Considerations for Design and In-Situ Calibration of High Accuracy Length Artifacts for Field Testing of Laser Trackers

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Abstract

Interim testing of laser trackers can be problematic due to the lack of high precision, long length artifacts that maintain their calibrated lengths during measurement. Gravitational loading, fixturing forces, and changes in the atmospheric conditions can have significant effects on the calibrated length of these artifacts. In particular, the length of these artifacts can change significantly during routine testing by simply reorienting the artifact in the varied positions that capture the different error sources in the laser tracker. This can be particularly problematic in the field where, in addition to the needs for high accuracy, testing time must be minimized to reduce costs and the changes in environmental conditions cannot be adequately controlled. This paper describes the design and evaluation of a prototype length artifact for field testing of laser trackers. To address the need for short testing time we describe a novel new method for in-situ field calibration of length artifacts using the integral ranging system of the tracker. The prototype was tested in laboratories at the National Institute of Standards and Technology (NIST); the results show that artifacts developed using the methods described in this paper can be fixtured in varied horizontal, vertical and inclined positions while maintaining a calibrated length within 5 ppm under field conditions. The new understandings of the design and fixturing of high precision long length artifacts described in this paper provide a process for laser tracker users to quickly evaluate their instrument performance in the field before costly or critical measurements are performed.

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Introduction

Laser trackers are portable, precision instruments which combine large measurement volume capabilities with varying levels of measurement uncertainty. This capability makes laser trackers especially valuable industrial and scientific tools, but can add to the difficulty in qualifying their performance [1-3]. Properly designed reference length artifacts can be calibrated with a sufficiently low uncertainty in relation to the measurement capability of the instrument that they can serve as useful tools for interim testing.

Background

In the ideal sense, the most comprehensive test of a laser tracker would (theoretically) be to compare measurement results at all points within the working volume against a zero uncertainty standard. The manufacturers' calibration laboratories are designed to mimic this ideal with many discrete orientations and ranges prescribed throughout the volume. The next levels of simplified tests are provided by international documentary standards, which are frequently used to provide a common set of tests which to compare the performance of different laser trackers [4-6]. Given concerns about changes in the state of the instrument due to its portable nature and the variety of field environments they are used in, interim field tests have been suggested by manufacturers and in the documentary standards, and comprise an even smaller set of final checks.

Field testing is important, since portable metrology instruments may be subjected to frequent shipping and handling and may be operated in challenging industrial environments. Small changes of alignment in the mechanical and optical components that may be thermally induced or caused by vibration or shock during shipping can lead to measurement errors. The ranging element, either a laser interferometer or absolute distance meter operates at a much higher level of accuracy. Larger systematic errors stem from components and operations related to angular encoders of the instrument. Existing standards allow for isolating the ranging element of the tracker to establish a reference length to evaluate the performance of the laser tracker.

Given the opportunity for errors in the field and the potential cost of inaccurate measurements, testing trackers in the field offers a significant benefit to the user by increasing confidence or identifying significant errors immediately prior to critical operations. These interim checks are not a substitute for more comprehensive tests, but used over time, serve as a baseline and indicator of tracker performance between periodic compensation.

Interim tests in the ASME B89.4.19 and draft ISO/DIS 10360-10 describe the use of reference lengths for testing laser trackers. Both of these documentary standards allow the use of 2.3m calibrated artifacts for this purpose [4, 5]. Portable, long length high-accuracy artifacts of this type have been difficult to realize because gravitational loading and fixturing forces can have significant effects on the length of these artifacts. In particular, the length can change significantly during routine testing by simply reorienting the artifact to the varied positions that capture the different error sources in the laser tracker. Stiffness and straightness become critical factors in the design because their influence on the change in length is magnified as the length is increased. Long length artifacts that meet the requirements for stiffness and straightness can be bulky and expensive to produce.

The goal of this research effort is to apply precision engineering fundamentals in the design and construction of a portable 2.3m length artifact for interim testing of laser trackers in the field. The artifact is designed to maintain its reference length through the course of the test in horizontal, vertical, and diagonal orientations with sufficiently small uncertainty to be of practical utility in the field. The artifact must accommodate a procedure that can be performed in under an hour, and the artifact must be portable and deployable in the field.

Design

Performance of the length artifact is contingent upon the combination of several distinct component groups that function as a system to accomplish fixturing and alignment tasks. The primary component groups are the stand, positioner, support beam, kinematic mounts, the length artifact, and alignment aids. The length artifact comprises a tube with three kinematic nests to hold targets, so three different lengths can be realized to test the laser tracker. Figure 1 is an annotated view of the system model.

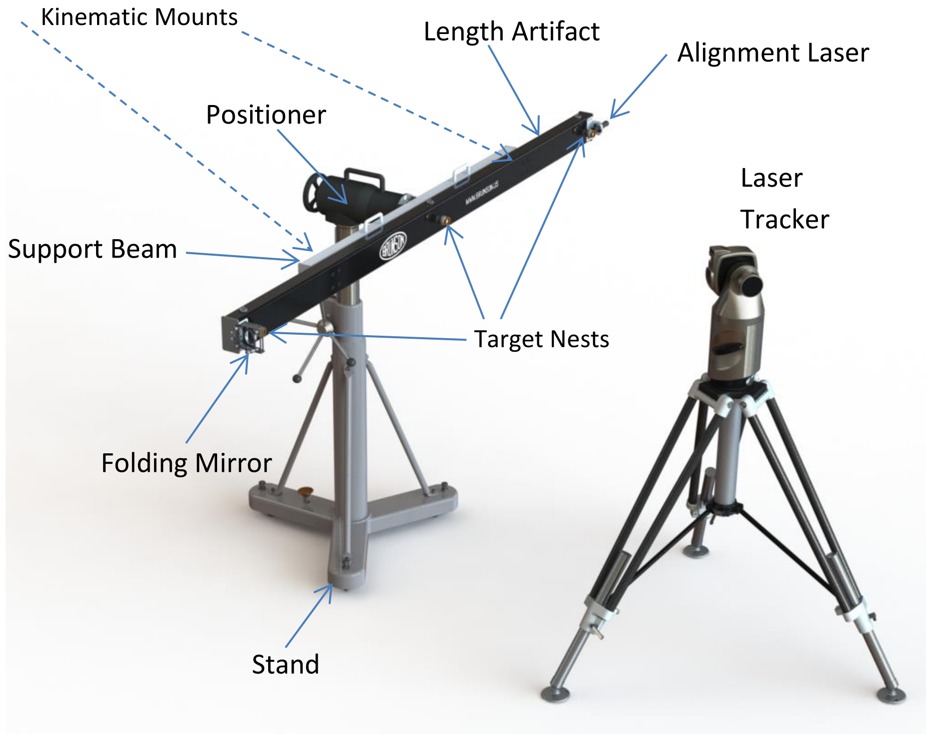

Figure 1. System Model

In the design of this length artifact, emphasis was placed on designing the location of length artifact supports, location of targets along the neutral plane of bending, foreshortening due to straightness variation, vibration, kinematic mounting, construction materials and their properties, support structure, and alignment.

Neutral Plane

The neutral plane for bending of a simply supported length artifact under self-loading is along the centroid of the cross-section. In this plane, the stresses are theoretically zero. Placing targets far from the neutral plane can amplify the effects of curvature in the deformed state, and cause changes in the observed reference length. Targets placed on the neutral plane necessarily reduce the maximum foreshortening error between either of two possible horizontal orientations [7].

Length artifact Supports

To embody the length artifact a rectangular cross-section tube constructed using carbon fiber reinforced polymer (CFRP) was chosen. CFRP was chosen for its high stiffness to density ratio (specific stiffness), and ability to be possesses a low coefficient of thermal expansion (CTE) [8]. Even with its high specific stiffness the CFRP tube can experience length changes when the length artifact's loading changes due to different testing orientations.

In the horizontal orientation, gravity acts on the length artifact as a distributed load. Deflections under that load, and the corresponding curvatures produced in the bar causes a change in the length between target nests placed near the ends of the tube. Airy stress points for a bar of constant cross section were chosen as an approximation of the ideal support points for the bar [9, 10]. The Airy points were selected because the SMR nests cannot be perfectly aligned along the neutral plane for bending during assembly. The Airy points define a set of supports that minimizes the errors in length caused by the measurement point, the center of the SMR, not lying precisely in the neutral plane. A simple laboratory check confirmed that foreshortening due to incremental concentrated loading in the horizontal orientation was less than one micrometer with the tube supported at the Airy points and loaded with a 6.8 kg load at mid span. Even though Airy stress points are calculated for a bar that experiences an equally distributed load, they still sufficiently limit length changes due to concentrated loading; a worst case scenario for this design. For comparison, the CFRP tube was also loaded with supports near the end points. See Figure 2 below, with test results shown in Figure 3. (The effect of the 6.8 kg loading far exceeds that caused by self or gravitational loading of the CFRP tube.)

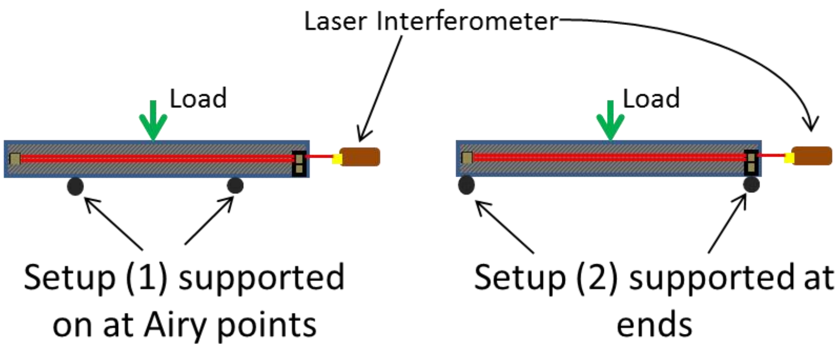

Figure 2. Test Arrangement of Laser Interferometer and CFRP tube Supported at End Points and Airy Points

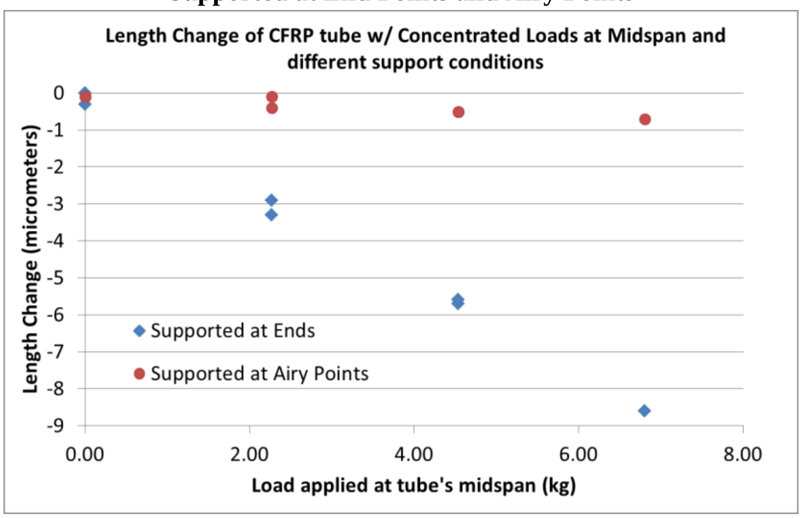

Figure 3. Change in Length of Supported Carbon Fiber tube with Concentrated Load at Midspan

Straightness

Errors in the straightness of long, slender artifacts are difficult to control, and gravitational load can contribute to additional errors in the reference length. As a length artifact is rotated about its roll, or long axis, straightness errors have been shown to cause changes in the length of reference artifacts [7]. While it may not be practical to eliminate straightness errors, it is possible to reduce their affect by changing the cross sectional geometry and using higher specific stiffness materials. The cross-section of the CFRP tube used to construct the length artifact was designed to have a high second moment of inertia (or second moment of area) to reduce effects of straightness to a negligible amount. Additionally, the choice of CFRP was beneficial because it has a higher stiffness to weight ratio compared to other materials considered.

Kinematic Mounts

To prevent the support structure from transmitting loads and deformations to the length artifact, and to fixture it in place, an exact kinematic constraint solution is used. Loads and deformations can include unequal thermal growth of the length artifact and support structure, warping or twisting especially in the long slender members, or clamping and indexing forces intended to position and fix the support structure to the stand. For these reasons kinematic mounts were designed to support, and precisely constrain the length artifact's six degrees of freedom (DOF). A variant design of a Kelvin clamp was chosen as a solution, and a specific implementation was derived to closely approximate the ideal 6-DOF constraint [10, 11] .

The kinematic components are fastened onto two sets of brackets to couple the aluminum support beam and length artifact together, as shown in Figure 4. The cone and flat are located at one set of brackets, and the vee is located at the other. The cone consists of a large sphere nested in three smaller-diameter spheres. The flat is created by a sphere contacting a flat surface. The vee is created by resting a cylinder between two spheres, and is oriented to allow for differential growth of the support beam and length artifact. Mechanical springs were used to pre-load the connections and create the assembly. The assembly containing the cone and flat is loaded through a mechanical spring and sphere contacting an additional flat. The preload is acting at a point, intersecting an axis that runs from the center of rotation of the cone to the point of contact of the flat. This loading arrangement minimizes misalignment torques due to the preload. The vee is preloaded through a mechanical spring and sphere contacting the surface of the cylinder at a point intersecting a circle which passes through the other two sphere contact points. The springs also provide sufficient restoring force to recover from misalignment during handling of the length artifact.

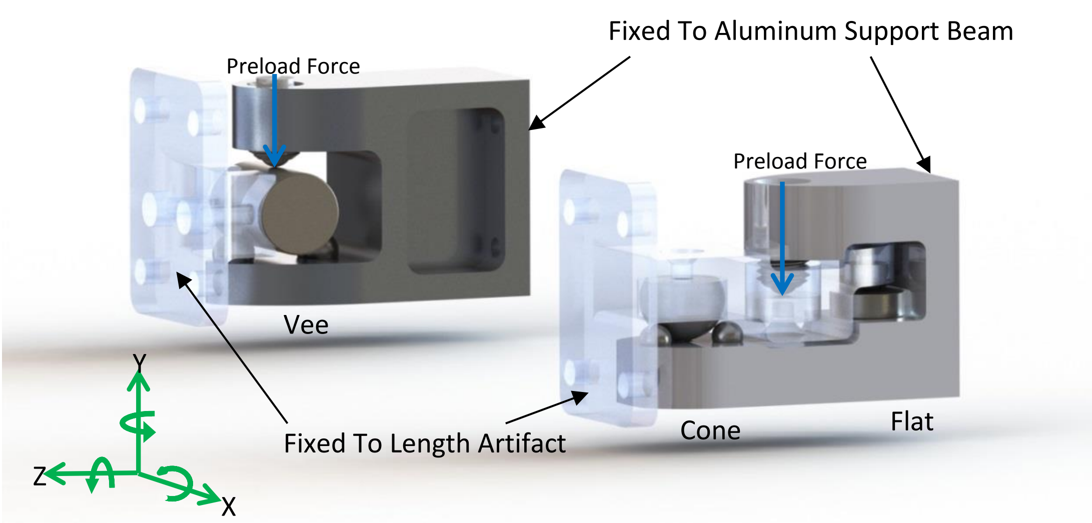

Figure 4. Implementation of Kinematic Mount (length artifact mounts onto the flanges)

Loading Conditions

The location and configuration of the kinematic mounts and orientations of the length artifact create three primary loads that must be resisted by the length artifact. The mounting location of the SMRs located on the front face of the length artifact creates a small torque about its long axis. Gravitational or self-loading causes bending and changes the length when the bar is mounted horizontally. When the length artifact is mounted vertically, the loading is axial and consequently the length change is different. To reduce distortion associated with the torsion, bending, and axial loads, a tube with a very high specific stiffness was chosen. A rectangular tube was selected, made from CFRP, comprised of layers of woven fabrics and longitudinal fibers. In addition to a high specific stiffness, CFRP has excellent vibration damping properties and capability for low coefficient of thermal expansion [12, 13].

Support Structure

Ultimately, the performance of most metrology systems is realized as the result of an entire chain of hardware, user, and environment. Contributors to this system include the support structure, positioner, and stand used to orient the length artifact within the environment. The primary goals of the support system are to elevate the length artifact to vertical positions relative to the laser tracker, provide rotation and indexing between rotational orientations, cantilever the length artifact beyond the envelope of the stand to allow rotation, and rigidly clamp the support beam to the stand assembly. These goals are balanced against the practical needs of portability, requiring smaller size and lighter weight.

Rapid Alignment

Following common procedures, the laser tracker would be bucked-in within a very small angular envelope to isolate the ranging system and establish the reference length of the artifact. This can require rigorous positioning of the tracker, only to re-position it once more for the volumetric test. A sighting laser and folding mirror were attached to the length artifact to allow the tracker to remain in place throughout the reference calibration and first volumetric check and reduce handling of the tracker and its stand. The sighting laser is mounted in alignment with the target nests, and reflected toward the tracker by the folding mirror, both shown in Figure 5. The beam is then positioned in the reticle of the tracker using a tip-tilt stage adjustment of the flat, folding mirror. Standard manufacturing tolerances of the assembly were sufficient to meet angular buck-in restrictions, typically +/-0.1 degrees.

Figure 5. Detailed View of Alignment Laser and Folding Mirror

Uncertainty Analysis

This section provides an uncertainty budget for a field calibration of an length artifact. While the artifact used for this example was actually measured at NIST, the uncertainty budget is deliberately documented in a manner similar to a calibration in the field. The artifact provides three lengths (the distances between the three pair combinations of SMRs out of the three SMRs that can be placed on the artifact). Each of the three measurands is defined as the distance between the sphere centers of two SMRs placed in kinematic nests (along the center of the length artifact) while the SMRs are pointed towards the reflected tracker beam (see Figure 6) at the existing ambient temperature and while the length artifact is in any orientation achievable using the provided positioner.

The measurand is intentionally defined in any orientation that the positioner allows, so that the same calibrated value may be used regardless of the artifact orientation. This convenience will be seen to result in a larger uncertainty of the calibrated length, since the distance between SMRs does, in fact, change with orientation due to gravity or self-loading.

If accuracy needs were sufficiently demanding, a lower uncertainty could be achieved by calibrating the lengths in every orientation that will be used during tracker testing. It may also be possible to perform a field calibration in one orientation and use information of orientation-induced length changes obtained prior to the field measurement to create corrections to the calibrated length for various orientations, but such an effort is beyond the intent of this paper.

Due to practicality, the lengths are calibrated with the artifact positioned in the horizontal orientation only, this orientation is assumed to be the easiest to measure. To calibrate in the vertical orientation, for example, would require the top target to be at least 2.3 m above the ground, which would necessitate the use of a ladder or step stool. Also, it is easier to break the beam while repositioning an SMR from one nest to another under those conditions.

To obtain a calibrated length, the coordinates of each nest (i.e., with an SMR in the nest) are obtained with a bucked-in laser tracker. The distance between the two acquired points in space is calculated and the measurement is repeated two more times. The average of the three distances obtained is the calibrated length. The choice of three measurements was a compromise between averaging out repeatability and limiting time the operator would be handling the scale bar, reducing the transfer of heat.

The uncertainty components outlined below are considered with respect to procedures that might be used to calibrate the scale bar in the field. Specifically, uncertainty components relative to temperature and orientation of the artifact might be handled differently if the tests were performed in a controlled laboratory environment.

The uncertainty components considered in the uncertainty evaluation are (1) the laser tracker measurement uncertainty, (2) repeatability effects, (3) loading due to gravity, (4) retroreflector imperfections, (5) temperature effects, and (6) stability. These components are treated individually in the paragraphs below.

Laser tracker measurement uncertainty, L_{tracker}

Measurement uncertainty arises from the laser tracker due to imperfect ranging and angular measuring systems. The performance of the laser tracker is documented in the manufacturer's specification sheet using Maximum Permissible Errors (MPEs). For the laser tracker used in our case (a Leica AT901-B), the relevant MPEs measuring a length L (in the ranging direction) or a transverse length at a distance R from the tracker are:

MPE_{range}	$\pm\,(0.4 + 0.3L/\text{m})$ µm
MPE_{angle}	$\pm\,(15 + 6R/\text{m})$ µm

Clearly the ranging performance of the instrument is significantly better than its angular performance. However, proper alignment during the calibration of the scale bar can effectively eliminate the effects of the angular errors, as is commonly employed in cases without the mirror when lengths measurements are aligned in the ranging direction. Specifically, the angle between the two laser directions obtained when acquiring the two targets can be aligned with modest effort to within 0.1 degrees, or 0.0017 radians (see Figure 6). This causes the angular errors that the tracker makes in locating the targets to result in a cosine error in the computed distance between the targets, meaning the computed distance is largely insensitive to angular errors. (At 0.1 degrees, the angular errors from the tracker propagate to an error much less than 0.1 µm in the computed distance, an insignificant amount.)

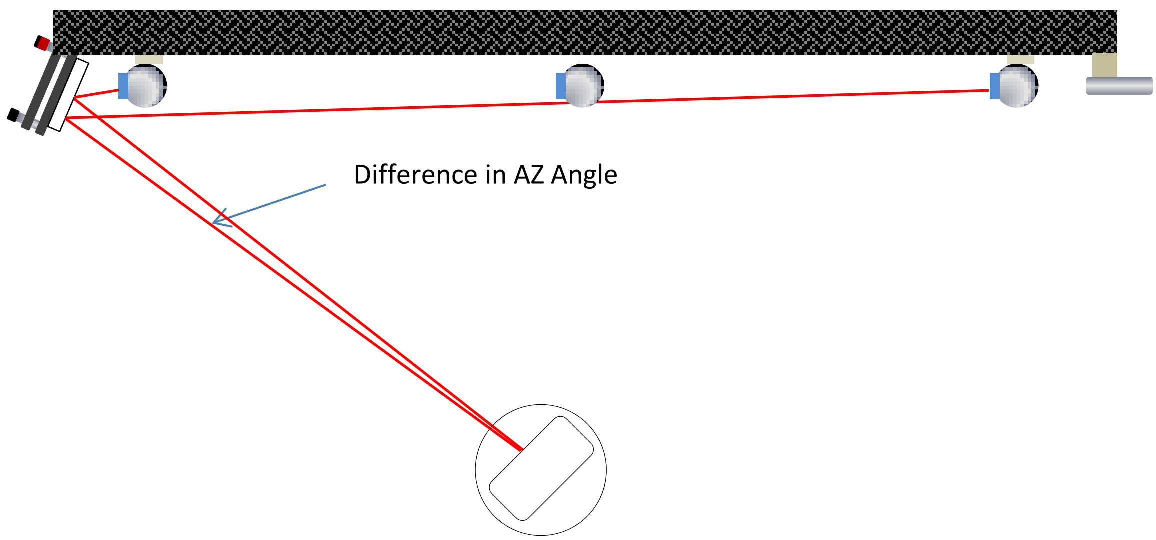

Figure 6. Top View of Azimuth Variation While Establishing the Reference Length

Assuming a uniform error distribution bounded by MPE_{range}, the standard uncertainty contribution due to the laser tracker errors is 0.4 µm for the 1.15 m length and 0.6 µm for the 2.3 m length. Since we do not have knowledge of the extent of the systematic nature of these errors, these uncertainty contributions are (conservatively) not reduced even though the calibrated length is the average of three measurements.

The offset of the apex in the SMR relative to the center of the SMR spherical housing was considered as a possible error source associated with optical alignment. However, this affect is minimized procedurally. During calibration of the reference lengths, the SMR is held in the same orientation relative to the measurement beam during all measurements. Because the apex of the SMR is not perfectly centered in the sphere, small unintended rotation of the ball when placed in the nest can lead to direct errors in the calibrated length. For SMRs with centering error of 2.5 µm, and assuming the SMR can be well aligned, this component is negligible.

Repeatability, L_{repeat}

When an SMR is moved and replaced in the kinematics, repeatability may be affected by the support structure, or the location of the sphere center may change slightly due to imperfect form of the nest or the sphere. Repeatability effects cause systematic errors when using the scale bar, because the placement of the SMRs into the nests will be different than the placements during calibration. Furthermore, during the actual use of the scale bar for laser tracker testing, the SMRs are placed in the nests and length measurements are not averaged. Any non-repeatability in seating of the SMR along the direction of the reference length must be accounted for in the uncertainty. Repeatability of the measured length was evaluated by calibrating the scale bar lengths six times over a short period of time and calculating the standard deviation. The standard deviation of the repeated measurements was approximately 1.5 µm. This gives a standard uncertainty in the length due to the repeatability of the nest, L_{repeat} = 1.5 µm with five degrees of freedom. We note that this uncertainty source is double-counting some of the effect of the laser tracker ranging performance. That is, the laser tracker ranging has some repeatability effects that are accounted for above, but show up in the computation of this standard deviation. However, due to lack of knowledge of the correlation, we (conservatively) treat the effects as independent.

Loading Due to Gravity, L_{gravity}

The gravity loading on the length artifact changes with orientation. For the measurements performed in this paper, the artifact is calibrated in the horizontal position with the handles on top of the support beam. From testing in the laboratory, the estimated maximum change in length due to orientation is no greater than 3 µm for the 2.3m length and 1.5 µm for the 1.15 m lengths. Conservatively assuming the worst case (that is, that the horizontal orientation yields a length at one of the extreme ends of the distribution) and assuming a uniform distribution, we determine the standard uncertainty resulting from orientation to be 1.7 µm for the 2.3 m length and 0.9 µm for the 1.15 m lengths. The uncertainty analysis in this paper is designed to provide information to help a user develop an uncertainty evaluation in the field where calibration of length artifacts in the vertical orientation may not be practical. But the uncertainty resulting from various orientations may be different for different scale bars, so the user should obtain the appropriate values for this effect prior to going to the field.

Retroreflector, $L_{\mathbf{retro}}$

During testing of the laser tracker, the targets are placed along the center-line of the artifact and oriented facing the laser tracker. Offsets of the retroreflector apex relative to the center of the sphere and transverse to the measurement beam cause a direct error in length. For the high grade SMRs used in the tests at NIST, the MPE for the retroreflector offset from the mechanical center of the ball is no larger than 2.5 µm. Projecting the error into the length direction the standard uncertainty from this contribution to the length between two such SMRs is L_{retro} = 1.4 µm.

Artifact Temperature, $L_{\mathbf{temp}}$

Uncertainty due to the artifact temperature has two components: (1) due to the uncertainty in the CTE of the artifact, and (2) due to the variation of the temperature of the artifact.

Uncertainty in the coefficient of thermal expansion of the scale bar and uncertainty in the temperature during testing are sources of uncertainty in the calibrated length of the artifact. The carbon fiber beam manufacturer's estimate of the coefficient of thermal expansion is 1.5 x 10^{-6} /°C. In this paper the actual value of the CTE is assumed to lie within ± 100 % of that value, i.e., uniformly within the interval $[0.0, 3.0] \times 10^{-6}/°\mathrm{C}$.

Assuming the estimate of the uncertainty in the CTE represents maximum error and assuming a uniform distribution gives one standard uncertainty of 0.9 x 10^{-6}/°C. For a change in artifact temperature of 1 degree between calibration and use in laser tracker testing, one standard uncertainty for the 2.3 m and 1.15 m lengths are approximately 2.0 µm and 1.0 µm, respectively.

It is assumed that the scale bar is allowed to thermally stabilize with the environment before calibration and testing. Further, it is assumed that the scale bar is calibrated at ambient temperature and no correction is applied to calculate the length at 20 °C. Lastly, it assumed that no thermal correction is applied during tracker testing. This means that changes in the artifact length due to changes in its temperature must be accounted for in the uncertainty budget. Assuming a maximum change in the scale bar temperature during testing of 1 °C, the standard uncertainty in the determination of the artifact temperature is 0.29 °C. For the 2.3 m and 1.15 m lengths, one standard uncertainty due to change in the artifact temperature are 2.0 µm and 1.0 µm respectively.

Combining the effect due to the uncertainty in the coefficient of thermal expansion and variation in artifact temperature in a root sum of squares gives one standard uncertainty in the calibrated length due to artifact temperature, $\boldsymbol{L}_{\mathbf{temp}}$, of 2.8 µm and 1.4 µm for the 2.3 m and 1.15 m lengths, respectively.

Length Stability During Testing, $L_{\mathbf{stability}}$

Another consideration includes the dimensional stability of the artifact during testing. This error can occur when the scale bar is not allowed to soak in the measuring environment before calibration. The carbon fiber beam used to build the artifact was fitted with a linear interferometer and retroreflectors as shown in Figure 7 below. The beam was simply supported and placed in a temperature controlled metrology laboratory. Over a two hour period, the length artifact was stable to less than one micrometer. Considering this data, the following assumptions were made: testing duration of 30 minutes, and a maximum error in the length due to dimensional stability of the carbon fiber of 0.8 µm for the 2.3 m length and 0.4 for the 1.15 m lengths. Assuming a uniform distribution for this uncertainty source gives one standard uncertainty for the 2.3 m and 1.15 m lengths of 0.5 µm and 0.2 µm, respectively.

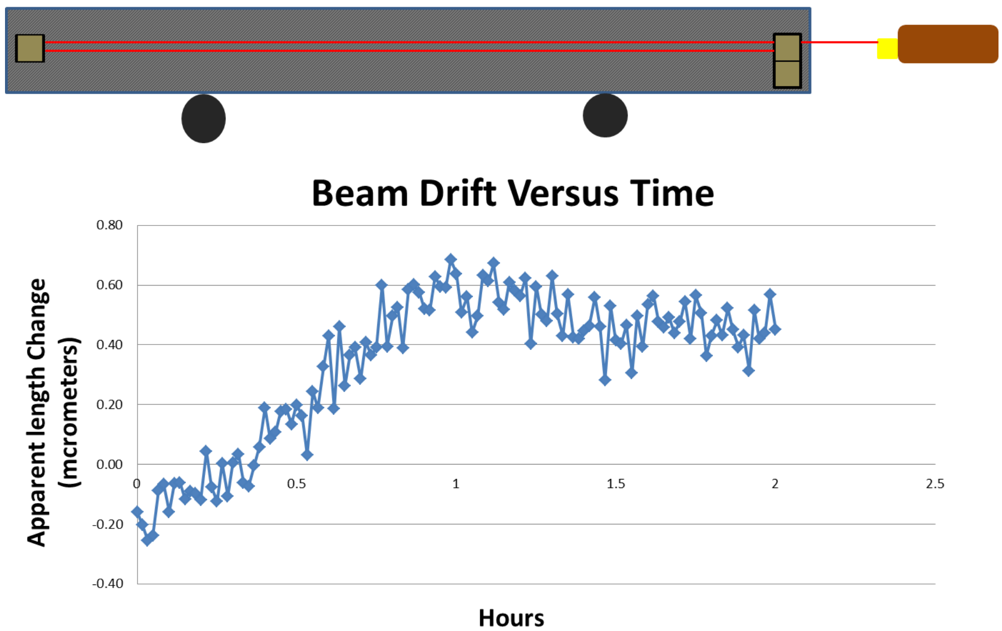

Figure 7. Drift Test of Carbon Fiber Beam in a Stable Environment

Combined Uncertainty

The components described in this section are considered to be uncorrelated, and are combined in a root sum of squares. Under assumed field conditions, the combined expanded uncertainty of the length artifact, *U*, where $k = 2$, is evaluated to be 7.9 µm and 5.4 µm for the 2.3 m and 1.15 m lengths, respectively. The components of uncertainty are summarized in Table 1, below.

Table 1. Uncertainty Budget for the Calibrated Length of the Artifact

Component	Symbol	1 standard uncertainty (um)	
		2.3 m	1.15 m
Optical Alignment, Interferometer and angular encoders	$L_{tracker}$	0.6	0.4
Repeatability of Nests	L_{repeat}	1.5	1.5
Gravity Loading	$L_{gravity}$	1.7	0.9
Retroreflector	L_{retro}	1.4	1.4
Artifact Temperature	L_{temp}	2.8	1.4
Length Stability During Testing Time	$L_{stability}$	0.5	0.2
	$U_{k=2}$	7.9	5.4

Conclusion

Interim testing of laser trackers is a useful step to maintain quality of critical measurements, and can be well served by a length artifact that is both portable and dimensionally stable. A 2.3m length artifact was designed and manufactured using fundamental engineering principles. Design considerations included: locations of beam supports, locations of targets along the neutral plane of bending, foreshortening due to straightness variation, vibration, kinematic mounting, beam properties and materials, support structure, and alignment. Testing was conducted at NIST to evaluate the performance of a prototype length artifact. The combined expanded uncertainty, for $k = 2$, under assumed field conditions was estimated to be 7.9 µm and 5.4 µm for the 2.3 m and 1.15 m calibrated reference lengths, respectively. The expanded uncertainties are sufficiently small to be of practical use in the field. These uncertainties can be further reduced by measuring the temperature of the scale bar and making temperature corrections during testing. Additionally, better estimates of the CTE and its uncertainty could be used to further reduce the uncertainty in the calibrated length.

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