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COMBINED BUOYANCY- AND PRESSURE-DRIVEN FLOW THROUGH A HORIZONTAL VENT: THEORETICAL CONSIDERATIONS

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ABSTRACT

Flow through a horizontal vent is considered where the vent-connected spaces near the elevation of the vent are filled with fluids of different density in an unstable configuration, with the density of the top space larger than that of the bottom space. With zero-to-moderate cross-vent pressure difference the instability leads to a bi-directional exchange flow between the two spaces. For relatively large cross-vent pressure difference the flow through the vent is unidirectional, from the high- to the low-pressure space.

For arbitrary specified cross-vent pressure difference, boundary value problems for the flow are formulated for cases where the fluid media in the two spaces are the same perfect gas, with relatively high and low temperature (corresponding to low and high density) in the lower and upper spaces, respectively. Two separate classes of problem are distinguished. In the first, the higher pressure is in the space above the vent. This enhances the downward component of the flow from the top to the bottom space, and diminishes, or reduces to zero, the upward flow. In the second, the higher pressure is in the lower space leading to enhancement of the upward flow, etc. Relationships between the two boundary value problems and their solutions are identified. These are useful for extending an available solution for one class of problem to that of the other and for unified understanding and correlation of experimental data for the two flow configurations.

Keywords: **building fires; compartment fires; computer models fire models; mathematical models; vents; zone models**

INTRODUCTION

Consider the problem of flow through a horizontal vent where the vent-connected spaces near the elevation of the vent are filled with fluids of different density, the density of the top space, ρ_{TOP}, being larger than that of the bottom space, ρ_{BOT}.

$$\rho_{TOP} - \rho_{BOT} = \Delta\rho > 0; \ \bar{\rho} = (\rho_{TOP} + \rho_{BOT})/2 \tag{1}$$

The configuration is unstable [1] in that it leads to a purely buoyancy-driven bi-directional exchange flow across the vent, with the more dense material flowing through the vent from the top to the bottom space and the less dense fluid from the bottom to the top space.

The above flow comes about when there is no cross-vent pressure difference. If there is a non-zero pressure difference then the flow rates will be altered. The flow from the high pressure side of the vent will increase and that from the low-pressure side will decrease. If the pressure difference is large enough then the flow through the vent will become uni-directional. When this happens, the effect of the relative buoyancy between the spaces will still have an effect on the flow rate. If the pressure difference is increased further, then, eventually, the effect of bouyancy will not be important and the problem can be understood simply by means of Bernouli-type momentum considerations.

Theoretical and experimental studies of the above effects are presented in references [2]-[9].

In its full generality, the above problem and its solution has a broad range of importance and applicability. Of interest here are the aspects of the problem related to flow, in the presence of arbitrary cross-vent pressure difference, between relatively cool and dense air in the top space and relatively high-temperature and buoyant air in the bottom space. This is a general problem associated with ventilation of enclosed, heated/cooled spaces. It is a problem of particular importance in the spread of smoke (i.e., fire-heated and -contaminated air) during fires in multi-room facilities.

This paper considers the boundary value problems for the above flows. Two problems, identical in all features except for which space has the higher of the two pressures (i.e., the top or bottom space) are formulated. Relationships between the two problems and their solutions are then identified. These relationships would be useful for extending an available solution for one class of problem to that of the other and for unified understanding and correlation of experimental data for the two flow configurations.

THE TWO CONFIGURATIONS

Figure 1 depicts the general flow scenarios being considered. Figure 1a, is designated as Configuration 1 (including a co-ordinate sytem with X_3 in the upward direction, i.e., opposite to the direction of the gravitational force) with associated Boundary Value Problem 1 (referred to below as Problem 1), Figure 1b as Configuration 2 (X_3 downward) with Boundary Value Problem 2 (Problem 2). At the elevation of the vent opening, but far removed laterally, the pressures in the top and bottom space of Figure 1a are p_{HIGH} and p_{LOW}, respectively, where

$$p_{HIGH} - p_{LOW} = \Delta p > 0 \tag{2}$$

The pressures in the top and bottom space of Figure 1b are p_{LOW} and p_{HIGH}, respectively. Note that p_{HIGH} and p_{LOW} are identical for Problems 1 and 2, whereas p_{TOP} and p_{BOT} are not. This fact becomes useful in the problem formulations that follow.

As is always the case in practical problems of present concern it is assumed that

$$\Delta p/\bar{p} << 1; \;\; \bar{p} = (p_{HIGH} + p_{LOW})/2 \tag{3}$$

The fluids in the two spaces are assumed to be the same perfect gas. In the two figures the temperatures at the vent elevation, but far removed laterally, are T_{TOP} and T_{BOT}. As would be determined from the equation of state, these are the temperatures associated with ρ_{TOP} and ρ_{BOT}, respectively, and with the appropriate pressure, p_{HIGH} or p_{LOW}. The constraint of Eq. (3) leads to

$$T_{BOT} - T_{TOP} = \Delta T > 0; \;\; \bar{T} = (T_{TOP} + T_{BOT})/2 \tag{4}$$

Thus, the smaller of the two densities, ρ_{BOT}, is associated with the higher temperature gas, T_{BOT}, in the bottom space, etc.

THE BOUNDARY VALUE PROBLEMS

The problems for the two configurations are formulated for a vent in the $X_3 = 0$ plane. D is the characteristic dimension of the closed curve which defines the shape of the vent opening. The depth of the vent is L and it is assumed that the vent is shallow in the sense that $L/D << 1$ is negligible. For problems of interest here, it is assumed that at and near $X_3 = 0$, but outside the the vent opening, heat transfer to the $X_3 = 0$ surfaces, both in the top and bottom spaces, is negligible compared to, say, the enthalpy of the gas flows convected through the vent.

Although the flows under consideration are expected to be strongly time-dependent, it is assumed that time scales which characterize their fluctuations are relatively small, i.e., it is assumed that meaningful average flow characteristics could be established, in principle, with integrals over time intervals which are relatively small compared to the characteristic times of the problems of interest.

For the purpose of this paper the important features of the governing equations are highlighted by describing the problems as steady-state boundary value problems rather than initial/boundary value problems. Neglecting pdV work and viscous dissipation in the energy equation leads to Problems 1 and 2 for Configurations 1 and 2. These are presented in Table 1 [Eqs. (5)-(6)]*.

It is assumed that specific heat, $C_p = C_p(T)$, dynamic viscosity, $\mu = \mu(T)$, and thermal conductivity, $k = k(T)$ are all analytic functions of T near $T = \bar{T}$ and in a region of the real T axis in the entire range $T_{TOP} < T < T_{BOT}$.

---

*In the tensor notation used here, for a term with an unrepeated subscript i or j, the i or j can take on any value 1, 2, or 3. A term with a repeated index i or j, indicates that the term is summed over the repeated index from 1 to 3.

For the purpose of establishing the relationship between ρ and T, p in Eq. (8) (Table 1) is approximated as being uniform throughout the flow region.

$$\rho T = \text{constant} = \bar{p}/R = \rho_{TOP}T_{TOP} = \rho_{BOT}T_{BOT} \qquad (8')$$

where R is the gas constant and T_{TOP} and T_{BOT} are modified (from the values that would be determined from the specified ρ_{TOP} and ρ_{BOT} and the equation of state) to satisfy Eq. (8′) exactly. This will be a good approximation provided

$$\Delta\rho g|X_3|/\bar{p} << 1 \text{ throughout the region of interest} \qquad (10)$$

Introduce the following perturbation ("primed") variables and dimensionless ("starred") variables:

Problem 1

$$\rho' = \rho - \bar{\rho};\; T' = \bar{T} - T;$$

$$p' = p - \bar{p} - \bar{\rho}f_3^{(1)}X_3;$$

$$\rho^* = \rho/\bar{\rho} = 1 + \rho^{*'}(\Delta\rho/\bar{\rho})$$

$$T^* = T/\bar{T} = 1 - T^{*'}(\Delta T/\bar{T})$$

$$F_i^{*(1)} = f_i^{(1)}/(2g) = (0, 0, -1/2)$$

Problem 2

$$\rho' = \bar{\rho} - \rho;\; T' = T - \bar{T};$$

$$p' = p - \bar{p} - \bar{\rho}f_3^{(2)}X_3;$$

$$\rho^* = \rho/\bar{\rho} = 1 - \rho^{*'}(\Delta\rho/\bar{\rho})$$

$$T^* = T/\bar{T} = 1 + T^{*'}(\Delta T/\bar{T}) \qquad (11)$$

$$F_i^{*(2)} = f_i^{(2)}/(2g) = (0, 0, 1/2)$$

$$\rho^{*'} = \rho'/\Delta\rho;\; T^{*'} = T'/\Delta T;$$

$$p^* = p/(2g\Delta\rho D);\; p^{*'} = p'/(2g\Delta\rho D);$$

$$U_i^* = U_i[\bar{\rho}/(2gD\Delta\rho)]^{1/2};\; X_i^* = X_i/D$$

Also, define

$$C_p^* = C_p(T)/C_p(\bar{T});\; M^* = \mu(T)/\mu(\bar{T});\; K^* = k(T)/k(\bar{T}) \qquad (12)$$

Now it can be shown from Eqs. (8′) that

$$\Delta\rho/\bar{\rho} = \Delta T/\bar{T} = \varepsilon < 2 \qquad (13)$$

and that for the two problems

Problem 1 | **Problem 2**

$$\rho^{*\prime} = [T^{*\prime}/(1 - \varepsilon T^{*\prime})][1 + G(\varepsilon)/(\varepsilon T^{*\prime})]; \qquad \rho^{*\prime} = [T^{*\prime}/(1 + \varepsilon T^{*\prime})][1 - G(\varepsilon)/(\varepsilon T^{*\prime})];$$

$$\rho^{*} = W(T^{*\prime}; \varepsilon);\ C_p^{*} = C_p^{*}(-\varepsilon T^{*\prime}); \qquad \rho^{*} = W(T^{*\prime}; -\varepsilon);\ C_p^{*} = C_p^{*}(\varepsilon T^{*\prime}); \tag{14}$$

$$M^{*} = M^{*}(-\varepsilon T^{*\prime});\ K^{*} = K^{*}(-\varepsilon T^{*\prime}); \qquad M^{*} = M^{*}(\varepsilon T^{*\prime});\ K^{*} = K^{*}(\varepsilon T^{*\prime});$$

where

$$W(x; \sigma) = 1 + [\sigma x/(1 - \sigma x)][1 + G(\sigma)/(\sigma x)]$$
$$G(\sigma) = G(-\sigma) = 4/[2 + (1 - \sigma/2)/(1 + \sigma/2) + (1 + \sigma/2)/(1 - \sigma/2)] - 1 \tag{15}$$

and where, in some region around x = 0

$$C_p^{*}(x) = 1 + \sum_{n=1}^{\infty} c_n x^n;\ c_n = \{\bar{T}^n/[n!C_p(\bar{T})]\}d^nC_p(T)/dT^n\Big|_{T=\bar{T}}$$
$$M^{*}(x) = 1 + \sum_{n=1}^{\infty} m_n x^n;\ m_n = \{\bar{T}^n/[n!\mu(\bar{T})]\}d^n\mu(T)/dT^n\Big|_{T=\bar{T}} \tag{16}$$
$$K^{*}(x) = 1 + \sum_{n=1}^{\infty} K_n x^n;\ K_n = \{\bar{T}^n/[n!k(\bar{T})]\}d^nk(T)/dT^n\Big|_{T=\bar{T}}$$

According to assumed analytic character of C, μ, and k, $C_p^*(x)$, $M^*(x)$, and $K^*(x)$ can be continued analytically along the real x axis in the range $-\varepsilon/2 \le x \le \varepsilon/2$.

The boundary value problems in their dimensionless form are presented in Table 2 [Eqs. (17)-(20)] where the Grashoff and Prandtl numbers introduced there are defined as

$$\bar{Gr} = 2gD^3\varepsilon/[\mu(\bar{T})/\bar{\rho}]^2;\ \bar{Pr} = C_p(\bar{T})\mu(\bar{T})/k(\bar{T}) \tag{21}$$

For specified values of ε, Π, $\bar{Gr}$, and $\bar{Pr}$ a solution to Problem 1 or 2 yields

$$U_i^{*(N)} = U_i^{*(N)}(X_i^*; \Pi, \varepsilon, \bar{Gr}, \bar{Pr});$$
$$p^{*\prime(N)} = p^{*\prime(N)}(X_i^*; \Pi, \varepsilon, \bar{Gr}, \bar{Pr}); \tag{22}$$
$$T^{*\prime(N)} = T^{*\prime(N)}(X_i^*; \Pi, \varepsilon, \bar{Gr}, \bar{Pr})$$

where the superscript N = 1 or 2 denotes a solution to Problem N. For Problem 1, the above would be solutions to the boundary value problem defined by the left and center equations of Eqs. (17)-(20). Solutions to Problem 2 would satisfy the right and center equations.

THE RELATIONSHIP BETWEEN PROBLEMS 1 AND 2 AND THEIR SOLUTIONS

Assume a solution is available for one of the two problems, say Problem 1, for specified values Π, $\bar{G}r$, $\bar{P}r$, and for arbitrary $\varepsilon > 0$. Now assume this solution can be extended analytically to $\varepsilon \leq 0$. Although such a negative-ε solution may be available, note that there is no *a priori* reason to suspect that it is physically meaningful. From Eqs. (17)-(20) it can now be seen that for a particular $\varepsilon = \varepsilon_1 < 0$ the available "non-physical" solution is also a solution to "physical" Problem 2, for $\varepsilon = -\varepsilon_1 > 0$. Thus, if $\varepsilon < 0$ in the presumed known solution to one of the problems is replaced by $-\varepsilon = |\varepsilon|$, then the boundary value problem satisfied by this available solution becomes identical to that of the other problem for $\varepsilon = |\varepsilon| > 0$. In other words, the available solution to the one "non-physical" problem is a useful, physically-meaningful solution to the other. In particular

$$
\begin{aligned}
U_i^{*(1)}(X_i^*;\ \Pi,\ +/-\ \varepsilon,\ \bar{G}r,\ \bar{P}r) &= U_i^{*(2)}(X_i^*;\ \Pi,\ -/+\ \varepsilon,\ \bar{G}r,\ \bar{P}r);\\
p^{*'(1)}(X_i^*;\ \Pi,\ +/-\ \varepsilon,\ \bar{G}r,\ \bar{P}r) &= p^{*'(2)}(X_i^*;\ \Pi,\ -/+\ \varepsilon,\ \bar{G}r,\ \bar{P}r);\\
T^{*'(1)}(X_i^*;\ \Pi,\ +/-\ \varepsilon,\ \bar{G}r,\ \bar{P}r) &= T^{*'(2)}(X_i^*;\ \Pi,\ -/+\ \varepsilon,\ \bar{G}r,\ \bar{P}r)
\end{aligned}
\tag{23}
$$

In general one would hope that the above ideas can be used as an aid in unifying solutions to Problems 1 and 2. For example, the problem formulation and the form of the $U_i^{*(N)}$, $p^{*'(N)}$, and $T^{*'(N)}$ can be used as a theoretical basis for correlating experimentally determined values of these solution variables and of properties of important quantities derived from these variables.

Examples of the latter referenced derived quantities are vent flow rates and vent flow coefficients, when the flow is uni-directional, and vent exchange flow rates, when the flows are bi-directional. For example, define $\dot{V}_{HIGH}^{(N)}$ as the volume flow rate, under uni-directional flow conditions, from the high pressure side of the vent to the low pressure side of the vent,

$$\dot{V}_{HIGH}^{(N)} = -\int_{A_V} U_i^{*(N)}(X_1^{(N)}, X_2^{(N)}, X_3^{(N)} = 0)dA_V = -A_V\bar{U}_3^{(N)} \tag{24}$$

In the above, the integral is over the area of the vent, A_V, and $\bar{U}_3^{(N)}$is the average value of $U_3^{(N)}$ at $X_3^{(N)} = 0$. Then, defining the Froude number

$$\bar{F}^{(N)} = (\dot{V}_{HIGH}^{(N)}/A_V)/(2gD\varepsilon)^{1/2} \tag{25}$$

it can be shown from the first of Eqs. (22) and (23) that

$$(\dot{V}^{(N)}_{HIGH}/A_V)/(2gD\varepsilon)^{1/2} = \bar{F}^{(N)}(\Pi, \varepsilon, \bar{G}r, \bar{P}r)$$
$$\bar{F}^{(1)}(\Pi, +/- \varepsilon, \bar{G}r, \bar{P}r) = \bar{F}^{(2)}(\Pi, -/+ \varepsilon, \bar{G}r, \bar{P}r) \quad (26)$$

SUMMARY AND APPLICATION

Flow through a horizontal vent was considered where the vent-connected spaces near the elevation of the vent are filled with fluids of different density in an unstable configuration, with the density of the top space larger than that of the bottom space. With zero-to-moderate cross-vent pressure difference the instability leads to a bi-directional exchange flow between the two spaces. For relatively large cross-vent pressure difference the flow through the vent is unidirectional, from the high- to the low-pressure space.

In its full generality, the problem and its solution has a broad range of importance and applicability. Of interest here are the aspects of the problem of predicting the flow, in the presence of arbitrary cross-vent pressure difference, between relatively cool and dense air in the top space and relatively high-temperature and buoyant air in the bottom space. This is a general problem associated with ventilation of enclosed, heated/cooled spaces. It is a problem whose general solution is required if one is to be able to predict the spread of smoke (i.e., fire-heated and -contaminated air) and the flow of fresh air (i.e., oxygen, which could sustain a fire, lack of which could extinguish a fire) during fires in multi-room facilities. Reference here is to smoke spread between contiguous rooms, or between a smokey room and the outside environment, separated by a horizontal partition (i.e., ceiling/floor) with penetrations (i.e., vents), where room-to-room or room-to-outside, cross-vent, pressure differences of arbitrary magnitude and direction can be generated by forced-ventilation HVAC systems, buoyancy forces (i.e., stack effect), and/or wind effects. The problem has application in fire scenarios involving top-vented atria, stairwells, shipholds, etc.

For arbitrary specified cross-vent pressure difference, boundary value problems for the flow were formulated for cases where the fluid media in the two spaces are the same perfect gas, with relatively high and low temperature (corresponding to low and high density) in the lower and upper spaces, respectively. Two separate classes of problem were distinguished. In Problem 1, the higher pressure is in the space above the vent. This enhances the downward component of the flow from the top to the bottom space, and diminishes, or reduces to zero, the upward flow. In Problem 2, the higher pressure is in the lower space leading to enhancement of the upward flow, etc. Relationships between the two boundary value problems and their solutions were identified. These relationships would be useful for extending an available solution for one class of problem to that of the other and for unified understanding and correlation of experimental data for the two flow configurations.

The theoretical results developed here are the basis of the analysis of Reference [10] which uses a variety of previously published experimental data to develop a unified solution to Problems 1 and 2 for flow through a shallow circular vent.

REFERENCES

[1] Taylor, G. I., The Instability of Liquid Surfaces When Accelerated in a Direction Perpendicular to Their Planes, Proc. Roy. Soc., A, 201, pp. 192-196, 1950.

[2] Cooper, L.Y., Calculation of the Flow Through a Horizontal Ceiling/Floor Vent, NISTIR 89-4052, NIST, Gaithersburg MD, 1989.

[3] Epstein, M., Buoyancy-Driven Exchange Flow Through Small Openings in Horizontal Partitions, Jl. Heat Transfer, 110, pp. 885-893, 1988.

[4] Brown, W.G., Natural Convection Through Rectangular Openings in Partitions - 2. Horizontal Partitions, Int. J. Heat Mass Transfer, 5, pp. 869-878, 1962.

[5] Epstein, M. and Kenton, M.A., Combined Natural Convection and Forced Flow Through Small Openings in a Horizontal Partition With Special Reference to Flows in Multicomparment Enclosures, Jl. Heat Transfer, 111, pp. 980-987, 1989.

[6] Tan, Q. and Jaluria, J., Flow Through Horizontal Vents as Related to Compartment Fire Environments, Rutgers University Report to NIST, Gaithersburg MD, Dec. 1991.

[7] Heskestad, G. and Spaulding, R.D., Inflow of Air Required at Wall and Ceiling Apertures to Prevent Escape of Fire Smoke, Proceedings of 3rd Intl. Symp. Fire Safety Science, pp. 919-928, Elsevier, 1991.

[8] Cooper, L.Y., An Algorithm and Associated Computer Subroutine for Calculating Flow Through a Horizontal Ceiling/Floor Vent in a Zone-Type Compartment Fire Model, NISTIR 4402, NIST, Gaithersburg MD, 1990.

[9] Yamada, T. and Cooper, L.Y., Experimental Study of the Exchange Flow Through a Horizontal Ceiling Vent in Atrium Fires, Abstracts: Annual Conference on Fire Research, October 13-15, 1992, NISTIR 4924, S.B. Smith, ed., NIST, Gaithersburg MD, pp. 35-36, 1992.

[10] Cooper, L.Y., Combined Buoyancy- and Pressure-Driven Flow Through a Shallow Horizontal Circular Vent, to appear as NISTIR, NIST, Gaithersburg, MD.

NOMENCLATURE

A_V	vent area
c_N	coefficients in a Taylor expansion, Eq. (16)
C_p	specific heat at constant pressure
C_p^*	dimensionless C_p, Eq. (12)
D	characteristic length of vent
$f_i^{(1)}$, $f_i^{(2)}$	body force acceleration vector for Configurations 1 and 2, Eq. (6)
$F_i^{*(1)}$, $F_i^{*(2)}$	dimensionless $f_i^{(1)}$, $f_i^{(2)}$, Eq. (11)
$\mathcal{F}_i^*$	generalized $F_i^{*(1)}$, $F_i^{*(2)}$, Eq. (18)
$\bar{F}^{(N)}$	Froude number for Problem N, Eq. (25)
$G(\sigma)$	Eq. (15)
$\bar{G}r$	Grashoff number, Eq. (21)

g	acceleration of gravity
K_N	coefficients in a Taylor expansion, Eq. (16)
K^*	dimensionless k, Eq. (12)
k	thermal conductivity
M^*	dimensionless μ, Eq. (12)
m_N	coefficients in a Taylor expansion, Eq. (16)
p	pressure
p_{HIGH}, p_{LOW}	far-field pressure on high-, low-pressure side of vent, near the vent elevation
$\bar{p}$	$(p_{HIGH} + p_{LOW})/2$
p'	perturbation pressure, Eq. (11)
p^*	dimensionless p, Eq. (11)
$p^{*'}$	dimensionless p', Eq. (11)
$p^{*'(N)}$	$p^{*'}$ for Problem N
$\bar{P}r$	Prandtl number, Eq. (21)
R	gas constant
T	absolute temperature
T_{TOP}, T_{BOT}	far-field T in top, bottom space
$\bar{T}$	$(T_{TOP} + T_{BOT})/2$
T'	perturbation T, Eq. (11)
T^*	dimensionless T, Eq. (11)
$T^{*'}$	dimensionless T'
$T^{*'(N)}$	$T^{*'}$ for Problem N
U_i	velocity
U_i^*	dimensionless U_j, Eq. (11)
$\bar{U}_3^{(N)}$	average U_3 at vent elevation for Problem N, Eq. (24)
$U_i^{*(N)}$	U_i^* for Problem N

$\dot{V}^{(N)}_{HIGH}$	volumetric flow rate from high- to low-pressure side of vent for Problem N, Eq. (24)
$W(x; \sigma)$	Eq. (15)
X_i	cartesian coordinates, Fig. 1
X_i^*	dimensionless X_i, Eq. (11)
Δp	$p_{HIGH} - p_{LOW}$
ΔT	$T_{BOT} - p_{TOP}$
$\Delta \rho$	$\rho_{TOP} - \rho_{BOT}$
ε	dimensionless $\Delta\rho$, ΔT, Eq. (13)
ρ_{TOP}, ρ_{BOT}	far-field ρ in top, bottom space
$\bar{\rho}$	$(\rho_{TOP} + \rho_{BOT})/2$
ρ'	perturbation ρ, Eq. (11)
ρ^*	dimensionless ρ, Eq. (11)
$\rho^{*'}$	dimensionless ρ', Eq. (14)
Λ	Eq. (6)
μ	dynamic viscosity
Π	dimensionless Δp, Eq. (20)

Problem 1 | **Problem 2**

Conservation of Mass

$$\partial(\rho U_j)/\partial X_j = 0 \quad (5)$$

Conservation of Momentum

Problem 1:

$$\rho U_j \partial U_i/\partial X_j - \rho f_i^{(1)} = \Lambda$$

$$f_i^{(1)} = (0, 0, -g)$$

Problem 2:

$$\rho U_j \partial U_i/\partial X_j - \rho f_i^{(2)} = \Lambda$$

$$f_i^{(2)} = (0, 0, g) \quad (6)$$

$$\Lambda = -\partial p/\partial X_i + \partial[\mu(\partial U_i/\partial X_j + \partial U_j/\partial X_i)]/\partial X_j - (2/3)\partial(\mu \partial U_j/\partial X_j)/\partial X_i$$

Conservation of Energy

$$\rho C_p U_j \partial T/\partial X_j = \partial(k\partial T/\partial X_j)/\partial X_j \quad (7)$$

Equation of State

$$p = \rho R T \quad (8)$$

Boundary Conditions: (9)

$\underline{X_3 > 0,\ \vec{r} \to \infty}$:

$$U_i \to 0$$

Problem 1	Problem 2
$p \to p_{HIGH} - \rho_{TOP} g X_3$	$p \to p_{HIGH} + \rho_{BOT} g X_3$
$\rho \to \rho_{TOP}$	$\rho \to \rho_{BOT}$

$\underline{X_3 < 0,\ \vec{r} \to \infty}$:

$$U_i \to 0$$

Problem 1	Problem 2
$p \to p_{LOW} - \rho_{BOT} g X_3$	$p \to p_{LOW} - \rho_{TOP} g X_3$
$\rho \to \rho_{BOT}$	$\rho \to \rho_{TOP}$

$\underline{X_3 = 0^+ \text{ or } 0^-\text{; outside the vent opening}}$:

$$U_i = \partial T/\partial X_3 = 0$$

Table 1. **Dimensional Boundary Value Problems 1 and 2, corresponding to Configurations 1 and 2 of Figure 1.**

Problem 1 **Problem 2**

Conservation of Mass

$$\partial(\rho^* U_j^*)/\partial X_j^* = 0$$

$$\rho^* = W(T^{*\prime}; \varepsilon) \qquad \rho^* = W(T^{*\prime}; -\varepsilon) \tag{17}$$

Conservation of Momentum

$$\rho^* U_j^* \partial U_i^*/\partial X_j^* =$$

$$\rho^{*\prime} F_j^* - \partial p^{*\prime}/\partial X_i^* + (1/\bar{G}r^{1/2})\{\partial[M^*(\partial U_i^*/\partial X_j^* + \partial U_j^*/\partial X_i^*)]/\partial X_j^* - (2/3)\partial(M^* \partial U_j^*/\partial X_j^*)/\partial X_i^*\}$$

$$F_j^* = (0, 0, -1/2) \tag{18}$$

$$\rho^{*\prime} = [W(T^{*\prime}; \varepsilon) - 1]/\varepsilon; \qquad \rho^{*\prime} = -[W(T^{*\prime}; -\varepsilon) - 1]/\varepsilon;$$

$$M^* = M^*(-\varepsilon T^{*\prime}) \qquad M^* = M^*(\varepsilon T^{*\prime})$$

Conservation of Energy

$$\rho^* U_j^* \partial T^{*\prime}/\partial X_j^* = [1/(\bar{P}r\bar{G}r^{1/2})](1/C_p^*)\partial(K^* \partial T^{*\prime}/\partial X_j^*)/\partial X_j^*$$

$$C_p^* = C_p^*(-\varepsilon T^{*\prime});\ K^* = K^*(-\varepsilon T^{*\prime}) \qquad C_p^* = C_p^*(\varepsilon T^{*\prime});\ K^* = K^*(\varepsilon T^{*\prime}) \tag{19}$$

Boundary Conditions:

$\underline{X_3^* > 0,\ \vec{r}^* \to \infty}$:

$$U_i^* \to 0;\ p^{*\prime} \to \Pi - X_3^*/4;\ T^{*\prime} \to 1/2;$$

$$\Pi = \Delta p/(4g\Delta\rho D) \tag{20}$$

$\underline{X_3^* < 0,\ \vec{r}^* \to \infty}$:

$$U_i^* \to 0;\ p^{*\prime} \to -\Pi + X_3^*/4;\ T^{*\prime} \to -1/2$$

$\underline{X_3^* = 0^+ \text{ or } 0^-\text{; outside the vent opening}}$:

$$U_i^* = \partial T^{*\prime}/\partial X_3^* = 0$$

Table 2. **Dimensionless Boundary Value Problems 1 and 2, corresponding to Configurations 1 and 2 of Figure 1.**