A constant from a mass, a mass from a constant

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Introduction

The International System of Units (SI) will soon be based on a set of fundamental constants with fixed, exact values known as defining constants [1,2]. For example, the unit of length was once derived from the space between two scratches on a metal bar, but now it can be derived anywhere at any scale using a defining constant, the speed of light. Similarly, it is planned that the unit of mass will cease to be derived from a single cylinder of platinum iridium (Pt-Ir) in a vault in Sevres, France near Paris, and instead it may be derived anywhere at any scale using the defining constants, most notably a fixed value of the Planck constant *h*.

The first step, of course, is fixing a value of *h*, which is now defined inexactly in terms of the existing standard of mass, the International Prototype of the Kilogram (IPK). This must be done based on experiments, with an uncertainty of less than 5 parts in 10^8 if the precision of mass is to be preserved after revising the units of measure. Multiple groups have recently crossed this threshold in precision using watt balance [3,4] and x-ray crystal density [5] approaches. At last, the long discussed decision whether to continue deriving *h* from the International Prototype of the Kilogram, or to instead derive the unit of mass from a fixed value of *h* [6], appears to be drawing to a close. A timeline has emerged: a pilot study on the realization of the kilogram from *h* is proposed for 2015, with revision of the SI slated for 2018.

The National Institute of Standards and Technology (NIST) recently used a watt balance known as NIST-3 to measure the Planck constant in terms of IPK with a relative uncertainty of approximately 4.5 parts in 10^8 [3]. Along the way to this new NIST value of *h*, the instrument was also employed to perform the reciprocal experiment: *h* was “fixed” and the unknown mass of a stainless steel mass standard was measured with reference only to standards of length, time (frequency), and electrical quantities, all derivable from fixed fundamental constants. This paper reviews the basic principles of a watt balance experiment and shares the results of this trial dissemination of mass directly from a set of defining constants, rather than from an artifact.

Watt Balance Principles

As illustrated in Figure 1, a watt balance has many elements in common with an ordinary compensation balance. There is a balance mechanism (a wheel pivoting on a knife edge in this case), a weighing pan suspended from the balance, and a moving coil actuator to apply compensation forces to hold a null position.

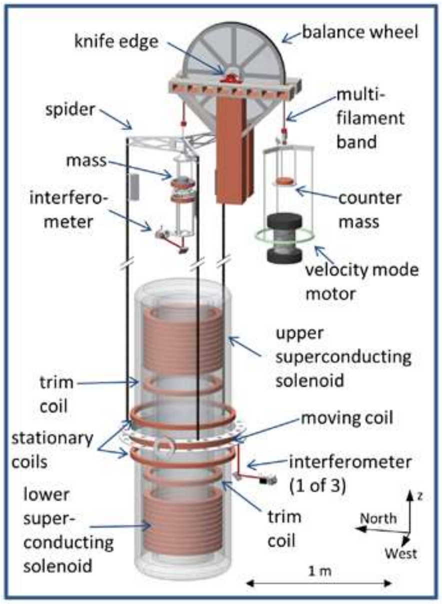

Figure 1 Schematic drawing of the NIST-3 watt balance.

In operation, such balances employ the substitution principle. The force on the mass pan is either generated by the gravitational force from the mass set on top of the mass pan or by an electromagnetic force created by the actuator located below the mass pan. However, what sets a watt balance apart from its compensation balance counterpart is its ability to operate alternately in two separated modes.

In the weighing mode, dc current I is fed back through a coil in a magnetic field to produce a force that nulls the position of the balance mechanics and exactly balances the gravitational force acting on the test mass m, so that $BLI=mg$, where B is the magnetic flux density, L is the wire length in the coil, and g is the local gravitational acceleration.

In the velocity mode, the geometric factor BL is precisely calibrated in terms of length, time, and electrical standards. The calibration is achieved by moving the coil through the magnetic field while measuring its velocity v and the voltage U_v induced across the coil, i.e., $U_v= BLv$.

Measurements from the two operational modes allow one to compute a virtual comparison of electrical power to mechanical power, $U_vI=mgv$, both measured in watts, hence the name watt balance. A small number of such balances have been constructed over the years, and more are in the planning stages, as evident in the recent special issue of the journal Metrologia [7]. In the next section, we will focus on the interesting connection the experiment makes between classical and quantum physics.

A constant from a mass

The virtual comparison of electrical power to mechanical power embodied in the watt balance relation $U_vI=mgv$ creates an opportunity to evaluate a fundamental constant of quantum physics in terms of a Newtonian quantity, gravitational mass. This opportunity arises because the references used to measure the induced voltage U_v and the applied current I are very precisely known in terms of the Planck constant.

The basis of modern voltage standards is the Josephson effect, named after Brian Josephson, who was awarded the 1973 Nobel prize in physics for its discovery in 1962 [8]. The Josephson effect describes the quantum mechanical behavior of electrons in a pair of superconductors separated by a thin, non-superconducting barrier. Taken as a whole, this superconductor sandwich is called a Josephson junction. The electrons in the superconducting elements of these junctions are characterized by quantum mechanical wavefunctions, the relative phases of which can be locked through interaction with electromagnetic waves (i.e., by exposing the junction to microwave radiation). Because of this phase locking of the wavefunctions, a fixed voltage $V= fnh/2e$ develops across an array of such junctions, where f is the frequency of the electromagnetic excitation, n is the number of Josephson junctions in the array, e the elementary charge, and h the Planck constant. The summed junction voltage is a quantum reference that is stable and repeatable to a few parts in 10^{10} and depends only on a frequency, everything else being a constant.

During the velocity mode of watt balance experiments performed at NIST, the voltage induced across the coil U_v is compared to a fixed reference voltage produced by a programmable array of such Josephson junctions that was developed by another group of researchers at NIST [9]. The NIST Programmable Josephson Voltage System works at a selectable microwave frequency and features software controls that allow the user to program the array of junctions to produce a variety of reference voltages, including limited waveform capabilities. In practice on the NIST watt balance, the system is used to generate a nominal 1 V reference. The velocity of the balance is then controlled to minimize the difference (error signal) between U_v and this reference as measured using a bridge type circuit arrangement. This establishes U_v in terms of f, n, and the ratio $1/K_J$ where $K_J=2e/h$ is referred to as the Josephson constant.

A second quantum mechanical phenomenon is necessary to completely link a watt balance to the Planck constant. This phenomenon is a special variant of the Hall effect known as the *quantum* Hall effect. Recall that the Hall effect describes the migration of charge carriers in a conductor to one side of the conductor in the presence of a perpendicular magnetic field. As a result, a voltage appears across the conductor in a direction perpendicular to both the current flow and magnetic field direction. In 1985, Klaus von Klitzing was awarded the Nobel Prize in physics for his experimental work showing that this Hall voltage becomes exactly quantized if the conductor is a two dimensional electron gas in the presence of a strong magnetic field [10]. The energy level of the electrons and hence the Hall resistance is quantized in this case at integer fractions of $R_K = h/e^2$, where R_K is known as the von Klitzing constant. At present, NIST uses a gallium arsenide heterostructure cooled to 0.3 K in order to create a two dimensional electron gas suitable as a quantum Hall resistance standard and to serve as the US representation of the ohm [11]. NIST has also made great progress recently in the development of graphene based quantum Hall devices [12] that show potential of working at higher temperatures and lower magnetic fields than available with present devices.

During the weighing mode of a watt balance experiment, the electric current to null the balance position as the test mass is placed on and off the balance is measured using Ohm's law by passing the current through a resistor and measuring the corresponding voltage drop, U_w. The value of U_w is once again measured via comparison to a Josephson array voltage standard, while the value of the resistor, R, is known, having been determined via comparison to a quantum Hall standard, $R=\alpha R_K$, where α is a proportionality constant measured using state-of-the-art bridge techniques [11].

With data available from both modes of the experiment, the electrical power is written as $P=UI=U_v U_w/R$. Because both of the voltages are measured via the Josephson effect, and the resistance is measured via the quantum Hall effect, one obtains for the electrical power

$$P=n_w n_v\ f_w f_v\ h\ /\ 4\alpha$$

In order to make a measurement of h, the mass m of a stable, Pt-Ir mass standard is determined by comparison to the US National prototype kilogram, then the local gravitational acceleration g is measured with an absolute gravimeter [13] and the electrical and mechanical powers are equated and solved for h

$$P = mgv \text{ or } h = 4\alpha mgv/(\ n_w n_v\ f_w f_v).$$

A value of h = 6.626 069 79 x 10^{-34} J s was determined over the course of six measurement campaigns in 2013 using a platinum iridium prototype. Typical data used for this determination are shown in Figure 2.

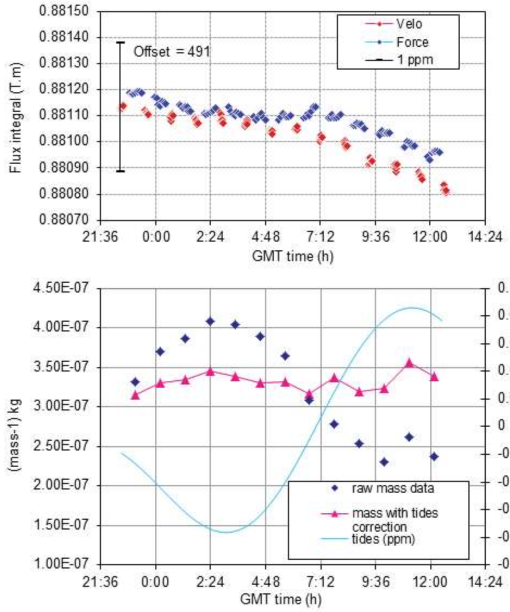

Figure 2. Upper plot is typical raw velocity mode and weighing mode data over a twelve hour period. Lower plot shows the magnitude of the relative variation of gravity due to tidal effects and the measured deviation of the mass before and after correction for this variation.

A mass from a constant

The equation for h in terms of mass can be rearranged to solve for mass in terms of h, or

$$m = n_w\ n_v\ f_w\ f_v\ h/(4\alpha gv),$$

where m, in this case, refers to the mass of an unknown test artifact to be determined. It is interesting to note that in this scheme, the unit of mass is no longer tied to a single cardinal value. The impact on scaling the unit of mass is beyond our scope here, but as observed in a recent essay by the lead author [14] it opens up the possibility of accurately weighing everything from "atoms to apples" in terms of the defining constants.

In June 2013, a stainless steel mass with a value unknown to the researchers at the watt balance was used on the NIST-3 watt balance. After 25 days of measurement on the watt balance, the mass of the artifact was determined using a previously recorded value h = 6.626 069 79 x 10^{-34} J s to be

$$m_{\mathrm{WBV}} = (1\ \mathrm{kg} + 338\ \mu\mathrm{g}) \pm 60\ \mu\mathrm{g}$$

The above estimate of the object's mass was determined in vacuum (0.5 Pa), in the absence of any significant buoyant forces. The value includes corrections for a variety of factors, including tidal corrections to gravity (e.g., see [3]). An important correction new to this work was the effect of magnetic forces arising from the weak magnetic properties (i.e., susceptibility and magnetization) of the stainless steel artifact.

Briefly, the magnetic properties of this artifact were determined using the balance apparatus. Changes in force on the balance were recorded as the magnetic flux densities around the balance pan were varied in a systematic fashion. Prescribed fields were applied using auxiliary coils positioned around the mass pan. From these force measurements, the volume magnetic susceptibility and the polarization of the artifact were deduced. The magnetic flux density at the weighing position due to the superconducting magnet and the vertical gradient of its field at this position were also determined. Combining these observations with an estimate of the artifact volume, the magnetic field was found to contribute an additional apparent mass of 11.6 μg ± 6 μg. This correction was already applied to the vacuum mass value given above.

Finally, the true mass of the artifact in air is slightly heavier than the vacuum mass, since an adsorbed layer of contaminants (mostly water) clings to mass artifacts when they are used in air. Because of this, we must explicitly estimate this sorption correction if we wish to compare the watt balance value to the true mass of the artifact that will be measured using conventional practices in air.

Based on data available in reference [15], which examined sorption factors for Pt-Ir and stainless steel kilogram mass artifacts using a vacuum mass comparator, we estimate the amount of material adsorbed on the surface to be ≈ 14 μg ± 5 μg, so that the true mass, as determined by the watt balance approach, is

$$m_{\mathrm{WB}} = (1\ \mathrm{kg} + 352\ \mu\mathrm{g}) \pm 60\ \mu\mathrm{g}$$

In contrast, before and after the watt balance experiment, the NIST Mass and Force Group measured this stainless steel artifact on a mass comparator at ambient air conditions using a weighing design that included the Pt-Ir US National standard, for a determination of the true mass in terms of the SI kg. This value, m_{MG}, includes a correction for the buoyant force of air acting on the artifact. In the most basic weighing comparison [16], this correction appears as follows

$$m_{\mathrm{MG}} = m_{\mathrm{PtIr}} - \rho_{\mathrm{a}}(V_{\mathrm{PtIr}} - V_{\mathrm{SS}}) - C$$

where m_{PtIr} is the mass of the US National standard, ρ_{a} is the density of air, V_{SS} and V_{PtIr} are the volume of the unknown stainless steel and the platinum iridium US National standard mass artifacts, respectively, and C is the balance reading. Observe that this determination of mass includes the mass of adsorbed contaminants implicitly, since the entire determination was performed in ambient air. The magnitude of such contamination (in terms of mass) can vary over time, depending on a variety of features of the metal surface and its surrounding ambient environment, including obvious factors such as the surface roughness of the artifacts and the chemical composition of the metal surfaces and the air.

The NIST Mass and Force Group obtained the following value of the true mass for the stainless steel artifact

$$m_{\mathrm{MG}} = (1\ \mathrm{kg} + 324\ \mu\mathrm{g}) \pm 14\ \mu\mathrm{g}.$$

The difference between this value, and that determined on the watt balance is

$$m_{\mathrm{WB}} - m_{\mathrm{MG}} = 28\ \mu\mathrm{g} \pm 62\ \mu\mathrm{g}.$$

No significant difference between the two measurement methods is observed. This demonstrates the utility of a watt balance for making direct calibration of the true mass of stainless steel mass artifacts, which are the most common type of standards employed for mass metrology.

Conclusion

The SI is poised for a major revision where the unit of mass will no longer be based on an artifact definition. The experiment described here illustrates that, as expected, a watt balance capable of measuring the Planck constant can be used effectively in a reciprocal mode, so that the realization of mass at the kilogram level is possible. In this regard, the experiment provides a useful preview of the methods and issues that surround the planned revision of the definition of the SI unit of mass. The experiment highlights the close coordination that is required between artifact mass metrology and the more abstract realization of the unit of mass engendered by the watt balance experiment, particularly during this transition from one unit definition to the other. That both methodologies yielded here the same outcome should lend confidence to the notion that a new definition of mass based on defining constants can succeed. Moving forward, it is apparent that the coordination between artifact metrology and watt balance experimentation must persist. After all, artifacts calibrated through reference to defining constants using a watt balance will be the means of disseminating the unit beyond NIST. Such artifacts are fully compatible with the existing mass infrastructure, and, until such time as watt balance instruments proliferate and become commonplace, traceability to the handful of such realizations will be achieved as it has always been, through exchanges of artifact mass standards.

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