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Application Of Chaotic Dynamics To Stochastic Resonance

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Abstract

For a class of systems with a periodic signal and noise, the improvement of the signal to noise ratio (SNR) achieved by increasing the noise intensity is referred to as stochastic resonance (SR). We show that, for a class of multistable systems, a chaotic dynamics approach to SR allows the assessment of the effect of the spectral density of the noise on the SNR. Using this approach, we also show that, for certain systems, the SNR can be improved more effectively by adding a harmonic excitation than by increasing the noise intensity. The latter result may be used to develope a practical nonlinear transduction device for enhancing SNR.

Introduction

We examine SNR for multistable system, e.g., the Duffing oscillator

$$\ddot{x}(t) = -\beta\dot{x} + x - x^3 + A_0 \sin(\omega_0 t + \varphi_0) + \sqrt{2D\beta}\mathcal{R}(t) + A_a \sin\omega_a t, \quad (1)$$

where β is the damping coefficient, $A_0 \sin(\omega_0 t + \varphi_0)$ is a periodic signal, $\mathcal{R}(t)$ is Gaussian noise with unit variance and spectral density $g(\omega)$, D is the noise intensity, and $A_a \sin\omega_a t$ is an added harmonic excitation. Classical SR corresponds to the case $A_a = 0$ (Moss 1992).

If $D = 0$, the necessary condition for the occurrence of chaos is that the system's Melnikov function have simple zeros (Wiggins 1992), that is,

$$A_0 S_M(\omega_0) + A_a S_M(\omega_a) > 4\beta/3, \; S_M(\omega) = \sqrt{2}\pi\omega \operatorname{sech}(\pi\omega/2) \, , \quad (2)$$

where $S_M(\omega)$ is the Melnikov scale factor. Let ω^* be the frequency for which $S_M(\omega)$ is largest. In this work we assume $\omega_0 \ll \omega^*$. We choose A_0 such that,

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for $A_a = 0$, the inequality in Eq.(2) is not satisfied. Motions with parameters $\beta = 0.316$, $A_0 = 0.095$, $\omega_0 = 0.0632$, $\omega_a = 1.1$, for $A_a = (0.263, 0.287, 0.332)$ yielded SNR's of $(18.9, 31.1, 22.8)$, respectively. For $A_a = 0.263$ and $A_a = 0.287$ see Fig.(1). These motions are chaotic with estimated mean rates of exit from a well $\nu = (0.00629, 0.0107, 0.0252)$. For $A_a = 0.287$ the mean exit rate ν is close to the signal frequency $\nu_0 = \omega_0/2\pi = 0.0101$, and the energy in the low-frequency broadband portion of the spectrum is depleted, while the energy at the signal's frequency is enhanced with respect to their respective counterparts for $A_a = 0.263$ and $A_a = 0.332$. In our example ω_a is close to $\omega^* \approx 0.8$ so that the added excitation is effective in inducing chaos. A choice of ω_a uninformed by knowledge of the Melnikov scale factor may not allow the occurrence of chaos conducive to the energy transfer we noted; e.g., for $\omega_a = 2.2$ the lowest amplitude for which chaotic behavior with exits was observed caused a mean exit rate $\nu = 0.152$ and a worsening of the SNR.

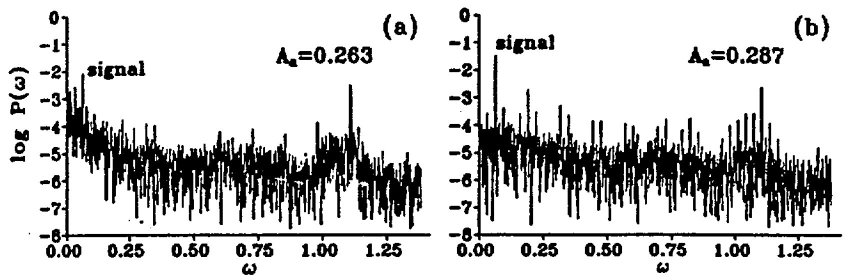

Fig.1) Examples of power spectra for $D = 0$.

Classical Stochastic Resonance

We now consider classical SR; i.e., $A_a = 0$, $D > 0$. Each realization of the random process $\mathcal{R}(t)$ can be represented as closely as desired by a sum of harmonic terms with random parameters (Rice 1954; Frey and Simiu 1993).

We assume $\mathcal{R}(t)$ has Lorentzian distribution $g(\omega) = \gamma\tau^{-2}(1+\omega^2\tau^2)^{-1}$ cut off at $\omega_{max} = 1.6$; τ is the correlation time and γ is a normalization constant such that $var[\mathcal{R}(t)] = 1$. Examples of averaged output spectra $P(\omega)$ for $A_0 = 0.3$, $\omega_0 = 0.069$, $\beta = 0.25$ are shown in Figs.2(a-c) for three values of the power D. Note that $A_0 < 4\beta/(3S_M(\omega_0))$; i.e., no chaos can be induced by the periodic signal alone. However, for the noise realizations used to obtain the results of Figs.2(a-c), the Melnikov criterion for chaos was satisfied, and the respective motions were chaotic. In this case also, energy transfer to the signal frequency occurred when the mean exit rate ν for the chaotic motion was close to ν_0.

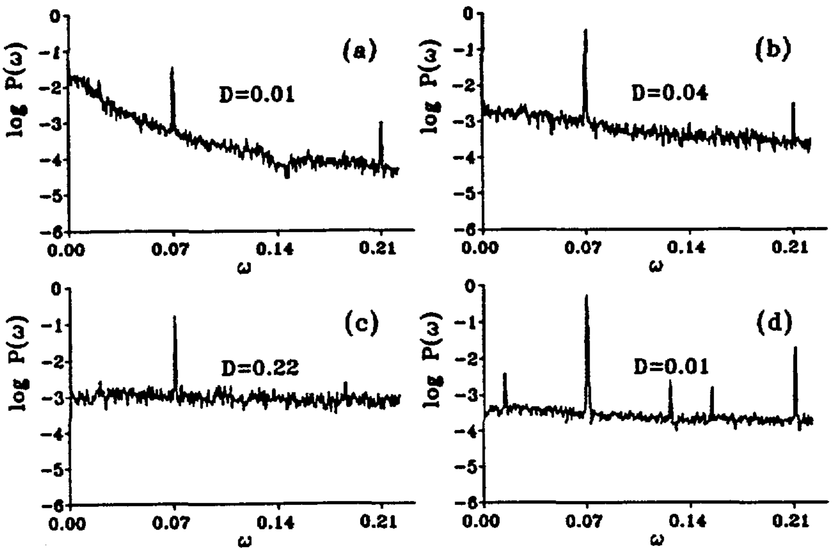

Fig.2) Averaged power spectra of output for stochastically excited system: (a-c) increasing noise intensity D and $A_a = 0$; (d) same noise intensity D as in (a) and $A_a = 0.23$. Noise correlation time $\tau = 0.2$ in all cases.

Effect of Noise Spectrum

The Melnikov scale factor is a measure of the degree to which a frequency component can be effective in inducing chaos. We now examine its effect on SR for noise spectra with the values $\tau = \tau_1 = 0.2$, which results in broadband

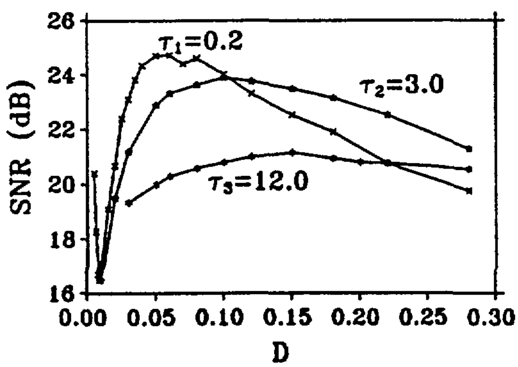

Fig.3) Signal to noise ratio vs. noise intensity D for the three noise correlation times τ.

noise with almost constant spectral density, and $\tau = \tau_2 = 3.0$, $\tau = \tau_3 = 12$, for which the spectra are increasingly peaked near the origin and increasingly weak elsewhere. The parameter values were $A_0 = 0.3$, $\omega_0 = 0.069$, $\beta = 0.25$. Fig.(3) shows that the peak SNR decreases and occurs at higher values of D as the correlation time τ increases from 0.2 to 12 or, equivalently, as the bulk of the energy spectra of the noise is further away from ω^*.

SNR Improvement by Adding Harmonic Excitation

It is clear that the greatest effectiveness in increasing SNR would be achieved by a single component with frequency equal or close to ω^*. This suggests the following method for improving SNR. Assume that $A_a = 0$, and that for a set of values A_0, ω_0, β and D the system has low SNR. We could improve the SNR by increasing D, as illustrated earlier. However, it is more effective to increase the SNR by keeping D unchanged and adding an excitation $A_a \sin(\omega_a t)$ such that (1) ω_a is equal or close to ω^* and (2) A_a is chosen so as to induce a chaotic exit rate ν comparable to the signal frequency ν_0. An example is shown in Fig.(2d), for which all parameters and the normalized spectrum $g(\omega)$ are the same as for Fig.(2a), except that the system is subjected to an added excitation with amplitude $A_a = 0.23$ and frequency $\omega_a = 1.1$. This approach to increasing SNR is seen to be quite effective. The added harmonic excitation induces subharmonics and superharmonics but these are well separated from the signal and can be filtered out by a suitable passband filter. We are currently investigating the development of a nonlinear transducing device based on the method just discussed.

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