

Uncertainty of nuclear counting

This content has been downloaded from IOPscience. Please scroll down to see the full text.

2015 Metrologia 52 S3

(<http://iopscience.iop.org/0026-1394/52/3/S3>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.6.99.22

This content was downloaded on 28/09/2015 at 20:07

Please note that [terms and conditions apply](#).

Uncertainty of nuclear counting

S Pommé¹, R Fitzgerald² and J Keightley³

¹ European Commission, Joint Research Centre, Institute for Reference Materials and Measurements (IRMM), Retieseweg 111, B-2440 Geel, Belgium

² National Institute of Standards and Technology, Gaithersburg, MD 20899-8462, USA

³ National Physical Laboratory, Hampton Road, Teddington, Middlesex, TW11 0LW, UK

E-mail: stefaan.pomme@ec.europa.eu

Received 24 October 2014, revised 21 January 2015

Accepted for publication 26 January 2015

Published 22 May 2015



CrossMark

Abstract

Nuclear counting is affected by pulse pileup and system dead time, which induce rate-related count loss and alter the statistical properties of the counting process. Fundamental equations are presented to predict deviations from Poisson statistics due to non-random count loss in nuclear counters and spectrometers. Throughput and dispersion of counts are studied for systems with pileup, extending and non-extending dead time, before and also after compensation for count loss. Equations are provided for random fractions of the output events, applicable to spectrometry applications. Methods for loss compensation are discussed, including inversion of the throughput equation, live-time counting and loss-free counting. Secondary effects in live-time counting are addressed: residual interference from pileup in systems with imposed dead times and errors due to varying count rate when measuring short-lived radionuclides.

Keywords: nuclear counting, dead time, pulse pileup, radionuclide metrology, counting statistics

(Some figures may appear in colour only in the online journal)

1. Introduction

Radioactivity measurements are most often based on the counting of individual interactions of emitted subatomic particles with the sensitive part of a detector. The counting of events is therefore a basic operation which has been extensively studied (see e.g. [1–23] and references therein). In particular, consideration of count loss through dead time and pulse pileup is of vital importance in any type of precision work. This is certainly the case with reference measurements which should be characterized by great accuracy and precision of the measured value as well as full statistical control of the experimental uncertainty.

At low and moderate count rates, the statistical uncertainty is readily available by taking the square root of the number of measured events, as follows conveniently from Poisson statistics. However, when the incoming event rate is very high,

count loss not only lowers the observed count rate, but also influences the statistical behaviour of the accepted counts by perturbing their time-interval density distribution. Therefore, at high count rates, interventions are required to reproduce the incoming event rate and to regain statistical control (see e.g. [1, 17, 23]).

Count loss in nuclear spectrometry through dead time and pulse pileup is a recognized problem. Typical contributors are the pulse height processing time in the analog-to-digital converter (ADC), the reset time in certain pre-amplifiers, the summing of coincident pulses due to their finite time width, the (optional) pileup rejection of closely spaced pulse pairs, and the paralyzing effects of large saturated pulses (e.g. from cosmic rays), electronic undershoots, and possible saturation of data transfer and storage (e.g. with digital acquisition in list mode). Several techniques have been applied to compensate the count loss, such as system live-time counting techniques [24, 25], including pileup correction [26], pulser techniques [27, 28], and ‘loss-free’ counting [29–34]. The simplest method of dead-time correction utilizes a live-time clock where an accurately timed pulse train is counted only in those



Content from this work may be used under the terms of the [Creative Commons Attribution 3.0 licence](https://creativecommons.org/licenses/by/3.0/). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

time intervals when the measuring system is free to accept or record events.

In the field of primary standardization of activity [35], counting is usually performed with a single-channel system and an artificial dead time of selectable length and type (extending or non-extending) imposed on every detected event. This imposed dead time dominates the effects of instrumental dead time, yielding greater control over the system live time. Even though the major contribution to count loss may be compensated for, additional extension of the artificial dead time may be required to cover long saturation pulses, delayed signals from metastable states and afterpulses (e.g. due to delayed fluorescence generated in liquid scintillation counting systems), pickup of electrical oscillations, etc.

Ultimate precision requires additional consideration of secondary effects. For example, in systems with imposed dead time there is a residual count loss effect due to pulse pileup which is mathematically equivalent to a series arrangement of dead times [36–45]. Another deviation to normal counting statistics occurs if the Poisson process is inhomogeneous, e.g. in the case of short-lived radionuclides for which the activity varies significantly during a measurement [7, 46–50]. Adaptive ‘loss-free’ counting systems can deal with the simultaneous variation of the count rate and the live-time fraction [29–34], but classical live-time counting applying an average dead-time correction is in error [44, 45].

In this work, three basic types of count loss mechanisms, their effects on a stationary Poisson process in the time domain, and statistical uncertainty formulas are discussed for both integral counting and for spectrometric analysis of a fraction of the event spectrum. Equations are presented to gain statistical control over nuclear counting, applicable from routine spectrometry to high-level standardization work.

2. Basics of nuclear counting

2.1. Pileup and dead time

In the literature it is often stated that there are two basic types of dead time, ‘extending dead time’ (EDT) and ‘non-extending dead time’ (NEDT), and that all nuclear counters can be described by one of these two types, or by a combination of them (see e.g. [51, 52]). However, this does not account for the statistical effects caused by pulse pileup (PU), which is the main loss mechanism in contemporary spectrometry chains [11, 13, 20].

‘Non-extending’ means that the dead-time period, τ_n , is not prolonged by the arrival of a new pulse during the dead-time period. The dead time created by an ADC is a typical example. For the so-called ‘paralysable’ systems, with dead time of the ‘extending’ type, each incoming event prolongs the system dead time by an amount τ_e . A clear distinction should be made between count loss by ‘pileup (rejection)’ and ‘extending dead time’. Pulse pileup occurs when a new pulse from the preamplifier is fed through the shaping amplifier before the ADC has had the chance to complete processing the previous pulse. In such cases, an amplifier-ADC combination with pileup rejection has the ability to inhibit

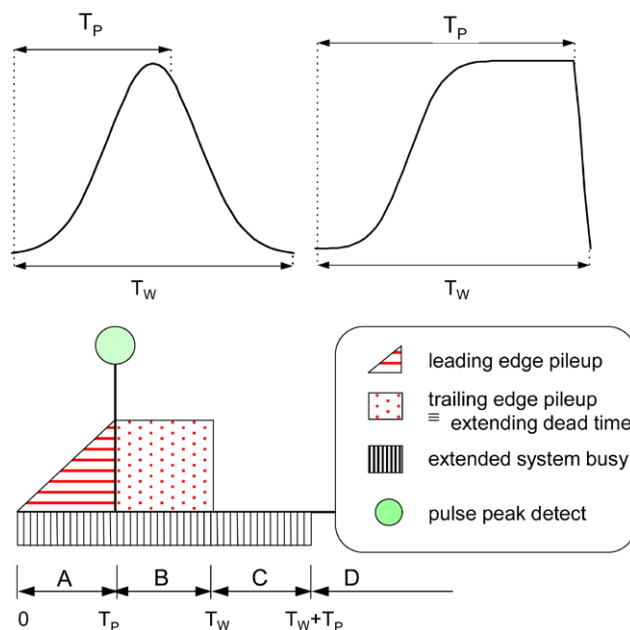


Figure 1. Typical pulse shapes and timing parameters involved in spectrometry with a semi-Gaussian (left) and a gated integrator (right) pulse shaping amplifier. T_W represents the pulse width and T_P the leading edge. Below: schematic representation of a detector pulse and its associated event loss mechanisms. Time zone A corresponds to pulse pileup, zone B to dead time, zone C is an extension of the system busy time (cf. LFC) to compensate for pileup and D is system live time.

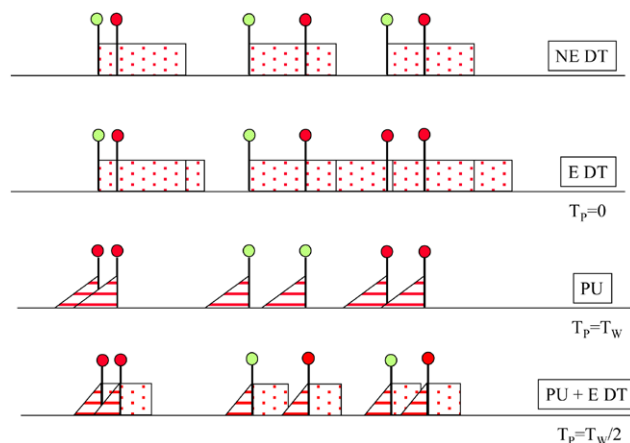


Figure 2. Event train passing through counters having different loss mechanisms; NEDT = non-extending dead time, EDT = extending dead time and PU = pileup rejection. The case of combined PU and EDT corresponds to a spectrometer with Gaussian pulse shaping and fast ADC with negligible dead time. The dark (red) circles correspond to events which are not counted.

the ADC from processing the composite pulses. Contrary to the case of ordinary dead-time effects, the ‘first’ incoming pulse is lost for spectrometric analysis, either by a built-in electronic pileup rejection system or by the pulse count being assigned to another spectrum channel [13]. Therefore we are facing at least three different types of pulse loss distortions of a Poisson process, each requiring a particular statistical treatment.

In figure 1 the pulse shapes of a semi-Gaussian (left) and a gated integrator (right) amplifier are shown, as well as a

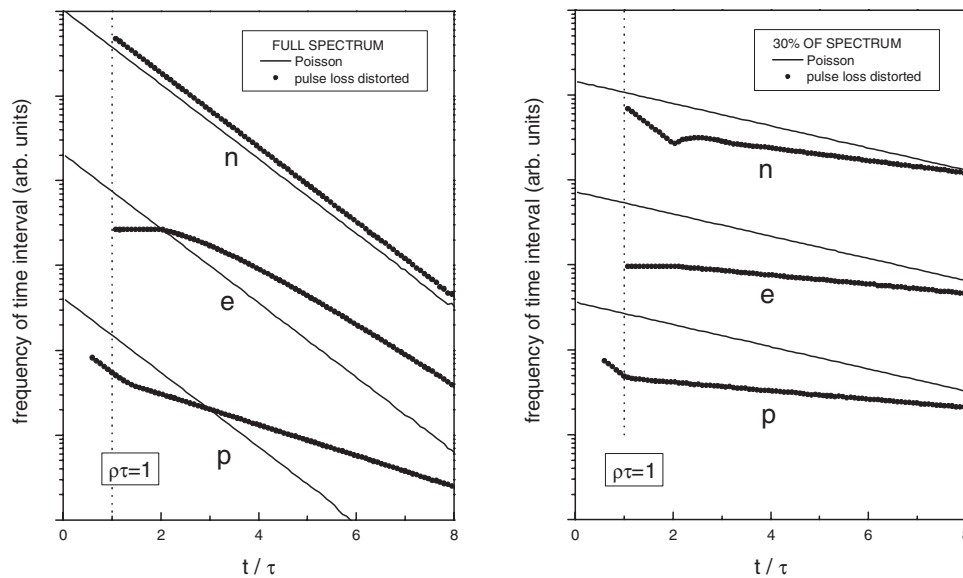


Figure 3. Typical time-interval distribution of consecutive events originating from a Poisson process at an incoming count rate of $\rho = 1/\tau$ and counted under different regimes of event loss; NEDT (n), EDT (e) and pileup (p); (left) for all incoming pulses, (right) for an arbitrary 30% fraction of the pulses.

schematic pulse representation (below). Pileup, with loss of at least two signals, occurs when the arrival of a pulse falls within the ‘leading edge’ of the previous pulse. This corresponds to the time zone ‘A’ in the schematic graph and has a characteristic duration T_P , i.e. from when the pulse rises above the noise level up to the peak height detection by the ADC. If the pulse falls within the ‘trailing edge’ portion, time zone ‘B’, the first pulse is successfully analysed and only the second pulse is lost. Therefore, pileup in the trailing edge is fully equivalent with EDT of duration $T_W - T_P$, where T_W is the pulse width.

Double count loss by pileup can also occur in the absence of a pileup rejector. In spectrometry, small fractions of the pulse height spectrum are usually considered. The composite pulse of coincident events will (most often) fall outside the region of interest (ROI) and hence be lost in a comparable manner as with pileup rejection. However, when performing ‘integral counting’ of all pulses above a threshold level, the summed pulse is valid as one count and pileup is equivalent to EDT. This applies to most of the primary standardization methods [35] and Geiger–Müller counters.

Figure 2 illustrates how the different loss mechanisms interact with the same pulse train. The moment of peak height detection of a pulse is represented by a circle. The triangles in front of the peaks represent their leading edge. The rectangle behind the peak represents dead time; it can correspond to the trailing edge of the pulse, equivalent to EDT, or to the NEDT of an ADC. Even though the same ‘effective dead time’ ($\tau = T_P + T_W$) (see equation (3)) is applied to all types of count loss, different events are counted or lost.

2.2. Time interval distributions

To understand counting statistics, i.e. the probability for observing a given number of events, one should look upon

the counting process as a sequence of events developing in time, where loss mechanisms directly affect the statistical distributions of the time intervals between successive events. These time intervals are independent, and form a renewal process [1, 5]. Multiple intervals, i.e. the waiting time for N subsequent events, are obtained from a N -fold convolution of the single time-interval density distributions. The transition from interval densities to counting probabilities can be done via differentiation of the corresponding Laplace transform (see e.g. [5, 13, 16]). Experimental as well as theoretical work for EDT or NEDT and their combinations can be found in [1, 5, 8, 52]. More recent work [11, 13, 16] includes the effects of pileup and limited fractions of spectra.

In figure 3, the time distortion of a Poisson process is shown for NEDT, EDT and ‘PU’ at an incoming count rate of $\rho = 1/\tau$. NEDT preserves the exponential shape of the interval distribution, except that events falling within the dead time τ are eliminated. EDT also perturbs the slope of the interval distribution after the dead-time period τ , which is most clearly visible in the time interval $[\tau, 2\tau]$ and piecewise fades out at higher time intervals. PU has yet another effect on the time-interval distribution and does not preserve the slope of the original Poisson process.

Figure 3 also shows the time-interval distributions for events belonging to a 30% fraction of the total spectrum, selected randomly in the time domain. Due to the lower probability of occurrence, the time-interval distribution is flatter than for the full spectrum. The perturbations—mainly noticeable at low time intervals—are therefore of less influence, which is a sign of convergence towards Poisson statistics. Other points of interest are the altered shape of the distortions, which is most peculiar in the case of NEDT (for further information see graphs in [16]), and the non-conservation of the exponential shape of the interval distribution.

2.3. Detector throughput

The time-interval distribution between successive counts determines the counting statistics. Expressions for the expectation value of the number of counts obtained during a fixed measuring time can be derived from the first moment of the N -fold time-interval distribution. For input pulses forming a Poisson process with rate ρ , the expected output rate R for a counter with non-extendable dead time τ_n is [1, 5]

$$R = \frac{\rho}{1 + \rho\tau_n}, \quad (1)$$

whereas with extendable dead time τ_e it is [1, 5]

$$R = \rho e^{-\rho\tau_e}, \quad (2)$$

and in the case of pileup it is also exponential [13]:

$$R = \rho e^{-\rho\tau_p}, \quad (3)$$

in which $\tau_p = T_p + T_w = 2T_p + (T_w - T_p)$. As far as the average throughput is concerned, pileup is equivalent to EDT in which the resolving time between two pulses is counted twice. The pulse width may vary as a function of pulse height [13], but in general the statistical equations in this paper apply to the mean characteristic width.

Correcting for count loss can be done by measuring in ‘live-time mode’, implicitly relying on the obtained real-time to live-time ratio correction factor, T_R/T_L :

$$\hat{\rho} = R \frac{T_R}{T_L} \quad (4)$$

or in the old-fashioned ‘real-time mode’ by explicitly applying the inverse throughput formula $X^{-1}(R)$ (equations (1)–(3)), assuming that the characteristic dead time is known. The expression for NEDT is straightforward:

$$\hat{\rho} = \frac{R}{1 - R\tau_n}. \quad (5)$$

For PU and EDT, the inversion of the throughput formula (equations (2) and (3)) is solved as a special case of Takacs’ formula [8]:

$$\hat{\rho} = R \sum_{k=0}^{+\infty} \frac{(k+1)^{k-1}}{k!} (R\tau_p)^k. \quad (6)$$

The dead-time correction method is sensitive to uncertainty on the characteristic dead times, which propagate to the corrected rate via

$$\frac{\sigma(\hat{\rho})}{\hat{\rho}} \approx \hat{\rho} \tau_n \frac{\sigma(\tau_n)}{\tau_n} \quad (7)$$

for NEDT (derivation in section 2.5), and

$$\frac{\sigma(\hat{\rho})}{\hat{\rho}} \approx \frac{\hat{\rho} \tau_p}{1 - \hat{\rho} \tau_p} \frac{\sigma(\tau_p)}{\tau_p} \quad (8)$$

for PU and EDT (derivation in section 2.6). A relative error of 10% on the characteristic dead times at a count rate of $\rho\tau = 0.3$ leads to errors of 3% (NEDT) and 4.3% (PU, EDT) on the count rate estimate.

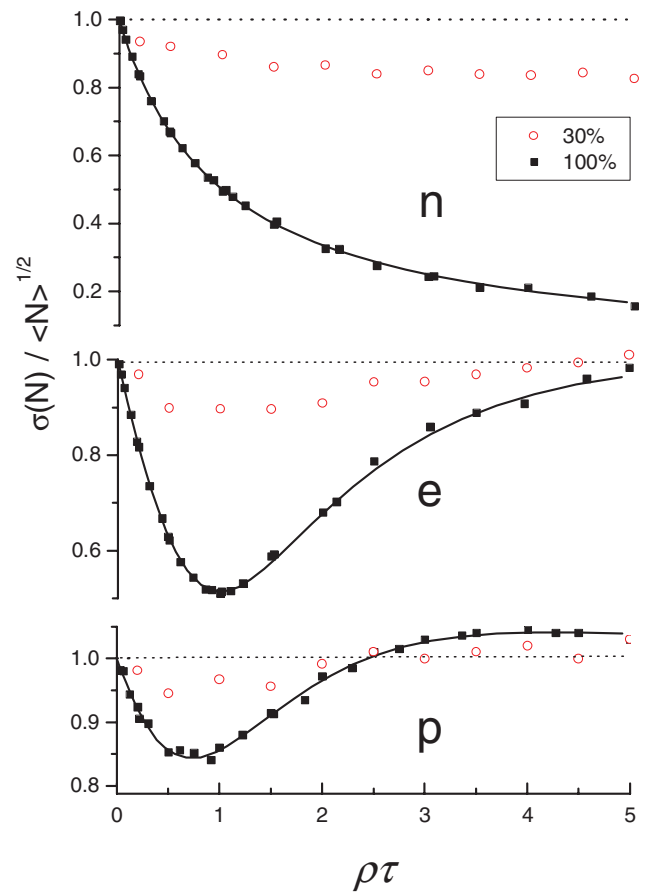


Figure 4. Standard deviation of counting a Poisson process with NEDT (n), EDT (e) and pileup (p) for the full spectrum and for a 30% subset. The squares ($f = 1$) and hollow circles ($f = 0.3$) are simulation results and the full lines represent theoretical calculations.

2.4. Dispersion

At high event rates, the scatter on the number of counted events N is relatively lower than for a pure Poisson process. In figure 4 the standard deviation is plotted as a function of the input rate for the three basic types of count loss. The functions were derived theoretically from the first two moments of the Laplace transform of the N -fold time interval distribution [5, 13]. The asymptotic counting uncertainty in a counter with NEDT is [1, 5]

$$\frac{\sigma(N)}{\sqrt{N}} = \frac{1}{1 + \rho\tau_n}, \quad (9)$$

and in the case of EDT it is [1, 5]

$$\frac{\sigma(N)}{\sqrt{N}} = \sqrt{1 - 2\rho\tau_e e^{-\rho\tau_e}} \quad (10)$$

on the condition that the measurement time t is much longer than the dead time τ ($t \gg \tau$).

Whereas the effect of dead time on the count rate dispersion has long been known, the mathematical solution for counting with pileup (rejection) was derived only fairly recently [13]:

$$\frac{\sigma(N)}{\sqrt{N}} = \sqrt{1 + 2e^{-\rho T_W} \{1 - [1 + \rho(T_P + T_W)] e^{-\rho T_P}\}}. \quad (11)$$

The $\sigma(N)/\sqrt{N}$ curve for pileup in figure 4 shows a resemblance with the one for EDT, but the deviation from $\sqrt{N}$ is smaller and even becomes positive for $\rho\tau_p > 2.5$ (here $\tau_p = 2T_P$). Such effects cannot be reproduced by a generalized dead time as proposed by Albert and Nelson [1, 3, 8, 53].

As a rule, counting with significant non-random count loss (in the time domain) is underdispersed relative to Poisson statistics. However, after proper correction for the count loss, the result is always overdispersed. The true input rate is found only after correction for count loss. It is a mistake to assume that one can directly apply the relative uncertainties represented in equations (9)–(11), since the proper uncertainty propagation factor has to be taken into account. Consequently, the uncertainty on the loss-corrected number of counts, N_t , always exceeds the value expected from Poisson statistics by at least the square root of the inverse throughput factor (equations (5) and (6)):

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \geq \sqrt{e^{\rho\tau}} \quad (12)$$

for counters with PU and EDT, and

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \geq \sqrt{1 + \rho\tau} \quad (13)$$

for counters with NEDT. Explicit equations are derived in the next section.

2.5. Uncertainty after count-loss correction (NEDT)

Measurements performed with a real-time clock, for a fixed measurement time $t_m = T_R$, have to be corrected for the non-linearity of the detector throughput to obtain the true count rate. The variance of the calculated counts N_t is obtained by uncertainty propagation on the variance of the measured counts N . The loss correction can be done either by applying the ratio between the measurement real time and the observed system live time, $\hat{N}_t = NT_R/T_L$, or by relying on the inverse throughput formula (equations (5) and (6)), $\hat{N}_t = X^{-1}(N)$, if the characteristic dead time τ of the counter is known.

Both methods yield equally precise results for NEDT. The uncertainty from the inversion of the throughput formula via equation (5) is calculated from [23]:

$$\begin{aligned} \sigma^2(N_t) &\approx \left(\frac{\partial N_t}{\partial N}\right)^2 \sigma^2(N) = \left(\frac{\partial N}{\partial N_t}\right)^{-2} \sigma^2(N) \\ &= (1 + \rho\tau_n)^4 \sigma^2(N), \end{aligned} \quad (14)$$

which, in combination with equation (9) for the dispersion, leads to the statistical uncertainty on the loss-corrected counting result in the case of NEDT:

$$\frac{\sigma(N_t)}{\sqrt{N_t}} = \sqrt{1 + \rho\tau_n}. \quad (15)$$

A systematic component has to be included for the uncertainty on the characteristic dead time using the propagation formula in equation (7), which can be derived directly from

$$\sigma^2(\rho) \approx (\partial\rho/\partial\tau_n)^2 \sigma^2(\tau_n) = \rho^4 \sigma^2(\tau_n).$$

The alternative method to correct for count loss, applying the real-time to live-time ratio T_R/T_L via equation (4) leads to a similar result. One applies the uncertainty formula for the product of two interdependent variables [23]

$$\frac{\sigma^2(N_t)}{N_t^2} = \frac{\sigma^2(N)}{N^2} + \frac{\sigma^2(T_R/T_L)}{(T_R/T_L)^2} + 2cor \frac{\sigma(N)}{N} \frac{\sigma(T_R/T_L)}{T_R/T_L}, \quad (16)$$

in which *cor* represents the correlation factor between the number of counts N and the T_R/T_L ratio, which is equal to one since $T_L = T_R - N\tau_n$ in the case of NEDT. Considering that

$$\frac{\sigma^2(T_R/T_L)}{(T_R/T_L)^2} = \frac{\sigma^2(T_L)}{T_L^2} = \frac{\tau^2 \sigma^2(N)}{T_L^2} = \frac{\sigma^2(N)}{N^2} (\rho\tau_n)^2, \quad (17)$$

one obtains from equation (16) the same asymptotic uncertainty as in equation (15). In practice this means that with NEDT the purely statistical dispersion (excluding systematic errors) on the estimated input rate $\hat{\rho} = N_t/T_R = N/T_L$ can be calculated directly from the number of counted events relative to the live-time:

$$\sigma(\hat{\rho}) = \sigma\left(\frac{N}{T_L}\right) = \frac{\sqrt{N}}{T_L}. \quad (18)$$

2.6. Uncertainty after count-loss correction (PU + EDT)

The throughput inversion method is not equivalent to live-time correction in the case of PU and EDT. Better statistical accuracy can be obtained by using the T_R/T_L ratio when available, since this contains additional information on the original count rate.

For a combination of PU and EDT ($\tau_p = T_P + T_W$), the uncertainty of the loss-corrected count integral, $\hat{N}_t = X^{-1}(N) = Ne^{\rho\tau_p}$ (equation (6)) is calculated from [23]

$$\begin{aligned} \sigma^2(N_t) &\approx \left(\frac{\partial N}{\partial N_t}\right)^{-2} \sigma^2(N) \\ &= \left(\frac{1 - \rho\tau_p}{e^{\rho\tau_p}}\right)^{-2} \{1 + 2e^{-\rho T_W} [1 - (1 + \rho\tau_p)e^{-\rho T_P}]\} N, \end{aligned} \quad (19)$$

which leads directly to [23]

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \approx \sqrt{\frac{e^{\rho\tau_p} + 2[e^{\rho T_P} - (1 + \rho\tau_p)]}{(1 - \rho\tau_p)^2}}. \quad (20)$$

The specific case of EDT follows from equation (20), by setting $T_P = 0$:

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \approx \sqrt{\frac{e^{\rho\tau_e} - 2\rho\tau_e}{(1 - \rho\tau_e)^2}}. \quad (21)$$

A systematic component has to be included for the uncertainty on the characteristic dead time using the propagation

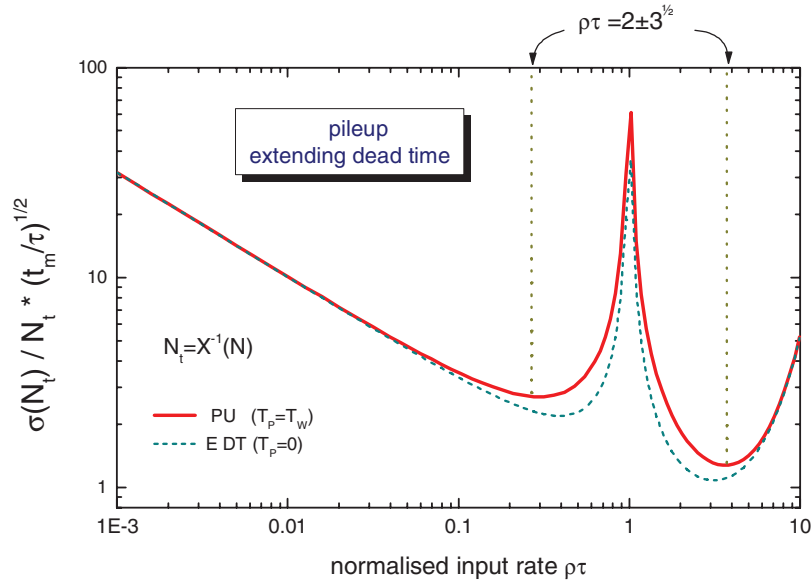


Figure 5. Relative statistical uncertainty on the number of counts corrected for loss by pileup or EDT using the inverse throughput formula, as a function of the normalized input rate.

formula in equation (8), which can be derived in a similar way as equation (19) via $\sigma^2(\rho) \approx (\partial R / \partial \rho)^{-2} (\partial R / \partial \tau_p)^2 \sigma^2(\tau_p)$
 $= ((1 - \rho\tau_p)e^{-\rho\tau_p})^{-2} (-\rho^2 e^{-\rho\tau_p})^2 \sigma^2(\tau_p)$.

Figure 5 shows theoretical uncertainties from equations (20) ($T_P = T_W$) and (21) ($T_P = 0$), pertaining to the count integral as a function of the input rate. At low input rates, the relative uncertainty is proportional to the inverse square root of the number of counts. Clear deviations from this trend occur at high count rates exceeding $\rho\tau = 0.1$. A singularity occurs at $\rho\tau = 1$, which is merely a technical consequence of taking the inverse of the derivative at one point: the maximum of the throughput curve. In practice one gets a finite uncertainty, depending highly on $\sigma(N)$. The count rates corresponding to optimum counting uncertainty are situated rather close to $\rho\tau = 2 \pm \sqrt{3}$.

Equations (20) and (21) should be used with caution for low count numbers, corresponding to a high relative uncertainty $\sigma(N)$, because the derivative introduced via equation (19) can be assumed constant only for relatively small variations of N , i.e. for sufficiently large count numbers.

When applying the live-time to real-time ratio T_R/T_L , the uncertainty is again calculated from equation (16). With loss of the ‘extending’ type, there is no one-to-one relationship between the counted events N and the system live time T_L . Hence the correlation factor generally does not equal 1. The uncertainties have been studied by computer simulation. For EDT ($T_P = 0$) one finds that the standard deviation of the loss-corrected number of counts in an arbitrary ROI equals the square root of the inverse throughput factor. However, the variance is higher with pileup rejection. Moreover, the relative deviation from Poisson statistics varies as a function of the considered spectrum fraction f and the T_P/T_W ratio. The following approximation can be applied for the statistical uncertainty on the loss-corrected counts for a spectrometer with PU and/or EDT:

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \approx r \sqrt{e^{\rho\tau_p}}, \quad (22)$$

in which r is a correction factor that was derived from a detailed analysis of counting statistics in live-time mode [14]:

$$r = 1 + \frac{A}{2} \left(\frac{2 + A'}{1 + A'} \right) f, \quad (23)$$

where $A = 1 - e^{-\rho T_P}$, $A' = 1 - e^{-f\rho T_P}$ and f is the fraction of the spectrum represented by the considered ROI. Dependence of counting statistics on f is discussed in section 3 (Spectrometry). The difference in statistical accuracy of both pulse loss correction methods is demonstrated in figure 6 for different combinations of PU and EDT (cf. T_P/T_W ratio).

3. Spectrometry

3.1. Detector throughput

In routinely used spectrometers (γ ray, x ray, alpha particle, etc.), the three types of count loss can be present at the same time. For a spectrometer counting electronic pulses with a finite width T_W and leading edge T_P followed by an ADC with a characteristic NEDT T_M , the throughput has been approximated by [52]:

$$R \approx \frac{\rho}{\exp[\rho(T_P + T_W)] + \max\{0, \rho[T_M - (T_W - T_P)]\}}, \quad (24)$$

in which ρ is the input rate and R the average output rate. This formula works perfectly when only one type of count loss is involved, since it leads to the classical expressions in equations (1)–(3) in the case of extending ($R = \rho e^{-\rho T_W}$) and non-extending ($R = \rho / (1 + \rho T_M)$) dead time and also for combinations of PU with EDT ($R = \rho e^{-\rho(T_P + T_W)}$). It is not rigorous for a combination of PU and EDT with NEDT at extremely

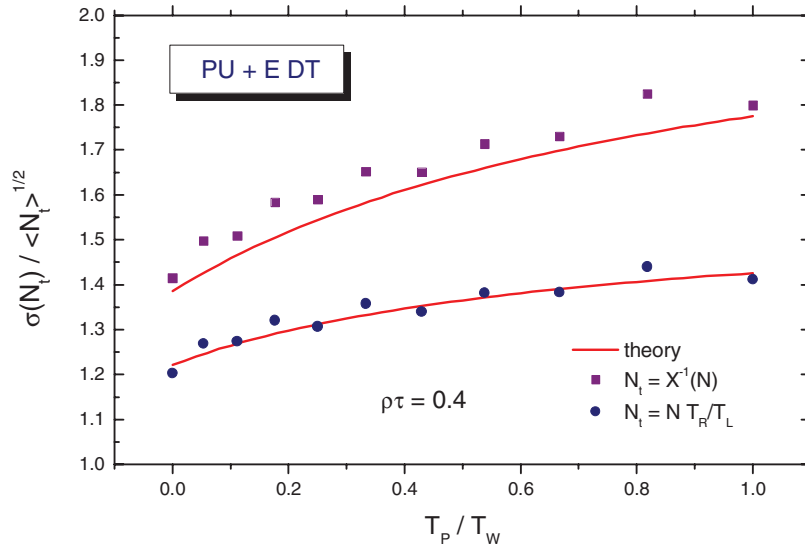


Figure 6. Theoretical and simulated standard deviation of the loss-corrected count integral in a spectrometer with various combinations of PU and EDT (cf. T_P/T_W ratio). The dispersion is larger for PU than for EDT. The live-time correction technique is systematically more precise than the inversion of the throughput formula.

high count rates. Such systems require specific calculations for dead times in series (see section 4).

3.2. Dispersion with NEDT

A spectrometer with narrow pulses and a slow ADC is considered ($T_M \gg T_W$), for which NEDT is dominant. From figure 3 it could be concluded that time-interval distributions for a fraction f of the energy spectrum is different from the full spectrum, hence also differences in statistical behaviour seen in figure 4 could be expected. Time interval distributions have been studied for random (in the time domain) fractions of the output events, with the aim of finding equations for the counting uncertainty in a ROI in the energy spectrum. This is of major concern for spectrometry measurements with a slow ADC.

The expression for the throughput for counting with NEDT in a ROI is $R = f\rho / (1 + \rho\tau_n)$, as expected from equation (1), and the asymptotic relative standard deviation on the number of counts follows rigorously from [16]:

$$\frac{\sigma(N)}{\sqrt{N}} = \frac{\sqrt{f + (1-f)(1 + \rho\tau_n)^2}}{1 + \rho\tau_n}. \quad (25)$$

In the limit of extremely small ROIs, with $f \approx 0$, the uncertainty in equation (25) approaches Poisson behaviour, i.e. $\sigma(N) = \sqrt{N}$. For the full spectrum, $f = 1$, equation (25) reduces to the well-known expression in equation (9).

An even more general formula can be applied for a ROI taken with a Wilkinson type ADC, in which the characteristic dead time increases linearly with the spectral channel number [19]:

$$\frac{\sigma(N)}{\sqrt{N}} \approx \frac{\sqrt{f + (1-f)(1 + \rho\langle\tau_n\rangle_{\neq})^2}}{1 + \rho\langle\tau_n\rangle}, \quad (26)$$

in which $\langle\tau_n\rangle_{\neq}$ is the average characteristic dead time for counts outside the ROI and $\langle\tau_n\rangle$ is the average for the full

spectrum. Equation (25) for a fixed dead time can be regarded as a special case of equation (26), in which $\langle\tau_n\rangle = \langle\tau_n\rangle_{\neq} = \tau_n$.

Computer simulation results reveal that the loss-corrected counting uncertainty with NEDT follows a surprisingly simple rule, which was confirmed with experimental tests using a Wilkinson type ADC [19]. Some evidence is shown in figure 7. The asymptotic statistical uncertainty on the loss-corrected number of counts in any ROI of a spectrum obtained by counting a Poisson process with a ‘slow’ Wilkinson ADC is given by

$$\frac{\sigma(N_t)}{\sqrt{N_t}} = \sqrt{1 + \rho\langle\tau_n\rangle}. \quad (27)$$

For spectrum channels corresponding to a long characteristic dead time, it seems that the relatively low count scatter $\sigma(N)$ is compensated by the strong correlation with the system live time (cf. application of T_R/T_L) or the observed throughput (cf. application of inverse throughput). The opposite is true for channels associated with a short dead time. In practice, equation (27) infers that the dispersion of the loss-corrected count rate in any ROI is simply calculated from the counted events per live-time in the ROI, applying equation (18).

3.3. Dispersion with pileup and EDT

Pileup (rejection) is the dominant loss mechanism in a spectrometer with a fast ADC ($T_W > T_M$). The asymptotic counting uncertainty in a ROI representing a fraction f of a spectrum obtained by a counter with pileup rejection (leading pulse edge T_P) and EDT (trailing pulse edge $T_W - T_P$) equals [13, 18]

$$\frac{\sigma(N)}{\sqrt{N}} = \sqrt{1 + 2fe^{-\rho T_W}[1 - (1 + \rho\tau_P)e^{-\rho T_P}]}, \quad (28)$$

in which $\tau_P = T_P + T_W$. The equation for EDT follows as a special case of equation (28) by introducing $T_P = 0$:

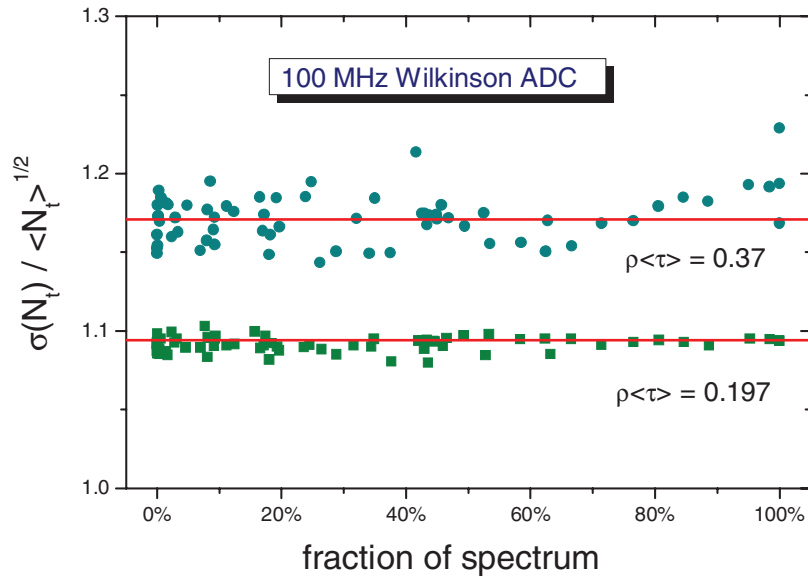


Figure 7. Standard deviation of live-time corrected counts $\sigma(N_t)$ relative to $\sqrt{N_t}$ in different parts of ^{137}Cs - ^{60}Co γ -ray spectra taken by an ‘Inspector’ multi-channel analyser with 100 MHz Wilkinson ADC at two input rates. The full line corresponds to theory, equivalent to $\sigma(N_t)/N_t = 1/\sqrt{N}$, i.e. the Poisson uncertainty on the directly counted events N .

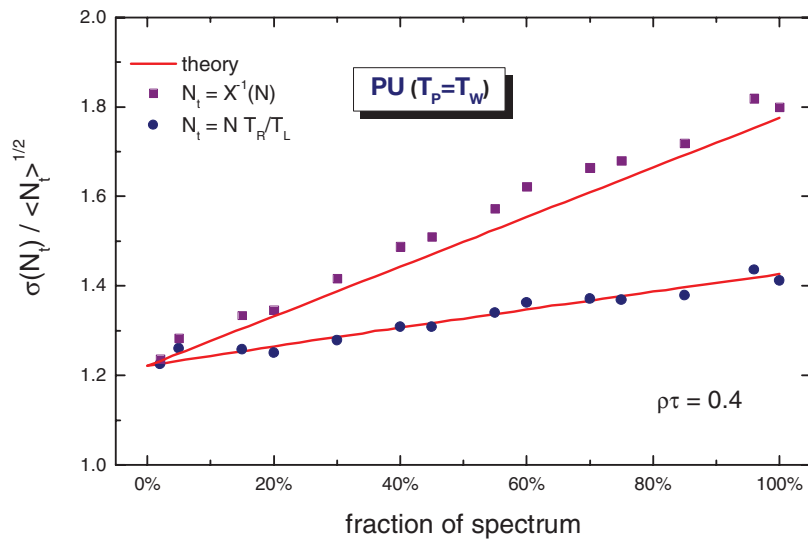


Figure 8. Dependence of the standard deviation of loss-corrected counts on spectral fraction, showing that pileup rejection broadens statistical distributions within extended live-time periods. Only in small parts of the spectrum can the uncertainty be calculated from the uncorrected number of events $\sqrt{N}$, while a correction factor r (equation (23)) is required for larger ROIs.

$$\frac{\sigma(N)}{\sqrt{N}} = \sqrt{1 - 2f\rho\tau_e e^{-\rho\tau_e}}. \quad (29)$$

Equations (28) and (29) are generalizations of equations (11) and (10), respectively. Together with equation (25) they reproduce the statistics of a 30% spectral fraction seen in figure 4.

Contrary to the case of NEDT, the dispersion of the loss-corrected count rate depends on the fraction of the spectrum represented. The dispersion for counting with live-time correction has already been presented in equations (22) and (23). When opting for the inversion of the throughput formula, a linear relationship as a function of the considered fraction ($0 < f < 1$) can be used as an approximation [23]:

$$\frac{\sigma(N_t)}{\sqrt{N_t}} \approx \sqrt{\frac{e^{\rho\tau_p} + 2[e^{\rho T_p} - (1 + \rho\tau_p)]}{(1 - \rho\tau_p)^2}} f + \sqrt{e^{\rho\tau_p}} (1 - f). \quad (30)$$

In figure 8 the statistical uncertainties obtained with both loss-correction methods, i.e. via $\hat{N}_t = X^{-1}(N)$ and $\hat{N}_t = N T_R / T_L$, are compared as a function of the size of the ROI. Live-time counting is more accurate (see also figure 6), but for small parts in a spectrum Poisson statistics is a good approximation and equation (18) can be applied.

3.4. Extended live-time clock

The Gedcke-Hale live-time clock method [52] is a modification of the simple live-time clock to include compensation for

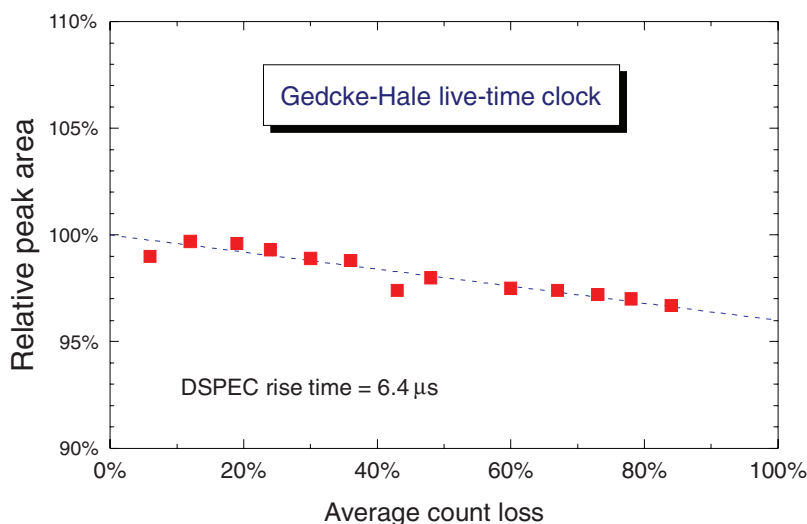


Figure 9. Relative area of the 1115.5 keV γ -ray peak of a fixed ^{65}Zn source as a function of the average count loss (varied by introducing a highly active ^{137}Cs source at variable distance to the HPGe detector) using a DSPEC (EG&G ORTEC) digital spectrometer for a live time of 200 s applying the Gedcke-Hale live-time clock.

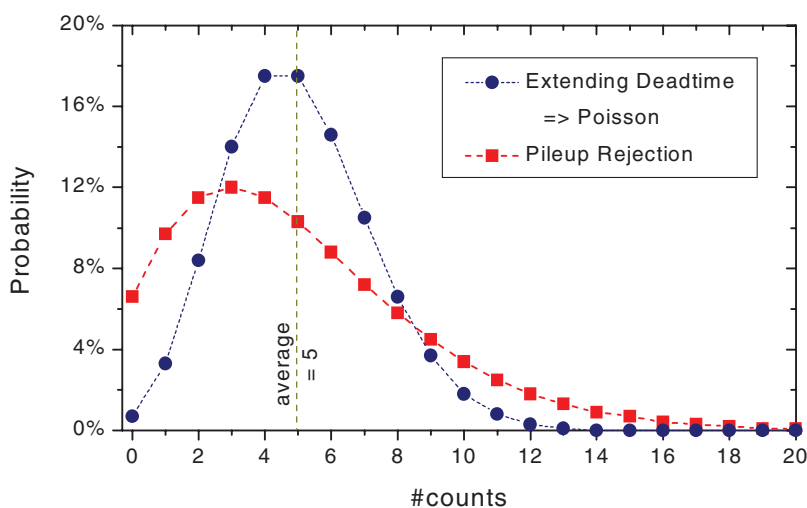


Figure 10. Theoretical probability distribution of the number of valid counts per fixed live-time period (or LFC inspection period) for counters with different types of count loss: EDT ($T_P = 0$) and PU ($T_P = T_W$). Whereas the average is 5 accepted counts in both cases, the distribution is broader for pileup rejection.

the two-fold count loss due to leading edge pileup. Whereas a simple clock accounts for the loss of a second pulse by turning off the clock during the duration of the first pulse, the Gedcke-Hale clock applies backward live-time recording during the 'leading edge', up to the pulse height identification of the first pulse.

The accuracy of the Gedcke-Hale pulse-loss correction method was tested on a digital spectrometer using the double-source method [18]. In figure 9, the relative peak area for a live time of 200 s is shown as a function of the average pulse loss. Up to a count loss of 30%, the observed peak area remains within 1% of the reference value (no loss). At extreme count-loss regimes, one notices a linearly increasing undercompensation amounting to about 3% at 80% count loss. This was within the specifications of the manufacturer.

The statistical behaviour of a digital spectrometer is identical to comparable analog systems. It has been confirmed by experiment that the statistical formulas in equations (28) and (30) are rigorously applicable to the dispersion of counted and loss-corrected number of events in selected ROIs as well as the full spectrum obtained with these digital spectrometers [18].

The artificial extension of the live-time by counting backwards has consequences on the count distribution of accepted events per live-time period. Whereas dead time does not affect the Poisson distribution of the number of accepted counts, a statistical broadening occurs with pileup. An example is shown in figure 10: for the same average count rate, relatively more low count numbers arise from double event losses, which is compensated by higher count numbers made possible due to the live-time extension. Detailed theoretical studies of these

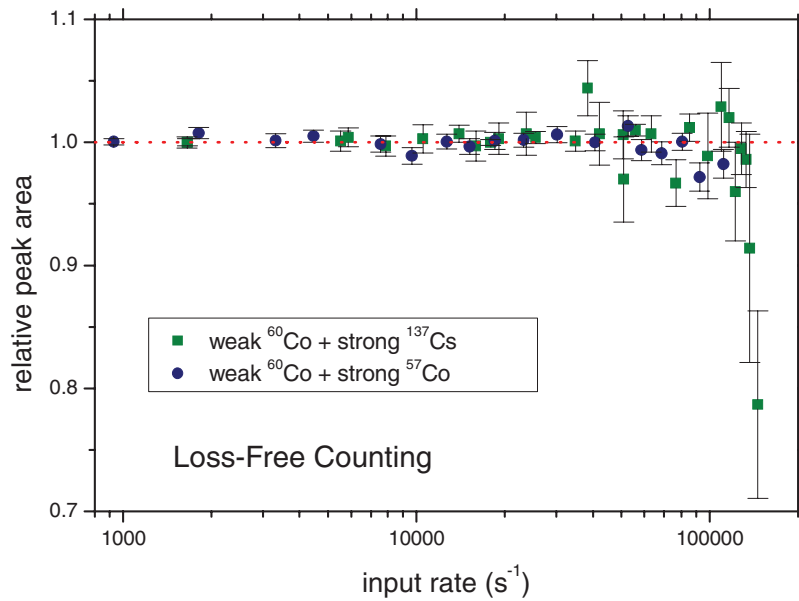


Figure 11. Loss-corrected peak areas by manually fine-tuned LFC of a weak ^{60}Co source measured with a HPGe detector in the presence of ‘background’ activity of a highly active ^{137}Cs or ^{57}Co source, relative to a measurement at low counting rate.

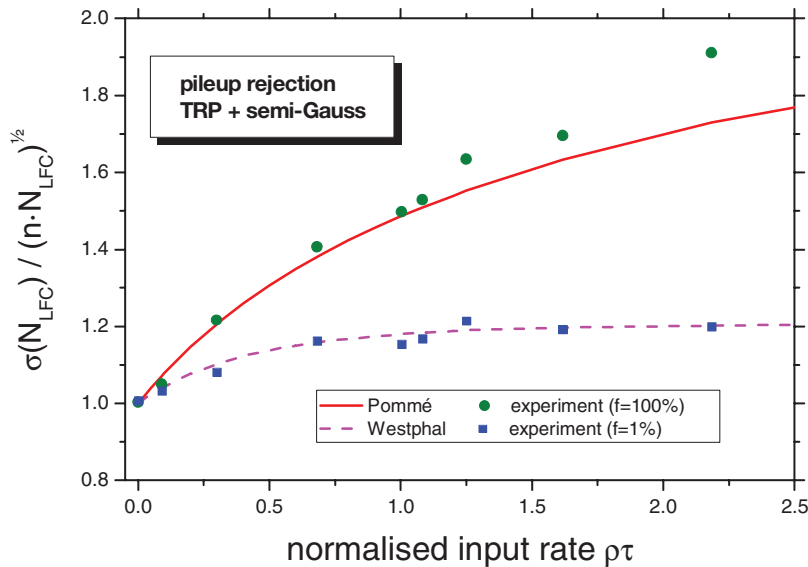


Figure 12. Measured and theoretical standard deviation of the number of counts in a small ($f = 1\%$) and large ($f = 100\%$) fraction of LFC spectra. The data are derived from about 2000 γ -ray spectra of ^{60}Co taken with a HPGe detector with a transistor reset preamplifier and semi-Gaussian shaping amplifier. The theoretical data were calculated from equation (31) by Pommé and the similar equation by Westphal, which misses the factor r to reproduce the additional dispersion in the full spectrum, but is valid for small fractions of the spectrum.

effects [14] are the basis of the correction factor r (equation (23)) appearing in live-time corrected counting statistics (cf. equations (22) and (31)).

3.5. Loss-free counting

‘Loss-free counting’ (LFC) [29–32, 34] and equivalently ‘zero dead-time’ (ZDT) counting [33, 34] is one of the most powerful tools for correcting rate-related losses in nuclear spectrometry. By performing ‘add- n ’ operations to the spectrum instead of ‘add-1’, the LFC principle is able to restore the linearity of the spectrometer throughput. Since the integer correction factor n is determined from the real-time to live-time

ratio over short time intervals, the LFC is able to deal with rapidly varying count rates. It is particularly well suited for application in neutron activation analysis, since the commonly enhanced loss of pulses from short-lived activation products is accounted for. Such is not the case with live-time correction techniques supplying one average correction factor for the entire measurement.

Pulse pileup can be compensated for provided that the width of the leading edge pulse T_P is added to the system busy time (as shown in figure 1) [14, 30–32, 52]. In an analog counting chain, T_P is introduced manually into the LFC module and fine-tuned for optimum loss compensation by means of the double source method. This is not required in

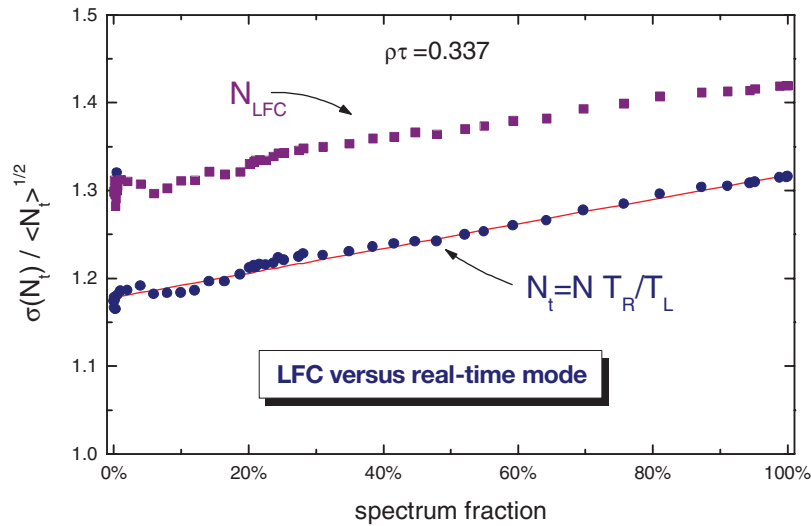


Figure 13. Standard deviation of loss-corrected data using the LFC spectrum and using live-time correction on the counted spectrum. Live-time counting is more precise at steady rate, but LFC is more accurate with rapidly changing count rates.

the digital implementation of the ZDT method [33], which uses the Gedcke-Hale live time clock to calculate the required short-term factor n .

Double source tests have demonstrated that manually fine-tuned LFC can correct for count losses—due to pulse pileup rejection and dead time—approaching 100% (see figure 11) [12] and a digital spectrometer performed well up to 60% count loss, but exhibited problems at more than 90% loss [20]. Extra care is advised when using detectors with reset preamplifier, since an incorrect setting of the inhibit signal can make the loss correction energy-dependent [12].

The main trade-off of LFC is an increase of the count scatter as the clustering of true and artificial events introduce unavoidable deviations from Poisson conditions. As demonstrated in figure 12, the standard deviation of loss-corrected counts from a Poisson process can be calculated from [14]:

$$\frac{\sigma(N_{\text{LFC}})}{\sqrt{N_{\text{LFC}}}} \approx r \sqrt{\langle n \rangle} \sqrt{1 + \left(\frac{\sigma(n)}{\langle n \rangle} \right)^2}, \quad (31)$$

in which r (equation (23)) is the statistical broadening due to pileup rejection (cf. section 2.6 and figure 10). The distribution of the factor n can be reproduced theoretically [14], and in the presence of pileup its variance depends critically on the length of the inspection period T_{IP} over which n is evaluated (and then applied for the next period). Short inspection periods allow fast response to changing count rates, but this is paid with higher statistical uncertainty (see figure 13). Heuristic equations for $\sigma(n)$ have been derived for different count loss mechanisms [14]. In the case of a system with NEDT and $\alpha = \tau_n / T_{\text{IP}} < 1$, a good approximation for the standard deviation on the LFC weighting factor is obtained from:

$$\sigma(n) \approx \sqrt{\rho \tau_n - (\rho \tau_n)^2 (1 - \alpha)^2}, \quad (32)$$

in which $\rho \tau_n$ can be obtained from the measured $\langle n \rangle$ -value: $\rho \tau_n = \langle n \rangle - 1$. For EDT and pileup rejection, a good approximation is:

$$\sigma(n) \approx \sqrt{(n-1) \left(\frac{n + 0.45 \alpha^{\beta+\Delta}}{1 + 0.45 \alpha^{\beta}} \right)}, \quad (33)$$

in which $\Delta = 0.6(1 - e^{-(n-1)})$ and $\beta = -2.6$.

An interesting development for the direct assessment of uncertainty with ZDT, is the implementation of a so-called ‘variance’ (Σn^2) spectrum next to the ZDT (Σn) spectrum [33]. Whereas the Σn^2 spectrum gives only an approximation of the true variance [21, 22], it is almost rigorously applicable to a small ROI ($r \approx 1$) when applying a relatively long inspection period ($\sigma(n)/n \approx 0$). Indeed, considering that the average correction factor $\langle n \rangle$ follows directly from the ratio of the number of counts in both spectra, $\langle n \rangle = \Sigma n^2 / \Sigma n$, it is clear that the ‘variance’ spectrum contains the value $\Sigma n^2 = \langle n \rangle \Sigma n = \langle n \rangle N_{\text{LFC}}$, which is a first order approximation of the variance in equation (27). It is comparably larger than the square root of the uncorrected number of counts: $\sigma(N_{\text{LFC}}) / N_{\text{LFC}} \approx \sqrt{\langle n \rangle / N_{\text{LFC}}} \geq \sqrt{1/N}$, since the average $\langle n \rangle$ is taken over the ZDT counts and not over the uncorrected counts [22].

4. Cascades of pileup and dead time

4.1. A series arrangement of pileup and dead time

In primary standardization of activity [35], it is common practice to count events triggered by detector pulses in a single-channel analyser and to impose a NEDT of fixed length that exceeds the pulse width of the detector signals. This principle can also be applied by software in the off-line analysis of list-mode files obtained with modern digital data acquisition methods. It is usually assumed that the recorded real-time to live-time ratio is an accurate correction factor for the incurred count loss. However, even if a very long dead time is selected, the effect of pileup is not completely compensated for, since any pulse occurring at the end of the imposed NEDT interval may block a partly overlapping

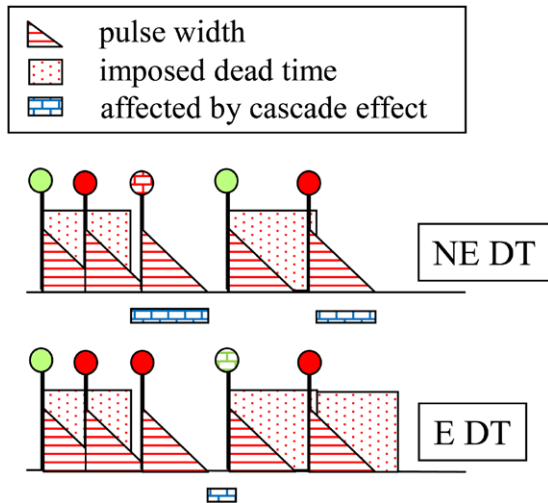


Figure 14. Pulses from a single-channel discriminator and their imposed dead time controlled by the live-time system. Due to the finite pulse width, pileup creates additional count loss not accounted for by the live-time clock.

event outside the dead time period that would normally be counted (see figure 14). This phenomenon is equivalent to a series arrangement of extending (EDT, i.e. pileup without rejection of the first pulse) and non-extending NEDT dead time [36, 40, 44]. At high count rates, if this effect is not properly taken into account, the live-time technique undercompensates for count loss.

Alternatively, one can impose an EDT instead. This dead time is in principle applied on every incoming event, whether it is falling within system live time and counted or within system dead time and excluded. In practice, most dead-time generators are triggered at the start of a pulse, which is recognized when the electronic detector signal rises above a threshold level, and are blind for a possible successive event until the trailing end of the pulse drops below the same threshold. Thus, only well-separated pulses prolong the system dead time, while piled-up pulses do not (see figure 14). Consequently, the system throughput will be slightly higher than what is expected from a perfect EDT counter. This situation is mathematically equivalent to a series arrangement of two EDTs [36–38, 42–44].

A convenient technical remediation of the problem is an EDT that is triggered not only at the start of the pulse, but also at the end [54]. By doing so, the dead time period is de facto extended by the full pulse width. Hence overlapping pulses also broaden the dead time period as if the counter had a perfect time resolution. Through this concept, the counter is insensitive to the pileup effect and acts as if it had a single EDT. One can also feed the live-time clock with an inhibit signal for the duration of each pulse. The alternative is a mathematical approach to correct for changes in the throughput, based again on studies of time interval distributions [36–44]. Last but not least, the need for corrections can be greatly reduced by triggering the discriminator and dead time with the differentiated waveform of the pulses, thus achieving a significantly better pulse-pair resolution and less impact from pileup.

4.2. Pileup and NEDT

For pulses with a width T_W and imposed NEDT τ_n (equivalent to EDT+NEDT in series), the throughput equation is [40, 44]:

$$R = \frac{\rho}{e^{\rho T_W} + \rho \max(0, \tau_n - T_W)}. \quad (34)$$

The relative error $\varepsilon(R)$ on the count rate by not taking into account the cascade effect of pileup with NEDT as a function of the expected count loss (when ignoring cascade effects) is calculated from the relative difference between the corrected (equation (34)) and uncorrected rate (equation (1)) [44]:

$$\varepsilon(R) = \left[\frac{1 + \rho \tau_n}{1 + \rho T_W - e^{\rho T_W}} - 1 \right]^{-1}. \quad (35)$$

The corresponding relative error on the loss-corrected rate $\hat{\rho}$ is found by multiplying equation (35) with T_R/T_L (cf. live-time mode) or the inverse throughput factor (cf. real-time mode), respectively [44]:

$$\varepsilon(\hat{\rho}) = \varepsilon(R) \frac{T_R}{T_L} = \frac{\varepsilon(R)}{1 - R\tau_n} = [e^{\rho T_W} - \rho T_W]^{-1} - 1, \quad (36)$$

whereas $\varepsilon(R)$ varies with τ_n as well as T_W , $\varepsilon(\hat{\rho})$ only varies with T_W . Consequently, the error cannot be reduced by imposing a longer NEDT. Figure 15 shows the error $\varepsilon(\hat{\rho})$ for $T_W = 2/3\tau_n$. The validity of equation (36) has been verified experimentally [45].

This error on $\hat{\rho}$ (equation (36)) can be reduced by correcting the count rate R mathematically via an iterative solution of the inverse of the mixed model ($\tau_n > T_W$) throughput formula in equation (34):

$$\hat{\rho} = \frac{R}{1 - R\tau_n} [e^{\rho T_W} - \rho T_W]. \quad (37)$$

As initial value for $\hat{\rho}$, one can use $R/(1 - R\tau_n)$ in real-time mode or $R T_R/T_L$ in live-time mode. The uncertainty on the thus obtained loss-corrected count rate is [44]

$$\frac{\sigma(N_t)}{\sqrt{N_t}} = X^{-1/2} \sqrt{\frac{1 - 2\rho T_W e^{-\rho T_W}}{(1 - \rho T_W)^2}}, \quad (38)$$

in which X is the throughput factor, hence $X^{-1/2} = \sqrt{\rho(\tau_n - T_W) + e^{\rho T_W}}$. Equation (38) is a generalized equation with respect to the simple cases of a counter with extending (equation (21)) or non-extending (equation (15)) dead time only. One has to include the propagation of the uncertainty on the pulse width T_W and in real time mode also on τ_n .

4.3. Pileup and EDT

The series arrangement of pileup and EDT, as described in section 4.1, is equivalent to an EDT–EDT cascade. Whereas the mathematical equation for the time interval distribution was found only recently [42, 43], the throughput factor could be derived earlier from probabilistic considerations [37, 38, 44], yielding:

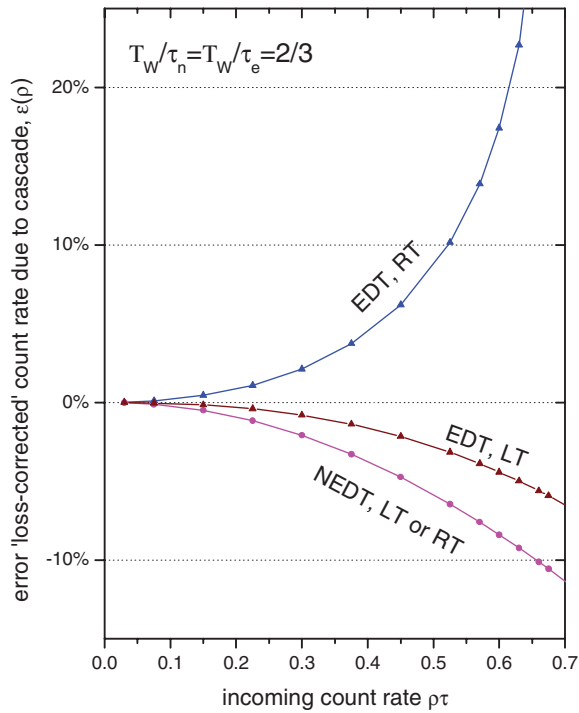


Figure 15. Relative error on the ‘loss-corrected’ count rate, $\varepsilon(\hat{\rho})$, due to ignoring the cascade effect of pulse pileup with extending (EDT) or non-extending (NEDT) dead time, when measuring in real-time (RT) or in live-time (LT) mode.

$$R = \rho e^{-\rho T_W} (1 - P_{\text{loss}}), \quad (39)$$

in which

$$P_{\text{loss}} = - \sum_{j=1}^J \frac{[-\rho(\tau_e - jT_W)e^{-\rho T_W}]^j}{j!}, \quad (40)$$

and J is the maximum integer between 0 and τ_e/T_W . Equation (39) turns out to be not smaller than the output rate for a system with a single EDT τ_e , i.e. $R' = \rho e^{-\rho \tau_e}$. The explanation is that some of the closely spaced pulses are eliminated by the pileup, thus preventing an extension of the second dead time to take place. As mentioned in section 4.1, this phenomenon can be suppressed by a double triggering of the EDT [54] or by feeding the pulse width as inhibit signal into the live-time clock.

Singly triggered EDT counters let more events pass through, but create less dead time. When measuring in live-time mode, one finds that both effects compensate each other. Yet there is a residual error due to similar loss effects as with NEDT (cf. equation (36)). Indeed, a chain of piled-up events can create additional dead time that exceeds the characteristic dead time τ_e imposed at the start of the first pulse. This probability reduces as the dead time τ_e is increased relatively to the pulse width T_W . The corresponding relative error in live-time mode has not been studied in detail, but can be estimated from the following rough approximation:

$$\varepsilon(\hat{\rho}) \approx [1 + \rho T_W - e^{\rho T_W}] \left(\frac{T_W}{\tau_e} \right), \quad (41)$$

which is confirmed by simulation results revealing some similarity with equation (36). In fact, the latter can be applied rigorously to the case $T_W = \tau_e$.

Contrary to NEDT, a distinction needs to be made between real-time and live-time mode counting. When working in real-time mode, one should consider the relative error on the output count rate R due to the increased throughput for an EDT–EDT cascade (cf. equation (18)):

$$\varepsilon(R) = \left[\frac{e^{-\rho T_W} (1 - P_{\text{loss}})}{e^{-\rho T_W}} \right] - 1. \quad (42)$$

Numerical examples [44] show that the results of equations (35) and (42) have opposite sign but similar magnitude for $T_W \leq \tau_e/2$ and combinations of τ_e and τ_n for which $\exp(\rho \tau_e) = (1 + \rho \tau_n)$. If one does not correct the output count rate R for the cascade effect, the error on the corresponding loss-corrected rate ρ is even enhanced via the inversion of the exponential throughput formula. Its propagation to the loss-corrected rate is well approximated by:

$$\varepsilon(\hat{\rho}) \approx \left| \frac{\partial R}{\partial \rho} \right|^{-1} \frac{R}{\rho} \varepsilon(R) \approx \frac{\varepsilon(R)}{1 - \rho \tau_e} \quad \text{for } \rho \tau_e \ll 1. \quad (43)$$

The expected error for $T_W = 2/3 \tau_e$ is shown in figure 15.

Also in the case of EDT ($\tau_e > T_W$), one can solve the inverse throughput relationship (equation (18)) by iteration:

$$\hat{\rho} = \frac{R e^{\rho T_W}}{1 - P_{\text{loss}}}, \quad (44)$$

using $R T_R/T_L$ in live-time mode or e.g. $R/[1 - R\tau_e - 0.5(R\tau_e)^2]$ in real-time mode as a first approximation for ρ and initial input for P_{loss} in equation (40). Here, it is implicitly assumed that the count rate is inferior to the rate at which the throughput reaches its maximum.

5. Statistics of a decaying source

5.1. Deviation from Poisson statistics

The validity of Poisson statistics is based on the hypothesis of a constant source activity, even though this assumption is contradictory to the concept of decay and only applies by good approximation if the half-life of the source is long compared to the measurement. Many radionuclides are short-lived and the influence of decay can have a non-negligible effect on counting statistics. The modified distribution law of the expected number of counts in a measurement has been derived theoretically and compared with experimental data, allowing even the derivation of the half-life from the shape of the distribution [7,48].

5.2. Dead time of a decaying source

The throughput is biased by the increasing live-time fraction as the source decays. Live-time counting generates a mean output rate R of a short-lived radionuclide with decay constant λ over a measurement interval T_R . To establish the relationship between R and the initial decay rate ρ_0 , the throughput

formulas have to be integrated over T_R , replacing ρ by the time-dependent decay rate of the source and a constant background rate ρ_b :

$$\rho(t) = \rho_0 e^{-\lambda t} + \rho_b. \quad (45)$$

The integrals have been carried out [7,46] and for NEDT, the throughput is

$$R = \frac{\rho_b}{1 + \rho_b \tau_n} + \frac{1}{\lambda T_R \tau_n (1 + \rho_b \tau_n)} \text{Ln} \left[\frac{1 + \rho_b \tau_n + \rho_0 \tau_n}{1 + \rho_b \tau_n + \rho_0 \tau_n e^{-\lambda T_R}} \right]. \quad (46)$$

Typically one has more control over the measurement duration than over the count rate, and thus can keep $\lambda T_R \ll 1$. In that case, and if the background is negligible, then the correction to the measured (and already conventionally corrected for decay and dead time) average rate can be approximated by [50]

$$f \approx 1 + \frac{1}{12} \frac{\rho_0 \tau_n}{(1 + \rho_0 \tau_n)} (\lambda T_R)^2, \quad (47)$$

which can be expressed in terms of measured throughput, R :

$$f \approx 1 + \frac{1}{12} R \tau_n (\lambda T_R)^2. \quad (48)$$

For EDT, the mean count rate is [7,46]

$$R = \frac{e^{-\rho_b \tau_e}}{\lambda T_R \tau_e} \left\{ e^{-\Lambda \rho_0 \tau_e} - e^{-\rho_0 \tau_e} + \rho_b \tau_e [E_1(\Lambda \rho_0 \tau_e) - E_1(\rho_0 \tau_e)] \right\}, \quad (49)$$

where $\Lambda = e^{-\lambda T_R}$ and the E_1 's are exponential integrals. The last term disappears for negligible background. From a serial expansion, a correction factor to the dead-time corrected rate was derived [50]

$$f \approx 1 + \frac{1}{12} \rho_0 \tau_e (\lambda T_R)^2. \quad (50)$$

For example, for an ^{18}F source (half-life of 1.83 h) measured for $T_R = 2 \times 10^3$ s, with a true count rate of $\rho_0 = 5 \times 10^4 \text{ s}^{-1}$, on a system with dead time of $\tau = 2 \times 10^{-5}$ s, we find correction factors for NEDT (equation (47)) and EDT (equation (50)) of $f = 1.0018$ and $f = 1.0037$, respectively.

These corrections would have uncertainties due to the expansion on the order of $\rho \tau (\lambda T_R)^2$, and may be most useful in designing experiments such that the correction is negligible, probably by limiting the counting duration, T_R , of individual measurements, averaging the resulting live-time and decay-corrected rates from numerous measurements if necessary.

6. Conclusions

Pulse pileup and dead time unavoidably influence the throughput of a counter and alter the statistical properties of the number of decays counted. Rate-related count loss can be corrected for by measuring in system live time, but uncertainties on the dead-time compensation have to be propagated to the resulting loss-corrected count rate. Software can reproduce

the role of live-time circuits in the analysis of list-mode data produced by digital spectrometers. Each pulse-loss correction method should be tested extensively at different count rates to assess rate-related errors and uncertainties. For high precision work, it is useful to utilize reduced count rates, e.g. via accurate dilutions. The statistical dispersion depends on the type of count loss, i.e. pileup, extending or non-extending dead time, or combinations of the three, but also on the considered fraction of the registered events. While Poisson statistics do not rigorously apply in nuclear counting, the inverse square root of the uncorrected number of counted events is still a good estimator for the relative standard uncertainty of the loss-corrected number in various practical situations.

References

- [1] ICRU 1994 Particle counting in radioactivity measurements *ICRU Report 52* (Bethesda, MD: International Commission on Radiation Units and Measurements) p 80
- [2] Libert J 1976 Comparaison des distributions statistiques de comptage des systèmes radioactifs *Nucl. Instrum. Methods* **136** 563–8
- [3] Libert J 1988 Détermination du taux de comptage en amont d'un temps mort généralisé *Nucl. Instrum. Methods A* **265** 537–40
- [4] Libert J 1989 Détermination du taux de comptage en amont de deux temps morts classiques en série et de types différents *Nucl. Instrum. Methods A* **274** 319–23
- [5] Müller J W 1973 Dead-time problems *Nucl. Instrum. Methods* **112** 47–57
- [6] Müller J W 1974 Some formulae for a dead-time distorted Poisson process *Nucl. Instrum. Methods* **117** 401–4
- [7] Müller J W 1981 Counting statistics of short-lived nuclides *J. Radioanal. Nucl. Chem.* **61** 345–59
- [8] Müller J W 1991 Generalized dead times *Nucl. Instrum. Methods A* **301** 543–51
- [9] Jordan D R and Mcbeth G W 1978 An experimental test of Müller statistics for counting systems with a non-extending dead time *Nucl. Instrum. Methods* **155** 557–62
- [10] Salma I and Zemlén-Papp É 1992 Experimental investigation of statistical models describing distribution of counts *Nucl. Instrum. Methods A* **312** 591–7
- [11] Pommé S 1998 Time distortion of a Poisson process and its effect on experimental uncertainty *Appl. Radiat. Isot.* **49** 1213–8
- [12] Pommé S, Alzetta J-P, Uyttenhove J, Denecke B, Arana G and Robouch P 1999 Accuracy and precision of loss-free counting in γ -ray spectrometry *Nucl. Instrum. Methods A* **422** 388–94
- [13] Pommé S, Denecke B and Alzetta J-P 1999 Influence of pileup rejection on nuclear counting, viewed from the time-domain perspective *Nucl. Instrum. Methods A* **426** 564–82
- [14] Pommé S 1999 How pileup rejection affects the precision of loss-free counting *Nucl. Instrum. Methods A* **432** 456–70
- [15] Pommé S 1999 Is loss-free counting under statistical control? *Biol. Trace Element Res.* **71–72** 415–22
- [16] Pommé S 1999 Time-interval distributions and counting statistics with a non-paralysable spectrometer *Nucl. Instrum. Methods A* **437** 481–9
- [17] Pommé S, Robouch P, Arana G, Eguskiza M and Maguregui M I 2000 Is it safe to use Poisson statistics in nuclear spectrometry? *J. Radiat. Nucl. Chem.* **244** 501–6
- [18] Pommé S and Kennedy G 2000 Pulse loss and counting statistics with a digital spectrometer *Appl. Radiat. Isot.* **52** 377–80

- [19] Pommé S and Uyttenhove J 2001 Statistical precision of high-rate spectrometry with a Wilkinson ADC *J. Radioanal. Nucl. Chem.* **248** 263–6
- [20] Pommé S 2001 Experimental test of the ‘zero dead time’ count-loss correction method on a digital gamma-ray spectrometer *Nucl. Instrum. Methods A* **474** 245–52
- [21] Pommé S 2002 A plausible mathematical interpretation of the ‘variance’ spectra obtained with the DSPECPLUS TM digital spectrometer *Nucl. Instrum. Methods A* **482** 565–6
- [22] Pommé S 2003 On the statistical control of ‘loss-free counting’ and ‘zero dead time’ spectrometry *J. Radioanal. Nucl. Chem.* **257** 463–6
- [23] Pommé S 2007 Dead time, pile-up and statistics *Applied Modeling and Computations in Nuclear Science (ACS Symposium Series 945)* ed T M Semkow *et al* (Washington, DC: American Chemical Society) pp 218–33
- [24] Cowell D F, Sandomire M M and Eichen M S 1960 Automatic compensation of dead time in pulse analysis equipment *Anal. Chem.* **32** 1086–90
- [25] Deighton M O 1961 Statistical errors arising from use of a gated pulse train for total live time measurement during pulse amplitude analysis *Nucl. Instrum. Methods* **14** 48–52
- [26] Bartošek J, Mašek J, Adams F and Hoste J 1972 The use of a pileup rejector in quantitative pulse spectrometry *Nucl. Instrum. Methods* **104** 221–3
- [27] Anders O U 1969 Experiences with the Ge(Li) detector for high-resolution gamma ray spectrometry and a practical approach to the pulse pileup problem *Nucl. Instrum. Methods* **68** 205–8
- [28] Wiernik M 1971 Normal and random pulse generators for the correction of dead-time losses in nuclear spectrometry *Nucl. Instrum. Methods* **96** 325–9
- [29] Harms J 1967 Automatic dead-time correction for multichannel pulse-height analyzers at variable counting rates *Nucl. Instrum. Methods* **53** 192–6
- [30] Westphal G P 1979 On the performance of loss-free counting—a method for real-time compensation of dead-time and pile-up losses in nuclear pulse spectroscopy *Nucl. Instrum. Methods* **163** 189–96
- [31] Westphal G P 1982 Real-time correction of counting losses in nuclear pulse spectroscopy *J. Radioanal. Chem.* **70** 387–410
- [32] Westphal G P 1985 Instrumental correction of counting losses in nuclear pulse spectroscopy *Nucl. Instrum. Methods B* **10/11** 1047–50
- [33] Upp D L, Keyser R M, Gedcke D A, Twomey T R and Bingham R D 2001 An innovative method for dead time correction in nuclear spectroscopy *J. Radioanal. Nucl. Chem.* **248** 377–83
- [34] Westphal G P 2008 Review of loss-free counting in nuclear spectroscopy *J. Radioanal. Nucl. Chem.* **275** 677–85
- [35] Pommé S 2007 Methods for primary standardization of activity *Metrologia* **44** S17–26
- [36] De Lotto I, Manfredi P F and Principi P 1964 Counting statistics and dead-time losses *Energ. Nucl.* **11** part 1: 557–64, part 2: 599–611
- [37] De Lotto I and Manfredi P F 1965 Counting-losses introduced by the cascade connection of two paralyzable-counters *Energ. Nucl.* **12** 102–3
- [38] Müller J W 1972 On the effect of two extended dead times in series *Internal Report BIPM BIPM-72/9* Sèvres, France
- [39] Müller J W 1973 Une nouvelle manière de calculer les pertes de comptage dues à un temps mort non cumulatif *Internal Report BIPM BIPM-73/4* Sèvres, France
- [40] Müller J W 1973 Sur l’arrangement en série de deux temps morts de types différents *Internal Report BIPM BIPM-73/9* Sèvres, France
- [41] Gostely J-J 1996 Pile-up effect for a counting channel with amplitude selection and imposed non-cumulative dead time *Nucl. Instrum. Methods A* **369** 397–400
- [42] Choi H D 2009 Counting statistics distorted by two dead times in series which end with an extended type dead time *Nucl. Instrum. Methods A* **599** 251–9
- [43] Choi H D 2011 Counting statistics modified by two dead times in series *Nucl. Eng. Technol.* **43** 287–300
- [44] Pommé S 2008 Cascades of pile-up and dead time *Appl. Radiat. Isot.* **66** 941–7
- [45] Michotte C and Nonis M 2008 Experimental comparison of different dead-time correction techniques in single-channel counting experiments *Nucl. Instrum. Methods A* **608** 163–8
- [46] Axton E J and Ryves T B 1963 Dead-time corrections in the measurement of short-lived radionuclides *Int. J. Appl. Radiat. Isot.* **14** 159–61
- [47] Baerg A P, Munzenmayer K and Bowes G C 1976 Live-timed anti-coincidence counting with extending dead-time circuitry *Metrologia* **12** 77–80
- [48] Müller J W 1979 Counting statistics of a decaying source *Internal Report BIPM BIPM-79/11* Sèvres, France
- [49] Lindstrom R M and Fleming R F 1995 Dead time, pileup, and accurate gamma-ray spectrometry *Radiat. Radiochem.* **6** 20–7
- [50] Fitzgerald R 2015 Decay-dead-time corrections for live-timed counting systems with extending and non-extending dead times ICRM (submitted)
- [51] Gilmore G and Hemingway J 1995 *Practical Gamma-ray Spectrometry* (Chichester: Wiley)
- [52] Jenkins R, Gould R W and Gedcke D 1995 *Quantitative X-ray Spectrometry* 2nd edn (New York: Marcel Dekker)
- [53] Albert G E and Nelson L 1953 Contributions to the statistical theory of counter data *Ann. Math. Stat.* **24** 9–22
- [54] Bouchard J 2000 MTR2: a discriminator and dead-time module used in counting systems *Appl. Radiat. Isot.* **52** 441–6