AN ANALYTICAL MODEL OF PYROLYSIS FOR A FINITE THICKNESS SAMPLE ON A SEMI-INFINITE BASE

Kathryn M. Butler
Building and Fire Research Laboratory
National Institute of Standards and Technology
Gaithersburg, Maryland 20899

Introduction To test the flammability of materials, a sample is placed on a substrate and exposed to a uniform heat flux. Measurements include sample mass and upper and lower surface temperatures as functions of time. Several analytical models[1,2,3] have been developed in an attempt to fully understand the relationship between properties of the tested materials and test results. Most of these models have assumed a semi-infinite sample, although the finite thickness of the sample is known to have a significant effect on pyrolysis behavior. An analytic solution that includes *finite sample thickness* and the material properties of the base is introduced in this abstract.

Model A sample of initial thickness L, thermal conductivity k_s, density ρ_s, and specific heat c_s is placed on a base of semi-infinite extent with thermal properties k_b, ρ_b, and c_b. The interface is located at $z = 0$, and the temperature is initially uniform at T_0. At time $t = 0$, a constant heat flux $\dot{q}''$ is applied to the upper surface of the sample. When the surface temperature has reached a critical temperature T_p, the material at the surface begins to pyrolyze at a rate that depends on the heat of vaporization $\triangle H_v$. The moving boundary is located at $z = Z(t)$ and the mass loss rate is $dm''/dt = \rho_s dZ/dt$. In-depth pyrolysis is ignored, so the temperatures within each material satisfy the diffusion equation for heat transfer.

In solving this problem, the nondimensionalized temperature is defined as $\theta = (T - T_0)/T_0$ and the nondimensionalized time as $\tau = \int_0^t (\alpha_s/Z(t)^2)\, dt$. The fixed coordinate system z is changed into a system that moves with the boundaries by defining $y = z/Z(t)$. This change of variables introduces a convective term into the heat transfer equation. A separate coordinate system for the base, $y_b = y\sqrt{\alpha_s/\alpha_b}$, matches the time coordinate in the base with that of the sample.

Other assumptions for this model are:

- Thermal properties independent of temperature
- Perfect thermal contact between sample and base, so that temperature and heat flux are continuous at the interface
- Surface temperature steady at T_p throughout pyrolysis
- Convective term in sample assumes the surface temperature gradient throughout the sample

Using these assumptions, analytic expressions can be found for temperature profiles in the sample and base during both preheating and pyrolysis stages. The nondimensional parameters that affect the temperatures in (y, y_b, τ) space are $\eta = \sqrt{k_s\rho_s c_s/k_b\rho_b c_b}$, the ratio of thermal inertias for the sample and base materials, $\beta = \dot{q}''L/k_sT_0$, a nondimensional parameter describing the incident heat flux, and $\gamma = (\triangle H_v/c_sT_0)$, which describes the heat loss due to vaporization. The parameter η appears in the solution in the form $\delta = (\eta - 1)/(\eta + 1)$, which ranges between -1 and 1 and orders the sets of infinite series that appear in the solution. For $\delta = 0$, properties of sample and base are identical, and the solution reduces to that for a semi-infinite sample. Pyrolysis begins when the temperature at the sample surface reaches the critical temperature $\theta_p = (T_p - T_0)/T_0$. To first order in δ, this occurs at time $\tau_p = \pi\theta_p^2/4\beta^2$.

Results An example demonstrates the characteristics of this model. In this problem, the sample is a material with thickness $L = 0.003175$ m, thermal properties $k_s = 16.9 \times 10^{-5}$ kW/m-°K, $\rho_s = 1190$ kg/m^3, and $c_s = 2.6$ kJ/kg-°K, pyrolysis temperature $T_p = 635$°K, and heat of vaporization $\triangle H_v = 850$ kJ/kg. The thermal properties of the base are $k_b = 7.8 \times 10^{-5}$ kW/m-°K, $\rho_b = 128.0$ kg/m^3, and $c_b = 0.76$ kJ/kg-°K. Ambient temperature is $T_0 = 300$°K, and the heat flux applied to the sample is $\dot{q}'' = 50$ kW/m^2. Heat losses due to re-radiation and convection from the hot surface are not included.

Preliminary results are shown in Figures 1 through 4. Figure 1 shows the nondimensional temperature θ as a function of y, the location normalized by the sample thickness, during preheating and pyrolysis. In

Figure 2, the same information is plotted for physical values of temperature and distance. Figure 3 plots the temperature as a function of time for locations from the sample/base interface to the upper surface of the sample. Note that the interface temperature starts to decrease after two minutes, indicating that the analytic solution is not valid beyond this point. At this time less than ten percent of the sample remains, and indepth pyrolysis becomes very important. The mass loss rate, determined from the spatial gradient of temperature at the sample surface, is plotted against time in Figure 4. The trend of this curve matches experimental behavior. More complete results will be discussed in the presentation.

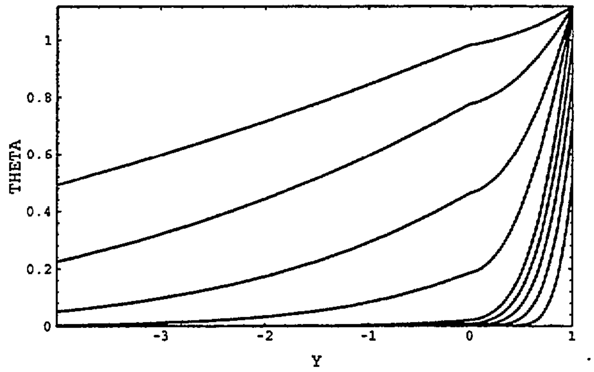

Figure 1: Nondimensional temperature (θ) vs. location (y) at times $\tau = \tau_p \times (0.2, 0.4, 0.6, 0.8, 1.0)$ during preheating and $\tau = (0.25, 0.5, 1, 2)$ during pyrolysis.

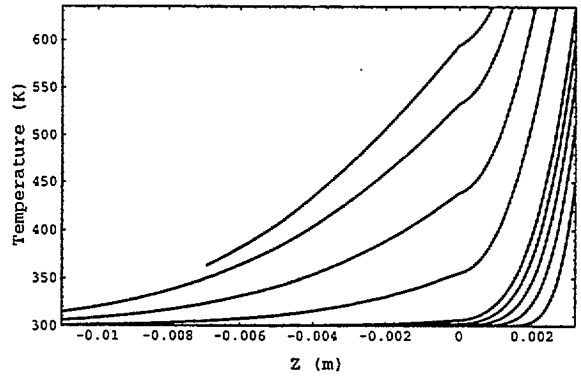

Figure 2: Same as Figure 1 but converted to physical temperature and location. Plots are at times (in seconds) t =(3.6, 7.3, 10.9, 14.6, 18.2, 41.5, 66.5, 93.9, 117.7).

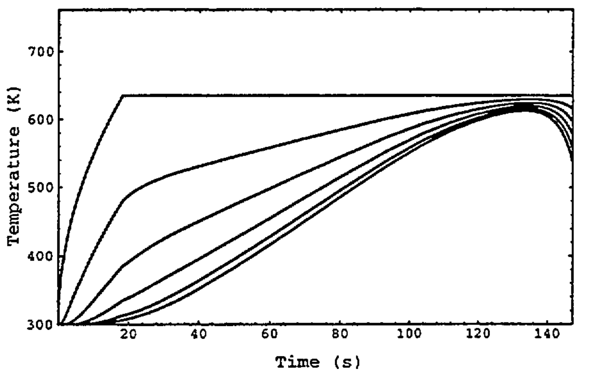

Figure 3: Temperature at evenly spaced locations between upper and lower surfaces of sample vs. time

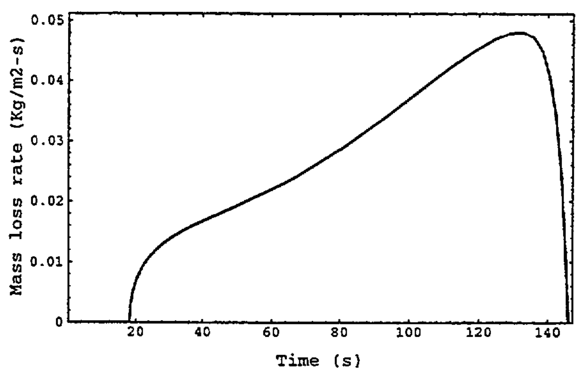

Figure 4: Mass loss rate of sample vs. time.

Conclusions An analytic solution has been determined for the pyrolysis of a sample of finite thickness resting on a substrate of a different material. Besides providing insight into pyrolysis behavior, this model may improve our capability to derive material properties from common flammability test measurements.

References

[1] Steckler, K.D., Kashiwagi, T. and Baum, H.R., "Analytical Model for Transient Gasification of Noncharring Thermoplastic Materials," *Fire Safety Sci. – Proc. of 3rd Intl. Symp.* 895–904 (1991).

[2] Chen, Y., Delichatsios, M.A., and Motevalli, V., "Material Pyrolysis Properties, Part I: An Integral Model for One-Dimensional Transient Pyrolysis of Charring and Non-Charring Materials," *Comb. Sci. and Tech.* 88:309–328 (1993).

[3] Quintiere, J. and Iqbal, N., "An Approximate Integral Model for the Burning Rate of a Thermoplastic-like Material," *Fire and Mat.* 18:89–98 (1994).