

# A METHOD OF DETERMINING SPHERE CENTER TO CENTER DISTANCE USING LASER TRACKERS FOR EVALUATING LASER SCANNERS

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## INTRODUCTION

The Dimensional Metrology Group (DMG) at the National Institute of Standards and Technology (NIST) is involved in the development of documentary standards for volumetric performance evaluation of laser scanners.

Testing using a calibrated grid of spheres is one of the proposed methods for evaluating the performance of laser scanners. Challenges in establishing a calibrated grid of spheres include the commercial availability of high quality spheres which are conducive to laser scanners. Surface finish and other properties such as stiffness, reflectivity, optical penetration and geometrical form affect the usage of such spheres for the performance evaluation of laser scanners.

This paper will present techniques explored at NIST for the purposes of establishing a grid of spheres, calibrated using a laser tracker. Such a grid can be used to evaluate the volumetric performance of laser scanners. These techniques determine the center-to-center distance of two spherical targets.

## MOTIVATION FOR ESTABLISHING DOCUMENTARY STANDARDS VIA ASTM E57.02

Laser scanners are 3D measurement instruments whose relative uncertainties are in the range of  $10^{-5}$  to  $10^{-4}$  and their measurement ranges are in the range of 1 m to 150 m. These instruments are used in a wide variety of industries, but are primarily used for reverse engineering, topographical reconstruction, forensics, historic documentation and preservation, etc.

Laser trackers are also 3D measurement instruments, and they offer uncertainties which are an order of magnitude lower than the current generation of laser scanners. However, these instruments require cooperative targets such as a spherical mounted retro-reflector (SMR).

Currently there are no existing standards for evaluating the volumetric performance of laser scanners. The ASTM E57.02 standards committee is attempting to prescribe methods for the performance evaluation of laser scanners in the medium range (2 m to 150 m). This evaluation could be performed by determining the measurement error between two derived points (volumetric performance) at many positions in the scanning volume of such systems [1].

Planar contrast targets that are preferred by some laser scanners are not dimensional standards; that is, the center of the target is determined by processing the returned intensity images and is not derived from 3D coordinates. Such contrast targets have been used to evaluate laser scanners in the past [2], but are unsuitable as a standardized target for laser scanner performance evaluation. To overcome this issue, a sphere is chosen as the target geometry, for which the derived point is its center and the measurand is the center-to-center distance between two spheres. A sphere is also a good candidate because of the prevalence of spheres as one of the most common targets for laser scanners.

## DESCRIPTION OF THE EXPERIMENT AND THE HARDWARE USED

The purpose of the paper is to determine the uncertainty in measuring the point-to-point distance between two geometrical

targets by using techniques that could be employed by a user who has access to a laser tracker.

We chose a 4 inch diameter steel sphere as a target because we have access to a high quality 4 inch SMR and the identical sizes minimizes the abbe errors. We note that this size SMR is generally not available to laser tracker users, but we use it to establish a calibrated reference value, with low uncertainty, of the center-center distance between two points measured using a laser tracker. This calibrated value will be compared to measurements involving a 1.5 inch SMR (commonly available to laser tracker users) over the 4 inch steel sphere (SMR-walking method). The difference between the center-to-center length measurement using the SMR-walking method and the calibrated reference value is used to establish the accuracy of using the SMR-walking method to calibrate the grid of spheres used for laser scanner evaluation.

The setup is based on existing hardware and the components are listed below<sup>†</sup>:

1. Laser tracker: Leica AT901-B
2. 4 inch ProSystems-1003 SMR
3. 1.5 inch Faro SMR
4. A 4 inch diameter, solid steel sphere, with a calibrated least-squares diameter of  $101.60924 \text{ mm} \pm 130 \text{ nm}$  ( $k=2$ ) and sphericity<sup>‡</sup> value of  $4 \text{ } \mu\text{m}$  (measured with over 400 points on the M48 Coordinate Measuring Machine at NIST)
5. Brunson model 230 tripods

A test method was developed to measure the center-to-center distance between two spherical targets using a laser tracker and is illustrated in FIGURE 1. Two tripods with kinematic nests are located at positions A and B. Tripod A is located closer to the tracker in its initial position. Pos0, Pos1 and Pos2 are the consecutive positions of the laser tracker. The tracker at Pos0 is in-line with a 4 inch SMR located in the kinematic nest of both the

tripods A and B (sequentially measured). Pos0 enables the measurement of the distance between targets in the kinematic nests on the two tripods without any significant contribution of errors due to the laser tracker's angular encoders.

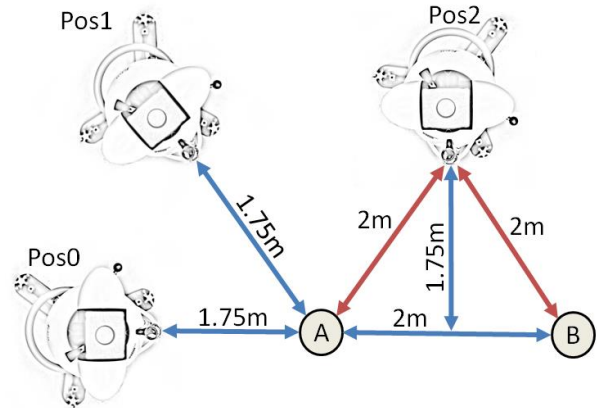


FIGURE 1: Experimental setup to determine center-to-center distance, with the laser tracker at Pos0, Pos1 and Pos2 and tripods at A & B.

The tracker is then moved to Pos1 and Pos2 where the targets are not in-line with the laser tracker. This is performed to evaluate the errors introduced due to the laser tracker's angular encoders.

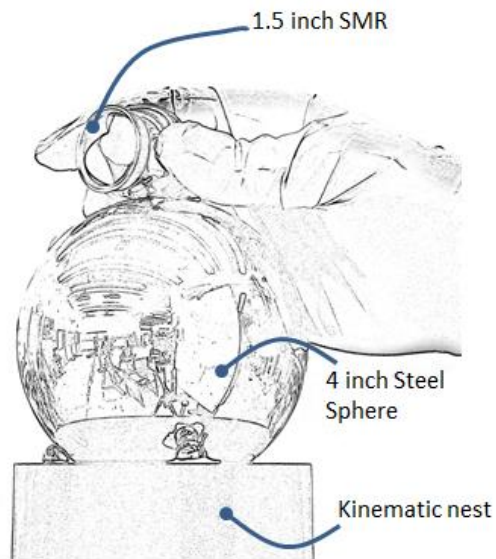


FIGURE 2: Procedure to record data points on the surface of the sphere.

The test procedure is described below:

1. Tripods A and B are aligned in such a way that the tracker at Pos0 and the 4 inch SMR on tripods A and B are in-line within  $10 \text{ } \mu\text{rad}$ . The nominal distance between the

<sup>†</sup> Disclaimer: Commercial equipment and materials may be identified in order to adequately specify certain procedures. In no case does such identification imply recommendation or endorsement by the National Institute of Standards and Technology, nor does it imply that the materials or equipment identified are necessarily the best available for the purpose.

<sup>‡</sup> Sphericity is defined as the smallest separation of two concentric spheres that contain all the points of the surface under consideration.

- tripods at A and B is 2 m. The tracker is placed at a nominal distance of 1.75 m from tripod A (diametrically opposite to tripod B). Care is taken that the tripods are lowered and secured to the floor before making any fine adjustments.
2. A repeatability test is performed on the distance measurement between the two kinematic nests on tripod A and B with the 4 inch SMR. The SMR is seated first on tripod A and the tracker's measurement is recorded. It is then unseated and placed in tripod B and the tracker's measurement is recorded. The process is repeated 10 times yielding 10 length measurements with the mean value ( $d_{SMR}$ ) and  $1\sigma$  standard deviation (repeatability) shown in TABLE 1. This mean value ( $d_{SMR}$ ) measured at Pos0 is used as the calibrated center-to-center distance.
  3. The SMR is removed from the tripod A, and the 4 inch solid steel sphere is placed in its kinematic nest. A set of 50 approximately equally spaced data points are recorded by walking the 1.5 inch SMR on the surface of the steel sphere that is mounted on the tripod (SMR-walking method). FIGURE 2 illustrates the procedure where the 1.5 inch SMR is positioned over a steel sphere, making contact, while being tracked by the laser tracker. If the beam breaks while recording the 50 data points, the 1.5 inch SMR is inserted back into the tracker's home position (commonly called the R0 position), the interferometer is zeroed and then the measurements on the steel sphere are resumed. At each measurement point, care is taken to ensure that there is only one point of contact between the 1.5 inch SMR and the steel sphere. Due to mechanical interference with the kinematic nest, only about 30% of the surface area of the sphere can be measured using the SMR-walking method. The steel sphere is then moved to tripod B and the procedure is repeated.
  4. The centers of the spheres are calculated using a sphere fitting algorithm based on non-linear least squares, by constraining the diameter to 101.60924 mm, and the center-to-center distance is then calculated and reported in TABLE 2.
  5. Step#3 and step#4 are repeated with the tracker at Pos1 and then at Pos2. Tests for repeatability using the 4 inch SMR were not performed at these positions.

## SUMMARY OF THE RESULTS

TABLE 1: Distance between the two tripods using the 4 inch SMR.

Tracker position	Distance (mm)	Repeatability ( $1\sigma$ , in $\mu\text{m}$ )
Pos0	1956.191 (mean)	1.1
Pos1	1956.198	-
Pos2	1956.225	-

TABLE 1 lists the distances measured using the 4 inch SMR. The mean distance measured by the tracker at Pos0 is 1956.191 mm ( $d_{SMR}$ ) and is the best estimate of the distance between the spheres with minimal errors due to the encoders. This value is used as the calibrated center-to-center distance. Distances measured by the tracker at Pos1 and Pos2, using the 4 inch SMR, have errors introduced due to the errors in the angular encoders of the laser tracker. No repeatability tests were conducted at Pos1 and Pos2 as these positions will have their angular errors and the tests will not represent the quality of the kinematic nest.

In a separate experiment, the SMR-walking method was repeated 10 times at Pos0 (using step#3 and step#4), and the repeatability ( $1\sigma$ ) of the distance measurement was 3.3  $\mu\text{m}$ .

TABLE 2 lists the distances measured by calculating the centers using a constrained fit algorithm (non-linear least squares fit, with a calibrated diameter of 101.60924 mm) on the 50 data points measured on the steel sphere. The distance errors in TABLE 2 are the difference in the distances calculated at each position ( $d_{Sphere}$ ) using the SMR-walking method and the calibrated distance ( $d_{SMR}$ ).

TABLE 2: Distance between the two tripods using the steel sphere SMR-walking method.

Tracker position	Distance (mm)	Error ( $d_{Sphere} - d_{SMR}$ ) ( $\mu\text{m}$ )	Repeatability ( $1\sigma$ in $\mu\text{m}$ )
Pos0	1956.192	1.9	3.3
Pos1	1956.200	9.5	-
Pos2	1956.207	16.3	-

FIGURE 3 illustrates the errors in the center-to-center distances at various tracker positions using two different algorithms; a constrained fit algorithm (where the diameter is constrained to 101.60924 mm) and an unconstrained fit algorithm (where the diameter is determined from the fit, along with its center).

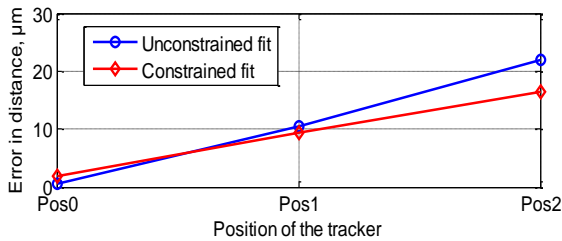


FIGURE 3: Error in the measured distances at each position of the laser tracker

## SOURCES OF UNCERTAINTY AND THE UNCERTAINTY BUDGET

### Measurement point density

The number of points that are measured using the SMR-walking method (shown in FIGURE 2) affects the determination of the center of the sphere. A simulation was performed on a synthetic sphere with errors introduced due to the form of the sphere and the laser tracker.

The form errors are introduced by superimposing sinusoidal waveforms (representing the sphericity and topography) along the radial direction of a sphere in a spherical coordinate system. The tracker errors are added based on MPE (Maximum Permissible Errors) values from the laser tracker manufacturer's specifications and drift tests.

Results for 300 simulations  
Errors in center location

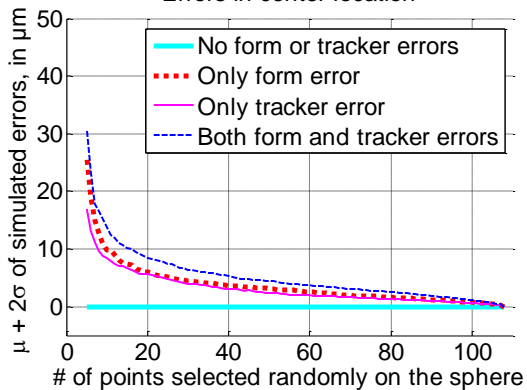


FIGURE 4: Error in the center location with varying number of simulated points on the sphere.

It was observed that the improvement in the sphere center errors diminished as the number of points increased and is shown in FIGURE 4. Based on this simulation, a value of 50 is chosen as the number of points that are measured on the surface of the sphere. Measuring more number of points on the sphere

is a labor intensive process, with diminishing returns.

### Laser Tracker's RMS Error

Each point on the sphere surface that is measured by the tracker is itself an average of 50 samples after the 1.5 inch SMR stabilizes. The laser tracker is programmed to reject any points outside a  $40 \times 10^{-6}$  RMS (Root mean square) threshold<sup>§</sup> and the RMS error that is within the  $40 \times 10^{-6}$  threshold is recorded.

In the setup illustrated in FIGURE 1, a  $40 \times 10^{-6}$  RMS threshold amounts to 70 µm at 1.75 m, 80 µm at 2 m and 150 µm at 3.75 m. A lower RMS threshold of  $20 \times 10^{-6}$  value results in numerous points being rejected due to higher RMS error caused by shaking hand. This RMS threshold value accounts for the stability of the measured point due to various sources of errors, of which the significant part is the human error (shaking hand).

A simulation was performed to understand the effect of the RMS error on the error in calculation of the center of the sphere. The recorded coordinates of the 1.5 inch SMR is the mean of the 50 samples (provided they are all within the RMS error threshold limit). The standard deviation of the mean of the 50 samples is approximately 0.14 ( $1/\sqrt{50}$ ) times that of the individual samples. Although we have the RMS error setting corresponding to 80 µm at Pos2, the largest observed RMS value at this position was 40 µm. Hence, the mean has a standard deviation of 5.6 µm.

Using this distribution for the point coordinate variation, the simulation of the steel sphere measurement (using the radius-constrained sphere fit) yields a sensitivity coefficient of 0.46. Thus the 4 inch steel sphere center varies by 2.6 µm for the tracker RMS error.

### Form of the target sphere

The form of the sphere affects the calculation of the center of the sphere. The SMR-walking technique covers only a portion of the sphere ( $\approx 30\%$  of the surface area); therefore, the calculation of the center of the sphere is also affected by the form of the sphere in the region of the measurement. A calculation was performed to understand the effect of the form of the sphere on the calculation of the sphere center. This calculation considers the sphericity with a frequency of 2 cycles per revolution (which represents the typical form of

<sup>§</sup> RMS threshold setting is programmed in the laser tracker's instrument software.

the spheres that are being used). This added to the radial distance from the center point of a sphere in a spherical coordinate system. Using a radius-constrained fit in the calculation yields a sensitivity coefficient (associated with the sphericity) of 0.204 for this particular form error. However, this sensitivity coefficient varies with the sampling strategy and nature of the form error of the steel sphere.

**Uncertainty budget for the measurement of the distance between sphere centers**

TABLE 3 lists the different sources of uncertainty, including the sources discussed in the previous sections. It also lists their standard uncertainty values, distributions and their corresponding contribution to the expanded uncertainty. It should be noted that some of the listed error sources result in the error in measurement of the position of a data point. For such sources (similar to the explanation given in section on laser tracker's RMS error), the error in calculating the center of the sphere would

reduce to 0.46 times the value of the error in the position of the point on the sphere.

Adding all the uncertainty terms in a root-sum-of-squares (RSS) manner will give a combined standard uncertainty ( $u_c$ ) of 14.92  $\mu\text{m}$ , and using a coverage factor of  $k=2$  yields expanded uncertainty of 29.84  $\mu\text{m}$ .

It may be observed that the major contributors to the uncertainty budget are the angular errors (in azimuth and elevation). It should also be noted that the form of the 4 inch steel sphere used in this experiment contributes very little to the overall uncertainty. However, obtaining such high quality spheres is very expensive. They are also very heavy and not amenable to mounting with a stem, oriented horizontally. Horizontal stem mounting is necessary to enable the laser scanners to measure a larger surface area of the sphere without being occluded by the mounting apparatus.

TABLE 3: Sources of uncertainty of the sphere center-to-center distance measurement at Pos2 in FIGURE 1

	Source of Uncertainty	Sphere A $u_i$ ( $\mu\text{m}$ )	Sphere B $u_i$ ( $\mu\text{m}$ )	Type	Distribution	Contribution <sup>‡</sup>
1	Laser tracker: Range <sup>†</sup>	0.27	0.27	B	Uniform	0.1 %
2	Laser tracker: Azimuth <sup>†</sup>	7.17	7.17	B	Uniform	46.2 %
3	Laser tracker: Elevation <sup>†</sup>	7.17	7.17	B	Uniform	46.2 %
4	Laser tracker: Thermal drift <sup>*</sup>	0.69	0.69	A	Gaussian	0.4 %
5	Laser tracker: R0 position	0.30	0.30	A	Gaussian	0.1 %
6	Laser tracker: RMS error	2.60	2.60	A	Gaussian	6.1 %
7	Repeatability of kinematic nest <sup>*</sup>	0.78	0.78	A	Gaussian	0.5 %
8	Form of the 1.5 inch SMR	0.31	0.31	B	Uniform	0.1 %
9	Thermal exp. of the 1.5 inch SMR	0.35	0.35	A	Gaussian	0.1 %
10	Form of the 4 inch sphere	0.47	0.47	B	Uniform	0.2 %
11	Thermal exp. of the 4 inch sphere	0.15	0.15	A	Gaussian	0.0 %
	Combined standard uncertainty (center calculation)	10.55	10.55			
	Combined standard uncertainty (center-to-center distance, $u_c$ )	14.92				
	Expanded uncertainty ( $U_{k=2}$ )	29.84				

<sup>‡</sup> Contribution percentage =  $(u_i/u_c)^2$

<sup>†</sup> From the manufacturer MPE values and accounting for the averaging effect explained in section on RMS errors. Rest of the sources of uncertainties are determined experimentally.

<sup>\*</sup> These sources correspond to the errors in the center distance of a sphere without any averaging effect.

## CONCLUSION

A method to determine the center-to-center distance of a sphere is developed and its uncertainty is described. This method is a building block in the development of a grid of spheres for evaluating the volumetric performance of laser scanners. All the uncertainty sources are listed and their corresponding contributions are calculated. The largest error observed using the 50 point sphere fit method was 16.3  $\mu\text{m}$ , which is well within the 29.84  $\mu\text{m}$  expanded uncertainty evaluated for the center-to-center distance of a 1956 mm length. We believe that this level of accuracy is sufficient to calibrate a grid of spheres for the evaluation of current technology of laser scanners.

Experiments are planned to reduce the major sources of uncertainty due to the angular errors of the laser tracker and also due to the anticipated errors due to lower quality spheres that may be used for the grid.

## REFERENCES

- [1] Scope document of the ASTM E57.02, "New test method for evaluating the point-to-point distance measurement error for a 3D imaging system" (Retrieved on July 14, 2014)  
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