percent, and σ_{TS} is the reproducibility of the respective transfer standard. Both the 5 cm and the 2.5 cm transfer standards had a reproducibility of $\sigma_{\text{TS}} = 0.021$ %, which is calculated by the standard deviation of repeated measurements made on the 15 kg/s LFS before and after the comparison.

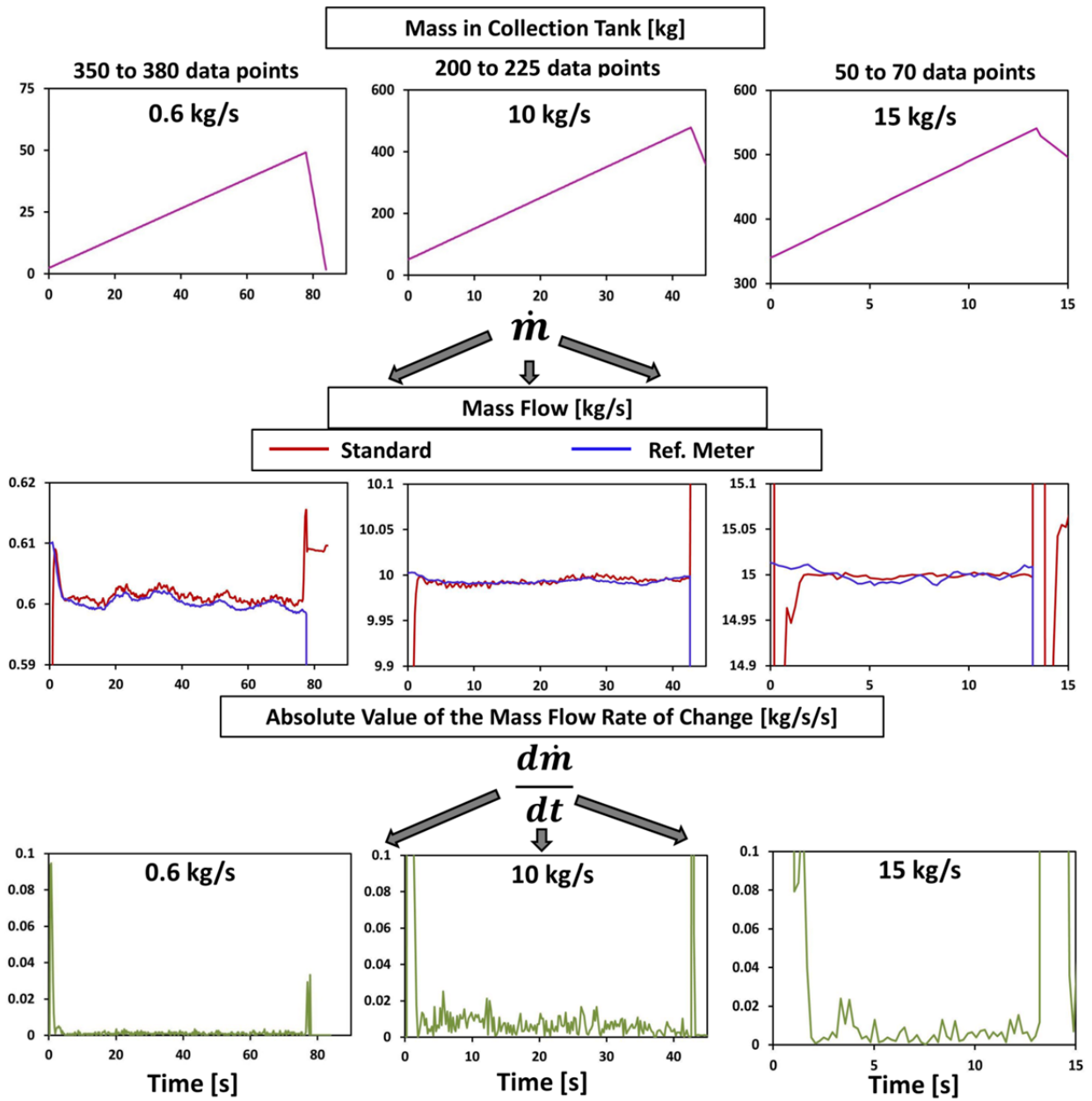


Fig. 6. Illustration of data selection.

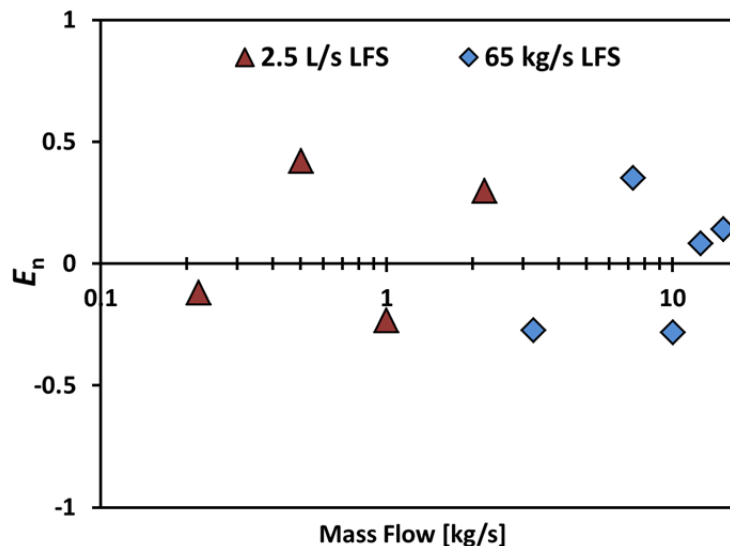


Fig. 7. Degree of equivalence for two meters used in a comparison between the 15 kg/s LFS and the 65 kg/s LFS (blue diamonds) and the 2.5 L/s LFS (red triangles). A 5 cm and a 2.5 cm coriolis meter was used for comparison with the 65 kg/s LFS and the 2.5 L/s LFS, respectively.

6. Uncertainty Analysis of the LFS

The significant factors that contribute to the uncertainty in the mass flow measurement of the 15 kg/s LFS are: 1) the determination of the slope from the scale mass vs. time data, 2) the buoyancy correction of the scale reading, and 3) mass storage in the connecting volume. The Monte Carlo method [4] was used to determine the overall uncertainty of the 15 kg/s LFS because of the complexity in performing an analytical analysis of Eqn. 9 to determine sensitivity coefficients for each variable. The expanded uncertainty is 0.021 % over the flow range of 15 kg/s to 0.22 kg/s. The following sections explain the contributing components to the mass flow uncertainty.

6.1 Slope determination

The weigh scale was calibrated over the range of 90 kg to 635 kg in January of 2013 and in August of 2014. The scale is linear and a single factor is used to correct its readings. The calibration factor determined in 2014 agreed with that in 2013 within 0.0003 %. Calibration of the weigh scale used a sequential, incremental loading method based on two 45 kg weights calibrated at NIST using the 65 kg weight set and 60 kg mass comparator from NIST's Volume calibration service [8]. The calibration procedure requires replacing steel weight of the mass standards with water and then adding mass standards to sequentially progress across the scale range. This is done because a sufficient number of mass standards are not available to complete the calibration without partial water fills. This method has been used to calibrate the weigh scale used in NIST's Water Flow calibration service via the 65 kg/s LFS, and is well documented within the Special Publication for that service [8]. A fit of the calibration data from the weigh scale to a straight line yields fit residuals within 0.00024 %. The resolution of the scale is 20 g. Therefore, the scale resolution is what determines the uncertainty in a single scale reading. However, because the resolution introduces noise in the scale readings that are centered on the mean value and because we are computing a slope from consecutive readings, the error introduced by the lack of resolution contributes $< 10^{-7}$ % to the overall uncertainty in the mass flow measurement.

The calculation of the slope (Eqn. 9) from the scale mass reading versus time data is the largest contributor to the uncertainty in the 15 kg/s LFS. It contributes more than 93 % to the

overall measurement uncertainty. The maximum allowed standard error in the slope calculation for a measurement to be considered for a calibration point is 0.009 %. To calculate the standard error of the slope, it is necessary to compute the standard error of regression, which is given by:

$$u_{\text{reg}} = \left[\frac{\sum_{i=1}^N (M_i - \dot{m}_{\text{LFS}} t_i - c)^2}{N - 2} \right]^{1/2} \quad (15)$$

and the normalization factor [4] S_{xx} :

$$S_{xx} = \sum_{i=1}^N t_i^2 - \frac{(\sum_{i=1}^N t_i)^2}{N}. \quad (16)$$

Combining Eqns. 15 and 16 gives the standard error of the slope $u_{\dot{m}_{\text{LFS}}}$:

$$u_{\dot{m}_{\text{LFS}}} = \left(\frac{u_{\text{reg}}^2}{S_{xx}} \right)^{1/2}. \quad (17)$$

Equation 15 estimates the standard deviation of the curve fit residuals of the M_i versus t_i data, and Eqn. 17 gives the standard error of the slope assuming no uncertainty in the t_i values.

The size of the boxcar filter (B) on the slope calculation was also tested. For the data considered here, the size of the filter did not alter the slope and contributes negligible measurement uncertainty.

6.1.1 Timing interval

The assumption that t_i values have no uncertainty is a good approximation. Tests of the scale oscillator and the data acquisition clock over a period of three days on three separate occasions showed a maximum deviation of 0.16 ms / hr, or < 0.00005 % standard uncertainty in the time increment per data point; which accounts for < 0.00002 % of the overall uncertainty of the standard.

6.2 Buoyancy corrections

The mass readings of the weigh scale are buoyancy corrected as shown in Eqn. 7. Buoyancy corrections applied to the scale mass readings are necessary for low uncertainty measurements. These corrections contribute up to 4.4 % to the overall measurement uncertainty. The density of air is given by [9]:

$$\rho_{\text{air}} \left[\frac{\text{kg}}{\text{m}^3} \right] = \frac{3.4848 \times 10^{-3}}{T} \left[P - 6.65287 \times 10^8 RH e^{(-5315.56/T)} \right], \quad (18)$$

where P is the atmospheric pressure in Pa, RH is the relative humidity expressed in percent, and T is the air temperature in K. The standard uncertainties in P , RH and T are 0.3 %, 10 %, and 0.07 %, respectively, as determined by control charts generated from years of calibrations by NIST working standards. This leads to a standard uncertainty of 0.31 % in ρ_{air} .

The density of water at 101 kPa is given by [10]:

$$\rho_{\text{water}} \left[\frac{\text{kg}}{\text{m}^3} \right] = a_5 \left[1 - \frac{(T+a_1)^2(T+a_2)}{a_3(T+a_4)} \right], \quad (19)$$

where T is the water temperature in the collection tank in °C, a_1 (°C) = -3.983035 ± 0.00067 , a_2 (°C) = 301.797 , a_3 (°C²) = 522528.9 , a_4 (°C) = 69.34881 , and a_5 (kg/m³) = 999.974950 ± 0.00084 . The stated standard uncertainties for a_1 and a_5 are the only significant contributors for the computed value of pure water. The uncertainty in the tank water temperature is 0.33 %. This uncertainty comes from the calibration of the sensor (10 mK) and an added uncertainty of 1 K for spatial variance in the collection tank. The 1 K was determined from temperature measurements made from the sensors in the 15 kg/s LFS and the environmental monitor. An added standard uncertainty of 0.01% was used for the density of the water used to validate the 15 kg/s LFS because tap water was used. This leads to a standard uncertainty of 0.03 % in ρ_{water} .

6.3 Mass storage in the connecting volume

The mass flow in the connecting volume between the MUT and the collection tank is computed continuously during a calibration. The contribution of this calculation to the overall measurement uncertainty is less than 3 %.

The density of water in the connecting volume is calculated as a function of both temperature and pressure:

$$\rho_{\text{water}} \left[\frac{\text{kg}}{\text{m}^3} \right] = \rho_{\text{ref}} [1 - \beta(T - T_{\text{ref}}) + \kappa(P - P_{\text{ref}})], \quad (20)$$

where ρ_{ref} is the density of water at 294 K and 101 kPa, β is the thermal expansion coefficient, and κ is the isothermal compressibility factor for water. The standard uncertainty in the reference density is 0.0083 % [10]. β (0.0002 / °C) is calculated from the best fit line to calibration data from 19 °C to 23 °C and has a standard uncertainty of 0.0015 % for pure water. REFPROP [11] is used to compute κ for pure water. A standard uncertainty of 10 % was given to both β and κ because the fluid in this work is tap water.

The pressure and temperature measurements in the connecting volume have uncertainties due to calibration and spatial resolution. The standard uncertainty in the temperature and pressure due to calibration is 0.01 % and 0.1 %, respectively. The uncertainty in the temperature and pressure measurements due to spatial resolution is determined by the difference in readings between the two temperature sensors in the connecting volume and from the difference of the pressure sensor downstream of the MUT and the atmospheric pressure, respectively. A 0.1 K difference has been observed between the two temperature sensors and therefore, this value is taken as the upper limit in a rectangular distribution. The standard uncertainty in the temperature is therefore 0.06 %. A difference of 445 kPa has been observed between the pressure at the MUT outlet and the atmospheric pressure. This value is taken as the upper limit of a rectangular distribution and gives the pressure measurements a standard uncertainty as high as 58 %. The standard uncertainty in the connecting volume density is therefore within 0.05 %.

The size of the connecting volume was determined by measuring the length and diameter of the piping between the MUT and the pipe exit leading to the collection tank (Fig. 2). This volume is approximately 0.0095 m³ with a standard uncertainty of 10 %. Therefore, the standard uncertainty in the connecting volume mass storage is within 17 %.

7. Summary

NIST's 15 kg/s LFS is a fully automated, dynamic system that utilizes a PID control loop for flow stability. The LFS uses only the dynamic method in order to avoid the expense and

complexity of a flow diverter. It can measure flows from 0.22 kg/s to 15 kg/s with an expanded uncertainty of 0.021 %. The standard has been validated using NIST's existing primary LFSs.

The uncertainty in the mass flow measurement comes primarily from the slope calculation from the mass vs. time data generated from the weigh scale, accounting for more than 93 % of the overall uncertainty. The two other significant contributing factors are the buoyancy correction to the scale readings and the mass storage in the connecting volume between the MUT and the standard's collection tank. The buoyancy correction accounts for up to 4.4 % of the overall uncertainty. The contribution from the connecting volume mass storage to the overall uncertainty is as much as 3 %.

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