

## **BENDING PERFORMANCE OF MASONRY WALLS STRENGTHENED WITH NEAR-SURFACE MOUNTED FRP BARS**

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### **Abstract**

Unreinforced masonry (URM) buildings constitute an important part of the building stock, and are vulnerable to earthquakes. Retrofitting of existing masonry walls with near surface mounted, non-corrosive fiber-reinforced polymer (FRP) bars, is an attractive option. Previous research has shown that it is difficult to develop the full tensile strength of the bars, which are attached with epoxy in grooves pre-cut in the masonry and mortar joints. In this study, epoxy strengthened with short glass fibers allows close to full strength development of 6.5 mm ( $\frac{1}{4}$  in) diameter glass FRP bars in 185 mm (7.3 in), or less than half a concrete masonry unit length. This fiber-reinforced epoxy should be effective for other types of bars as well.

With full anchorage assured, FRP bars provide an efficient method of strengthening masonry walls against out-of-plane bending. Three narrow (2.85 m  $\times$  0.40 m  $\times$  0.20 m or 112 in  $\times$  16 in  $\times$  8 in), grouted, concrete masonry beams reinforced to 45 % of balanced ratio and four wide (2.85 m  $\times$  0.80 m  $\times$  0.20 m or 112 in  $\times$  32 in  $\times$  8 in) beams reinforced to 66 % of balanced ratio were tested in four-point bending. All seven beams exhibited consistent flexural behavior, with ultimate failure precipitated by tensile rupture of the reinforcement at an average ratio of span to maximum deflection of 42.

The ACI (530-02) equations predict the flexural strength of these FRP externally reinforced concrete masonry beams conservatively, with a mean ratio of measured to predicted strength of 1.37 and a coefficient of variation of 0.057. There was no difference in behavior between reinforcement parallel or perpendicular to the mortar bed joints. The unexpected shear failure of one FRP reinforced, ungrouted beam needs further investigation.

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## Introduction

Unreinforced masonry (URM) buildings constitute an important part of the building stock, and are vulnerable to earthquakes. According to Tumialan et al. (2001), 96 % of the URM buildings in California need seismic retrofitting, at a cost of US \$ 4 billions. Retrofitting has proven to be effective: a survey of URM bearing wall buildings after the Northridge Earthquake (1994) showed that, of the inspected buildings, 67 % of the unretrofitted buildings suffered some damage, compared with 55 % of the retrofitted buildings (Lizundia et al. 1997). Although masonry walls have failed out-of-plane much more frequently than they have in-plane, more research and testing have aimed at improving in-plane than out-of-plane wall capacity. The focus of the present research is the strengthening of concrete masonry walls against out-of-plane bending using external fiber-reinforced polymer (FRP) bars embedded in grooves cut into the face of the masonry and mortar joints. Following a review of the relevant literature, the paper will cover results of bond and bending tests, and compare measurements with predictions of ultimate strength.

## Literature Review

Two traditional techniques that address out-of-plane deficiencies in walls that are too high for their thickness are diagonal bracing and strongbacks. More recent wall strengthening methods, such as applying shotcrete to a wall surface, or grouting reinforcing bars within vertical cores drilled through an URM wall, have been extensively implemented on the West Coast of the US. Other techniques, such as adhering high-strength fabric or surface coating, have seen more limited use.

Hamilton and Dolan (2001) reinforced two tall walls ( $1.2 \text{ m} \times 4.7 \text{ m}$  or  $48 \text{ in} \times 185 \text{ in}$ ) and four short walls ( $0.6 \text{ m} \times 1.8 \text{ m}$  or  $24 \text{ in} \times 72 \text{ in}$ ) to 26 % and 13 % of their balanced ratio respectively. The specimens were loaded by an air bag and failed by delamination and/or fracture of the E-glass fabric. The tall walls achieved a ratio of span to maximum ultimate deflection of 55 to 59. General flexural strength design equations similar to those used in reinforced concrete design overpredicted the actual capacity by no more than 20 %.

Albert et al. (2001) reinforced ten walls ( $4 \text{ m} \times 1.2 \text{ m}$  or  $157 \text{ in} \times 48 \text{ in}$ ) with carbon strips or fabric and tested them under four-point bending. Despite the large shear span to depth ratio of 6, flexure-shear was the mode of failure for most of the tests. Reinforced walls could carry up to 50 times more load than unreinforced ones, and achieved a ratio of span to ultimate deflection of 40. The authors used a triangular compressive stress distribution in the flange of the masonry unit, but their predictions for loads and deflections did not agree well with experimental measurements.

Based on the results of seven half-scale brick masonry walls reinforced with glass-fiber fabric and tested with air bags in cyclic out-of-plane bending, Velazquez-Dimas and Ehsani (2000) concluded that ultimate strength analysis similar to ACI 318 methods for the bending of reinforced concrete (RC) beams did not work. This is not surprising since the tensile reinforcement never reached the tensile strength used in the analysis. Triantafillou (1998) presented design equations and normalized interaction equations for the strength of masonry

walls reinforced with externally bonded FRP strips and subjected to out-of-plane bending, in-plane bending, or in-plane shear in conjunction with axial stresses.

Nanni and his colleagues at the University of Missouri-Rolla mounted FRP bars near the surface of concrete beams and masonry walls, in pre-cut grooves, to strengthen them in bending and shear. This “structural repointing” requires less surface preparation, and results in less appearance change than the adhered fabric technique (Tumialan et al. 2001). De Lorenzis and Nanni (2002) bonded 9.5 mm (3/8 in) and 13 mm (1/2 in) glass or carbon fiber bars, ribbed or sand-coated, to square grooves cut in the tensile face of concrete beams. They measured the bond between concrete and bar by hinged beam tests and observed three modes of failure: splitting of the epoxy cover, cracking of the concrete surrounding the groove, and pullout of the FRP bars. Ribbed bars exhibited better bond than sand-coated ones. Larger groove size, and thus, thicker cover, led to higher bond strength when failure was controlled by splitting of the epoxy cover, but had no effect when failure occurred by pullout. Optimum groove sizes were 19 mm (3/4 in) for 9.5 mm (3/8 in) bars and 25 mm (1 in) for 13 mm (1/2 in) bars. If the groove was deep enough to cause failure to occur in the concrete, then concrete tensile strength became a significant parameter. The authors proposed a bond stress-slip relationship and recommended further study of epoxy resins with higher tensile strength.

The objective of the present research was to establish the effectiveness of and provide design guidelines for near surface mounted rods in strengthening concrete masonry walls for out-of-plane flexure, and in a second phase, in-plane shear. Preliminary tests aimed at measuring the tensile strength (not reported here due to page limitation) and bond strength of the FRP bars used. Next, flexural tests were performed, and results compared with predictions.

## Bond Tests

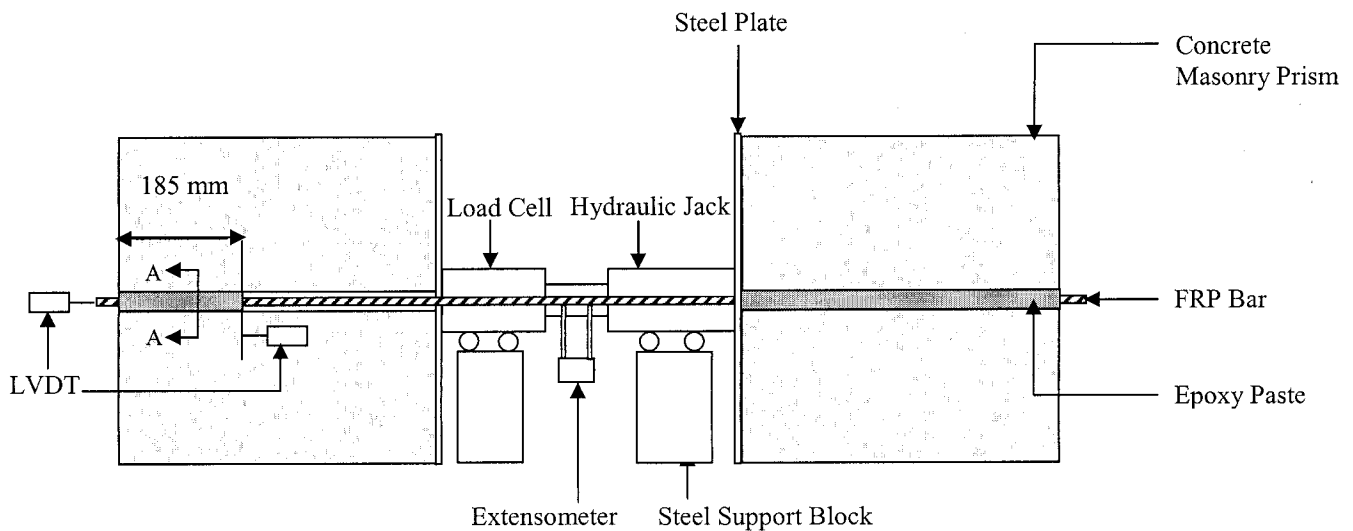


Figure 1: Test setup for determining bond strength

Since a loss of bond would render the FRP bars ineffective in strengthening, it is important to ensure sufficient anchorage length and to improve bond if necessary, in order to develop the tensile strength of the bar. Fig. 1 shows the set-up for the bond tests performed in the present research as a prerequisite to using FRP bars for external strengthening of concrete masonry walls in flexure and in shear. Each test specimen consisted of two masonry prisms placed at a clear spacing of 405 mm (16 in) and connected by two parallel bars 1220 mm (48 in) long. Each prism consisted of two concrete masonry units joined by mortar. FRP bars of two different sizes, 9.5 mm (3/8 in) and 6.5 mm (1/4 in) in diameter from two different manufacturers were bonded to the prisms in grooves cut along the mortar joint. The grooves were square in cross section and measured 19 mm (3/4 in) to the side for the 9.5 mm (3/8 in) bars and 13 mm (1/2 in) for the 6.5 mm (1/4 in) bars. The grooves were cut wide enough so all mortar was removed, thus ensuring that bars were bonded to masonry units on two sides of the groove. The FRP bar was bonded to one prism along the full length of the mortar joint, but to the other, only along a length of 185 mm (7.3 in), or slightly less than half the masonry unit length. This length was chosen because shear walls often crack along mortar joints, and thus, in running bond masonry, the minimum distance from the edge of a shear wall to the first vertical crack is half the length of a masonry unit. It is therefore desirable to develop the tensile strength of the bar over half a unit length. (As mentioned earlier, one of the objectives of this research program is to investigate the use of FRP bars in strengthening masonry shear walls.)

After the adhesive had cured as specified by the manufacturer, the concrete prisms were pushed apart with a hydraulic jack placed in series with a load cell. Symmetrical loading was ensured by monitoring the strain on each bar with extensometers and re-positioning the load if necessary. Further, linear variable differential transformers (LVDT) measured the displacement and slip of each bar with respect to the masonry unit, at the loaded and the free ends of the bonded segment.

A first test series was aimed at measuring the relative performance of three systems: two epoxies and a latex-modified mortar made in the following proportion: 2.04 kg sand; 0.68 kg cement; and 0.379 L acrylset polymer. Table 1 lists the compressive strength of the mortar mixture measured according to ASTM C39 after 14 days of air cure, and the mechanical properties of the two epoxies according to the manufacturer. Table 2 shows the bond test results for bar A#3 to concrete masonry using the three systems (one test per system). Epoxy #2, being superior, was selected for further work.

Table 1: Mechanical Properties of Epoxies and Mortar Mixture

Adhesive	Tensile Strength MPa (psi)	% Elongation	Compressive Strength MPa (psi)
Epoxy #1	13.8 (2000)	4.0	55.3 (8020)
Epoxy #2	34.6 (5020)	1.8	67.7 (9820)
Mortar mixture	---	---	29.5 (4280)

Table 2: Bond Test Results for Various Adhesives with Bar A#3

Adhesive	Ultimate Load, kN (lbf)	% of Bar Strength	Failure Mode
Epoxy #1	17.8 (4000)	29	Epoxy split
Epoxy #2	20.2 (4540)	33	Epoxy split
Mortar mixture	17.6 (3960)	28	Bond failed

Table 3: Bond Strength of FRP Bars with Concrete Masonry using Epoxy #2

FRP Bar	Ultimate Load, kN (lbf)	% of Bar Strength	Failure Mode
A#3-1	20.2 (4540)	33	Epoxy split
A#3-2	20.0 (4500)	32	
B#3-1	23.9 (5370)	44	
A#2-1	15.8 (3550)	54	
A#2-2	16.9 (3800)	58	
B#2-1	18.4 (4140)	68	

Table 4: Bond Strength of B#2 FRP Bars Bonded with Short Fiber Reinforced Epoxy

FRP bar	Ultimate Load, kN (lbf)	% of Bar Strength	Failure Mode
B#2-1	25.2 (5670)	93	Bar ruptured; adhesive and concrete split.
B#2-2	26.3 (5910)	97	
B#2-3	21.0 (4720)	78	Concrete failed in shear.

A second series of tests evaluated the bond performance of Epoxy #2 with the four types of FRP bars. The results, which are summarized in Table 3, showed that smaller bars achieved higher bond strength relative to their tensile strength. This is expected, as smaller bars have a higher ratio of perimeter to cross sectional area than larger bars. Test results also showed that bars B, which were sand-coated with a helical fiber tow on the surface, achieved higher bond strength than bars A, which had circular ribs on a smooth surface finish. No slip occurred at the free end of the bars. In all cases, half a masonry unit length was insufficient to develop the strength of the bars.

In a third series of tests, the epoxy was reinforced with 3 mm (0.12 in) long glass fibers at a volume fraction of 5 %. Test results are shown in Table 4. In two tests, the bars failed in tension, at 93 % and 97 % of the uniaxial tensile strength. In a third test, failure was in the concrete masonry unit, by shear near the bond line. Thus, reinforcing the epoxy with short glass fibers enhances significantly its strength and allows the 6.5 mm diameter bars to develop close to their full strength in half a masonry unit length.

### Bending Tests

Masonry walls can bend out-of-plane when subjected to lateral loads caused by wind, earthquake or blast. Since masonry is weak in tension, reinforcing is sometimes required to enhance flexural resistance.

## Nominal Flexural and Shear Strengths

The flexural strength of masonry beams under-reinforced with steel can be predicted by the ACI ultimate strength method (ACI 530-02), which is adapted here to FRP reinforced masonry beams:

$$\text{Force equilibrium: } A_f f_u = (0.80 f'_m)(0.80 c) b \quad (1)$$

where  $A_f$  is the cross sectional area of reinforcement with tensile strength  $f_u$ ,  $f'_m$  is the masonry compressive strength,  $b$  the beam width, and  $c$  the compression depth.

Ultimate moment, assuming reinforcement ruptures before concrete crushes:

$$M_n = A_f f_u (d - 0.40 c) \quad (2)$$

where  $M_n$  is the nominal flexural strength and  $d$  the beam depth.

The balanced ratio of reinforcement is obtained by setting the compressive strain in the masonry equal to 0.0025 and the tensile strain in the FRP equal to 0.0196 simultaneously. The value of ultimate tensile strain was the average of three tensile tests performed according to Castro and Carino (1998). The compression depth in this case is 21.9 mm (0.86 in) compared to a flange nominal thickness of 32 mm (1.25 in).

$$\text{Load at flexural failure: } P_n = \frac{2M_n}{g} \text{ where } g \text{ is the beam shear span.} \quad (3)$$

Nominal shear strength of running bond masonry not solidly grouted, not reinforced in shear,

$$\text{and not loaded axially (ACI 530-02): } \frac{V_n}{\text{N}} = 0.386 \frac{A_n}{\text{mm}^2} \text{ or } \frac{V_n}{\text{lbf}} = 56 \frac{A_n}{\text{in}^2} \quad (4)$$

where  $A_n$  is the net cross sectional area of the masonry beam.

Nominal shear strength of running bond masonry grouted solid, not reinforced in shear, and

$$\text{not loaded axially (ACI 530-02): } \frac{V_{ng}}{\text{N}} = 0.621 \frac{A_n}{\text{mm}^2} \text{ or } \frac{V_{ng}}{\text{lbf}} = 90 \frac{A_n}{\text{in}^2} \quad (5)$$

Table 5 shows the various parameters and the calculated nominal strengths for a narrow beam (two-course wide) and a wide beam (four-course wide). The wide beam has 1.5 times the ratio of reinforcement of the narrow beam. Experimentally, the ultimate moment and shear force are caused by the applied load and the dead load of the beam:

$$M'_e = M_e + \frac{wL^2}{8} = \frac{P_e g}{2} + \frac{wL^2}{8} \quad (6)$$

$$V'_e = \frac{P_e + wL}{2} \quad (7)$$

where  $M'_e$  is the ultimate moment,  $M_e$  the ultimate applied moment,  $w$  the dead load per unit length,  $L$  the beam span,  $P_e$  the applied load, and  $V'_e$  the ultimate shear force.

In Table 5,  $\rho_{bu}$  and  $\rho_{bg}$  are the balanced reinforcement ratios for ungrouted and grouted beams respectively, whereas  $\rho_u$  and  $\rho_g$  are the corresponding actual reinforcement ratios. The dead load is calculated using a density of 1920 kg / m<sup>3</sup> (120 lb / ft<sup>3</sup>) for both the masonry units and grout. The masonry compressive strength was measured according to ASTM C140 and averaged 11.3 MPa (1640 psi) for three specimens.

Table 5: Beam Flexural and Shear Strengths

	Units	Narrow beam	Wide beam	Equation
$g$	mm (in)	1 016 (40)	1 016 (40)	
$b$	mm (in)	394 (15.5)	797 (31.4)	
$w_g$	kN/m (lbf/ft)	1.56 (107)	3.11 (213)	
$A_f$	mm <sup>2</sup> (in <sup>2</sup> )	34 (0.053)	102 (0.158)	
$f_u$	MPa (ksi)	790 (115)	790 (115)	
$f_m'$	MPa (psi)	11.3 (1640)	11.3 (1640)	
$A_n$	mm <sup>2</sup> (in <sup>2</sup> )	25 000 (38.8)	50 000 (77.5)	
$A_g$	mm <sup>2</sup> (in <sup>2</sup> )	76 400 (118)	154 600 (240)	
$\rho_{bu}$	%	0.31	0.31	
$\rho_{bg}$	%	0.10	0.10	
$\rho_u$	%	0.14	0.20	
$\rho_g$	%	0.045	0.066	
$c$	mm (in)	9.5 (0.37)	14.0 (0.55)	1
$M_n$	kN·m (kip·ft)	5.07 (3.74)	14.97 (11.05)	2
$P_n/2$	kN (lbf)	4.99 (1122)	14.74 (3314)	3
$V_n$	kN (lbf)	9.67 (2173)	19.30 (4340)	4
$V_{ng}$	kN (lbf)	15.53 (3492)	31.03 (6975)	5

### Specimen Description

The FRP reinforcing bars were either parallel (Series 1), or perpendicular (Series 2) to the mortar bed joints. For each series, two narrow beams (two course wide) 2.85 m × 0.40 m × 0.20 m (112 in × 16 in × 8 in) and two wide beams (four course wide) 2.85 m × 0.80 m × 0.20 m (112 in × 32 in × 8 in) were tested. The narrow beams were each reinforced with a single bar in the middle of the tensile face, while the wide beams were each reinforced with three bars (Fig. 2). The bars were installed in 13 mm (½ in) square grooves and bonded along the entire length of the beams with epoxy reinforced with short glass fibers as described above.

### Test Set-Up

The beams were tested under four-point bending, with the bottom supports 2.65 m (104 in) apart and the top loads 0.61 m (24 in) apart. A steel tubular beam was used to spread the load from the testing machine to the two loading points. Steel rollers and steel bearing pads 102 mm × 102 mm × 13 mm (4 in × 4 in × ½ in) bonded to the masonry with a fast setting gypsum cement transferred the loads and reactions to the test beam.

The testing machine was of the screw driven type and the beams were loaded at a cross-head speed of 1.5 mm/min, except for Beam 1 which was loaded at 0.6 mm/min. An LVDT measured the midspan deflection and three strain gages mounted at midspan and 0.61 m (24 in) from the bar ends monitored the strains in each FRP bar. Data were recorded by a data acquisition system operating at a sampling frequency of 10 Hz. For the first beam that was tested, the cells in the masonry units at the support points and load points were grouted in order to prevent localized failure of the face shells. That beam failed unexpectedly in shear, so all subsequent beams had *all* their cells grouted to prevent that problem from recurring.

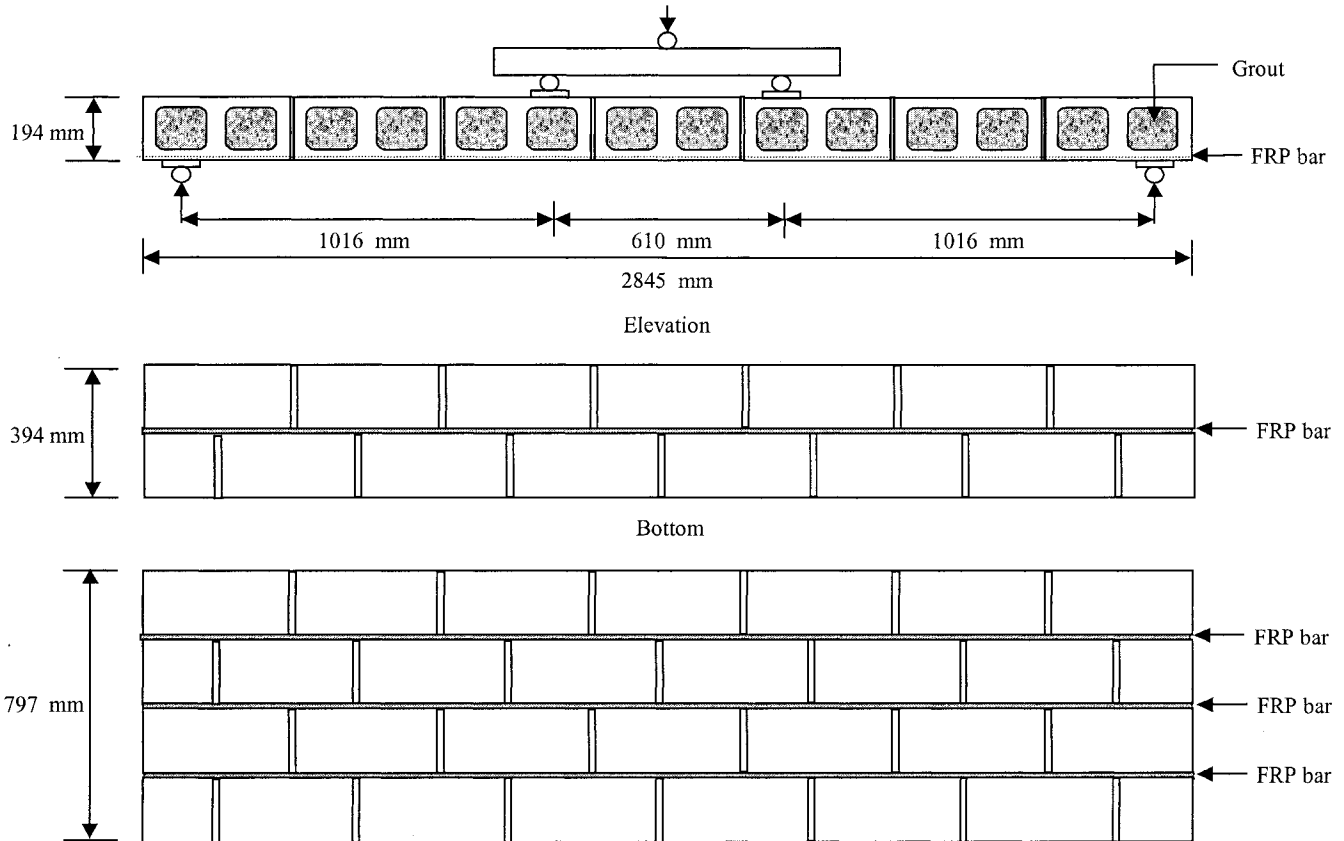


Fig. 2—Out of-Plane Bending Test of Concrete Masonry Walls, Series 1  
Narrow Beam and Wide Beam with FRP Bars Parallel to Mortar Bed Join

### Test Results

Figures 3 and 4 show the load-midspan deflection curves for the narrow and wide beams, respectively. Cracks developed in the tension flange of the units due to flexure and flexure-shear and caused the load to drop momentarily because testing was done at constant cross-head speed. The first beam to be tested was a narrow beam from Series 1, and had grout only in the cells directly loaded. Flexure and flexure-shear cracks reduced the area available to resist shear to the compression flange only, resulting in unexpected shear failure of the masonry at an applied load of 6.2 kN (1390 lbf) or only 43 % of the nominal shear strength



(Eq. 4) and a midspan deflection of 49 mm (1.93 in). These corresponded to three times the dead load of the masonry and a ratio of span to deflection of 54.

All subsequent beams were fully grouted to increase their shear resistance and they all failed in flexure by sudden rupture of the FRP tensile reinforcement. Table 6 shows the ultimate load and moment, ultimate midspan deflection  $\delta_u$ , FRP bar strains and ratios of measured to predicted ultimate moments or loads for all beams tested.

Table 6: Ultimate loads, moments, deflections and strains

Beam	$P_e$ kN (lbf)	$M_e'$ kN·m (kip·ft)	$\frac{M_e'}{M_n}$	$\frac{V_e'}{V_n}$	$\delta_u$ mm (in)	$\frac{L}{\delta_u}$	$\epsilon_m$ , % Middle	$\frac{\epsilon_m}{\epsilon_u}$ % Middle	$\epsilon_m$ , % End
1 //n*	6.2 (1390)	3.79 (2.80)	0.75	0.43	49.0 (1.93)	54	0.397	20	0.347
2 //n#	11.1 (2500)	7.00 (5.16)	1.38		61.2 (2.41)	43	Not recorded		
3 //w	32.7 (7350)	19.33 (14.26)	1.29		53.0 (2.09)	50	1.889	96	0.903
4 //w	34.2 (7690)	20.09 (14.82)	1.34		60.0 (2.36)	44	1.675	86	0.034
5 ⊥n	12.5 (2810)	7.71 (5.69)	1.52		73.0 (2.87)	36	1.878	96	0.118
6 ⊥n	11.6 (2610)	7.25 (5.35)	1.43		69.6 (2.74)	38	1.887	96	0.040
7 ⊥w	33.2 (7460)	19.58 (14.44)	1.31		66.0 (2.60)	40	1.757	90	0.945
8 ⊥w	34.4 (7730)	20.19 (14.89)	1.35		62.0 (2.44)	43	1.435	73	0.924
Mean <sup>+</sup>			1.37		63.5 (2.50)	42	1.754	89	
St.dev			0.079		6.63 (0.26)	4.6	0.179	9.1	
COV			0.057		0.10	0.11	0.10	0.10	

\* Failed in shear. #Reinforcing bars were either // or ⊥ to bed joints; n = narrow, w = wide.

<sup>+</sup> Mean, standard deviation and coefficient of variation (COV) were calculated for beams 2 to 8 for moment ratio and deflection, and for beams 3 to 8 for bar strain.

On average, for the beams that failed in flexure by bar rupture, the bars attained at midspan  $\epsilon_m / \epsilon_u = 89$  % of the ultimate strain achieved in a pure tension test.  $\epsilon_m$  is the FRP bar strain at ultimate moment, and  $\epsilon_u$  is the tensile strain capacity of FRP bars. The non-uniformity in loading within each bar, where load was transferred from the masonry through the epoxy by

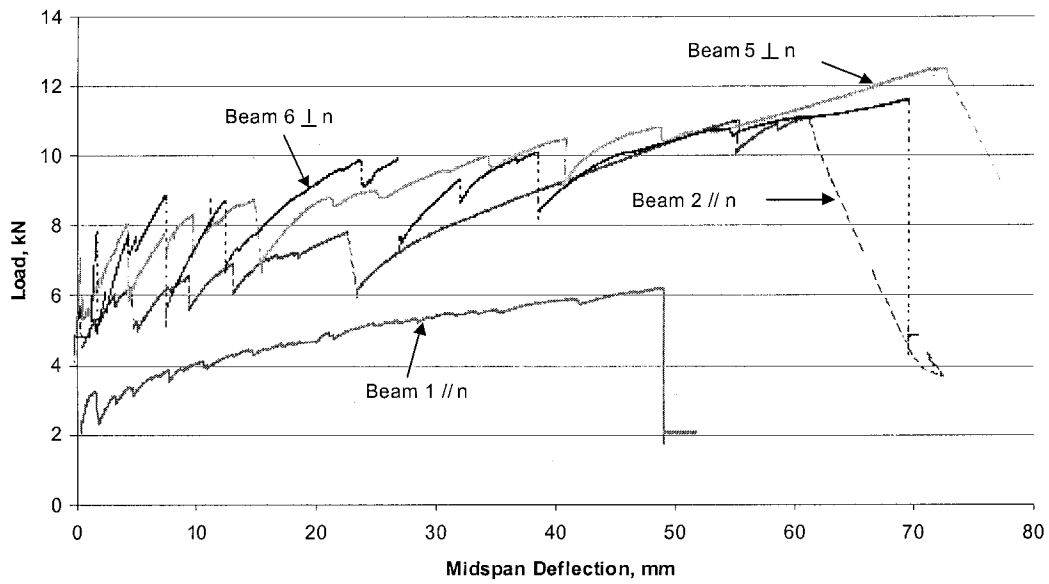


Fig. 3 Load-Deflection Curves for Narrow Beams with One FRP Bar Parallel (Series 1) or Perpendicular (Series 2) to Mortar Joints

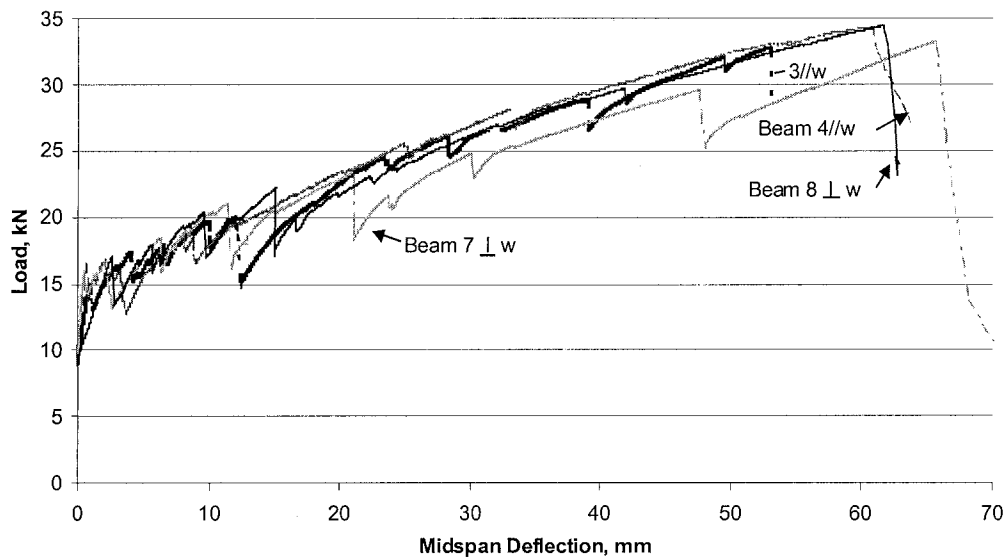


Fig. 4 Load-Deflection Curves for Wide Beams with Three FRP Bars Parallel (Series 1) or Perpendicular (Series 2) to Mortar Joints

shear on  $\frac{3}{4}$  of the bar circumference, contributed to lowering the failure strain below the bar ultimate tensile strain.

The ACI 530-02 equations, which are based on tests of steel-reinforced masonry, provide a conservative estimate of the flexural strength of FRP externally reinforced concrete masonry beams. The ratio of measured to calculated strength was 1.37 on average, with a coefficient of variation of 5.7 %. Substantial deflection was achieved at ultimate (mean ratio of span to maximum deflection = 42, coefficient of variation = 11 %). There was no difference in behavior between reinforcement parallel or perpendicular to the mortar bed joints. The unexpected shear failure of ungrouted Beam 1, at only 43 % of the nominal shear strength, requires further investigation.

## Conclusions

The strengthening of epoxy with short glass fibers allows close to full strength development of 6.5 mm diameter glass-FRP bars in 185 mm (7.3 in) or less than half a concrete masonry unit length. This fiber-reinforced epoxy should be effective for other types of bars as well.

With full anchorage assured, externally bonded FRP bars provide an efficient method of strengthening masonry walls against out-of-plane bending. For under-reinforced beams (45 % and 66 % of balanced ratio for the narrow and wide beams tested), flexural failure was consistently initiated by tensile rupture of the reinforcement.

Based on these few tests, it is recommended to extend the use of the ACI 530-02 equations to the ultimate flexural strength design of concrete masonry beams and walls reinforced with near-surface mounted FRP bars.

## Notation

$A_f$	cross sectional area of flexural reinforcement
$A_v$	cross sectional area of shear reinforcement
$A_{vg}$	shear cross sectional area of grouted beam
$b$	beam width
$c$	beam compression depth
$d$	beam depth, from extreme compression fiber to centroid of reinforcement
$f_u$	tensile strength of FRP reinforcing bar
$f_m$	masonry compressive strength
$f_{mg}$	grouted masonry compressive strength
$g$	beam shear span
$L$	beam span
$M_e$	ultimate applied moment
$M_e$	ultimate moment, including applied and dead loads
$M_n$	calculated nominal flexural strength
$P_e$	applied load
$P_n$	load corresponding to $M_n$
$V_e$	ultimate shear force, including applied and dead loads
$V_n$	calculated nominal shear strength
$w$	dead load per unit length

$\delta_s$	deflection at midspan under service loads
$\delta_u$	midspan deflection at ultimate load
$\epsilon_m$	FRP bar strain at ultimate moment
$\epsilon_u$	ultimate tensile strain of FRP bar
$\rho_{bu}$	balanced reinforcement ratio for ungrouted beam
$\rho_{bg}$	balanced reinforcement ratio for grouted beam
$\rho_u$	reinforcement ratio for ungrouted beam
$\rho_g$	reinforcement ratio for grouted beam

## References

- Albert, M. L., A. E. Elwi, and J. J. R. Cheng, "Strengthening of Unreinforced Masonry Walls Using FRP," *ASCE Journal of Composites for Construction*, v. 5, no. 2, May 2001, pp. 76-84.
- American Concrete Inst. International, "Building Code Requirements for Masonry Structures and Commentary (ACI 530-02/ASCE 5-02/TMS 402-02)," Farmington Hills, MI. USA, 2002
- American Society for Testing and Materials, "Standard Methods of Sampling and Testing Concrete Masonry Units," ASTM C140-96, West Conshocken, Pa. 1996
- American Society for Testing and Materials, "Test Method for Compressive Strength of Hydraulic Cement Mortar," ASTM C109, West Conshocken, Pa. 1995
- Castro, P.F., and N. Carino, "Tensile and Nondestructive Testing of FRP Bars," *ASCE Journal of Composites for Construction*, Feb. 1998, v. 2, no. 1, pp. 17-27
- De Lorenzis, L., and A. Nanni, "Bond Between Near-Surface Mounted Fiber-Reinforced Polymer Rods and Concrete in Structural Strengthening," *ACI Structural Journal*, Mar-April 2002, v. 99, no. 2, pp. 123-132.
- Hamilton, H.R., and C.W. Dolan, "Flexural Capacity of Glass FRP-Strengthened Concrete Masonry Walls," *ASCE J. of Composites for Construction*, v. 5, no. 3, Aug. 2001, pp. 170-178.
- Lizundia, B., W.T. Holmes, M. Longstreth, and A. Kren, "Development of Procedures to Enhance the Performance of Rehabilitated URM Buildings," 1997, NIST GCR 97-724-1.
- Triantafillou, T., "Strengthening of Masonry Structures Using Epoxy-Bonded FRP Laminates," *ASCE Journal of Composites for Construction*, v. 2, no. 2, May 1998, pp. 96-104.
- Tumialan, G., F. Micelli, and A. Nanni, "Strengthening of Masonry Structures with FRP Composites," *Structures Congress*, Washington D.C., May 2001.
- Velazquez-Dimas, J. I., and M. R. Ehsani, "Modeling out-of-plane Behavior of URM Walls Retrofitted with Fiber Composites," *ASCE Journal of Composites for Construction* v. 4, no. 4, Nov. 2000, pp. 172-181.