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A classical channel model for gravitational decoherence

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Abstract

We show that, by treating the gravitational interaction between two mechanical resonators as a classical measurement channel, a gravitational decoherence model results that is equivalent to a model first proposed by Diosi. The resulting decoherence model implies that the classically mediated gravitational interaction between two gravitationally coupled resonators cannot create entanglement. The gravitational decoherence rate (and the complementary heating rate) is of the order of the gravitationally induced normal mode splitting of the two resonators. Failure to see this in an experiment would rule out treating gravitational interactions as purely classical.

Keywords: decoherence, optomechanics, quantum measurement, quantum control

1. Introduction

The ability to optically cool macroscopic mechanical oscillators close to their ground state, from which highly non-classical superposition states may be prepared, provides a platform in which to study the interplay between gravitational and quantum physics [1–3]. The objective is to engineer quantum states of mechanical systems in which gravitational effects must be taken into



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account if we are to account for the dynamics. Karolyhazy [4], Penrose [5] and also Diosi [6] have proposed that in such a setting gravity would lead to a new kind of decoherence and, correspondingly, a new source of noise acting on the quantum degrees of freedom.

There are a number of ways to see why decoherence plays a special role in gravitational interactions. The central point can best be seen by contrasting the case of gravitational mediated interactions to that of electromagnetically mediated interactions for which a full quantum description is available in QED [7]. Indeed ion trap quanta computing shows that one can entangle the motional states of many particles using the coulomb interaction. The current limits on this are due to fluctuating charges on nearby electrons but in principle one can shield from all unwanted Coulomb interactions. This is not the case for the gravitational field of a massive object. There are many ways to measure the gravitational field of a large mass: scattering of test particles, detection of light, red shifts of clocks, contraction of rulers. Admittedly these effects are weak but in principle there is always an open measurement channel: one cannot shield against gravity. A gravitational source exhibits open measurement channels through its effect on space-time geometry. If one cannot close off a measurement channel then there necessarily remains a source of noise and decoherence even if the measurement results are never known. The key question is, how large is this effect?

If we had a quantum theory of gravity, the appearance of an additional source of noise would not be remarkable: it would ultimately arise from quantum fluctuations in the underlying field that mediates the gravitational interaction between quantum mechanical degrees of freedom [7]. Such effects, it is claimed, are likely to become important at the Planck scale and thus seem unlikely to arise in table-top opto-mechanical experiments for which the Newtonian description of gravitational interactions would seem to suffice. Surprisingly, the proposals of Penrose and Diosi would indicate that this is not the case and that, given sufficient quantum control over macroscopic mechanical degrees of freedom, opto-mechanical systems might reveal gravitational decoherence.

In this paper we use the recent proposal [8] for classically mediated long range interactions applied to gravitational interactions. They introduce a picture in which gravitational interactions are regarded as a two-way communication channel. Within this picture we contrast a conventional unitary treatment of the mutual gravitational interaction of two masses with a classical channel modelled as a weak continuous measurement of the position of each mass. Such a channel is classical in so far as the stochastic measurement record may be duplicated through fan-out without introducing noise. A classical measurement channel of this kind is an example of LOCC (local operation with classical communication) and cannot entangle the two masses. If gravitational interactions are of this form it implies one cannot entangle particles through purely gravitational interactions. Henceforth when we refer to a classical channel we mean one that cannot be used to entangle systems.

The continuous measurement record is used to control a reciprocal classical force on each mass via a feedforward control. In essence, the classical measurement record informs each mass of the other's position and applies the corresponding gravitational force. By requiring the effect to be reciprocal, we show that the gravitational decoherence rate is completely determined by the gradient of the gravitational force between the two masses. This is equivalent to the gravitational decoherence models proposed by Penrose and Diosi. We calculate the size of this effect for an experiment based on two gravitationally coupled oscillators. We show that the decoherence rate saturates the bound required for a classical channel to not entangle the oscillators. The key point is this: a decoherence rate below this level would enable one to entangle particles through gravitational interactions and imply that a quantum theory of gravity must realize a quantum channel.

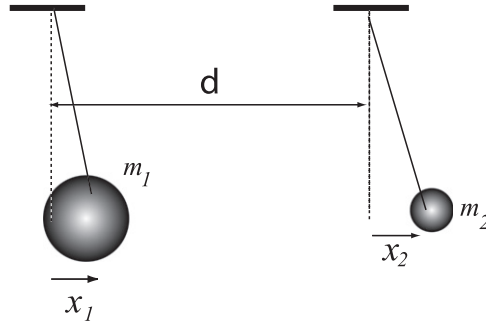


Figure 1. A gravitationally coupled system of two harmonic oscillators comprising two suspended masses m_1 , m_2 .

In [9] Diosi introduces a very similar model to the one we are proposing here for the case of relativistic quantum fields. In that paper a fundamental and universal measurement process measures the field configuration and the measurement results are fed back to modify the future dynamics of the field. In the Markov and non-relativistic limit a special case is made for the mass distribution being subject to the universal measurement. In many ways the model of this paper is a particular demonstration of the physical consequences of this assumption.

2. Combining quantum and gravitational physics

Consider two masses, m_1 , m_2 freely suspended so as to move (approximately) harmonically along the x -axis, figure 1. The gravitational interaction between the two masses couples these harmonic motions. The displacement of mass m_k from equilibrium is denoted x_k . The equilibrium positions are determined by the gravitational attractions between the two suspended masses. The interaction potential energy between the masses, expanded to second order in the relative displacement, may be written

$$V(x_1, x_2) = V_0 - \frac{Gm_1m_2}{d^2}(x_1 - x_2) - \frac{Gm_1m_2}{d^3}(x_1 - x_2)^2 \quad (1)$$

The term linear in the displacement represents a constant force between the masses and simply modifies the equilibrium position of the masses to $\bar{x}_1 = Gm_2/(d^2\omega_1^2)$, $\bar{x}_2 = -Gm_1/(d^2\omega_2^2)$. We will absorb this into the definition of the displacement coordinates. The quadratic terms proportional to x_k^2 can be incorporated into the definition of the harmonic frequency of each mass. The total mechanical Hamiltonian is then given by

$$H_{qm} = H_0 + K\hat{x}_1\hat{x}_2 \quad (2)$$

where

$$H_0 = \sum_{k=1}^2 \frac{\hat{p}_k^2}{2m_k} + \frac{m_k\Omega_k^2}{2}\hat{x}_k^2 \quad (3)$$

with

$$\Omega_k^2 = \omega_k^2 - K/m_k \quad (4)$$

and

$$K = \frac{2Gm_1m_2}{d^3} \quad (5)$$

and the usual canonical commutation relations hold $[\hat{x}_k, \hat{p}_j] = i\hbar\delta_{kj}$.

The model thus reduces to the very well understood case of two quadratically coupled simple harmonic oscillators. The resulting classical and quantum dynamics is then described as two independent simple harmonic oscillators, the normal modes, which are linear combinations of the local co-ordinates $q_+ = (x_1 + x_2)/\sqrt{2}$ is the *centre-of-mass* mode and $q_- = (x_1 - x_2)/\sqrt{2}$ is the *breathing* mode with frequencies $\omega_{\pm}$ given by

$$\omega_{\pm}^2 = (\Omega_1^2 + \Omega_2^2)/2 \pm \frac{1}{2} \left[(\Omega_1^2 - \Omega_2^2)^2 + 4K^2/(m_1m_2) \right]^{1/2}. \quad (6)$$

In what follows we will consider the symmetric case for which $m_1 = m_2 = m$ and $\Omega_1 = \Omega_2 = \Omega$. In that case the normal mode frequencies become

$$\omega_+ = \omega; \omega_- = \omega \left[1 - \frac{2K}{m\omega^2} \right]^{1/2}. \quad (7)$$

In most situations of laboratory relevance, the gravitational coupling is weak and the difference in frequency between the two normal modes, the normal mode splitting, can be written

$$\Delta \equiv \omega_+ - \omega_- \approx \frac{K}{m\omega}. \quad (8)$$

What is the relevance of the normal modes of two coupled oscillators and gravitational decoherence? Many discussions of gravitational decoherence are concerned with the ability to prepare a single massive object in a pure quantum state of its centre of mass degree of freedom, for example, with a delocalized wave function of the form $\psi(x) = \psi(x+d) + \psi(x-d)$ where d is a macroscopically distinguishable displacement [30]. The ground state of two normal modes is a superposition state of the configuration space variables of two centre-of-mass degrees of freedom (the local modes) and as such the ability to prepare such a state through purely gravitational interactions would be a test of gravitational decoherence. The wave function of the normal mode ground states, $|0\rangle_+ \otimes |0\rangle_-$, in the coordinate basis of the local centre-of-mass coordinates, is a Gaussian two-mode squeezed state [14],

$$|0\rangle_+ \otimes |0\rangle_- = \int \int dx_1 dx_2 \psi(x_1, x_2) |x_1\rangle \otimes |x_2\rangle, \quad (9)$$

where the wave function is

$$\psi(x_1, x_2) = \mathcal{N} \exp \left[-\vec{x}^T L \vec{x} \right], \quad (10)$$

where $\vec{x}^T = (x_1, x_2)$ and

$$L = \frac{m\omega}{4\hbar} \begin{pmatrix} 1 + \sqrt{1-\beta} & 1 - \sqrt{1-\beta} \\ 1 - \sqrt{1-\beta} & 1 + \sqrt{1-\beta} \end{pmatrix} \quad (11)$$

and $\beta = 2K/m\omega^2$. Decoherence will suppress the off-diagonal components of this state in the local co-ordinate basis. This state also carries a superposition of correlations through the off

diagonal elements of L , that is to say it is also entangled. Thus decoherence will tend to reduce entanglement.

3. Gravity as a classic measurement channel

Kafri and Taylor [8] have recently proposed a simple way to test if a long range interaction between two particles is mediated by a quantum or a classical channel. They define a quantum channel by introducing an ancillary degree of freedom, a harmonic oscillator. The coherent interactions between two local systems and the channel lead, under appropriate circumstances, to an effective direct non-local interaction between the two local systems. This kind of process is used in geometric phase gates to simulate non-local interactions between internal states of trapped ions, with the ionic vibrational modes serving as the ancilla [12, 13]. The key of course is the ability to implement controlled entangling operations between the electronic and vibrational degrees of freedom of each ion. A classical channel can then be defined by simply allowing the ancillary oscillator to be continually measured.

In this paper we will take a different, although equivalent, approach to defining a classical mediated interaction by using methods from quantum stochastic control theory [16]. Rather than a direct quantum interaction of the form $\hat{x}_1\hat{x}_2$, we assume the interaction is mediated by a *classical channel*. That is, the gravitational centre of mass co-ordinate, $\hat{x}_i$, of each particle is continuously measured and a classical stochastic measurement record, $J_k(t)$, carrying this information acts reciprocally as a classical control force on the other mass. The effect on the dynamics of the systems is to produce a Hamiltonian term of the form,

$$H_{\text{grav}} = \chi_1 \frac{dJ_1(t)}{dt} \hat{x}_2 + \chi_2 \frac{dJ_2(t)}{dt} \hat{x}_1. \quad (12)$$

As we will see, the units can be chosen such that the units of χ_k are Jm^{-2} , the same as the units of K . In the case of continuous weak measurements of $\hat{x}_k$ the measurement record obeys a stochastic differential equation of the form [16]

$$dJ_k(t) = \langle \hat{x}_k \rangle_c dt + \sqrt{\frac{\hbar}{2\Gamma_k}} dW_k(t), \quad (13)$$

where Γ_k is a constant that determines the rate at which information is gained by the measurement, while $dW_{1,2}$ are independent, real valued Wiener increments. The units of $\Gamma_k/\hbar$ are $m^{-2} s^{-1}$. The average $\langle \hat{x}_k \rangle_c$ is a *conditional* quantum mechanical average conditioned on the entire history of measurement records up to time t .

The conditional quantum dynamics of the coupled oscillator system is given by the stochastic master equation

$$d\rho_c = -\frac{i}{\hbar} [H_c, \rho_c] dt - \sum_{k=1}^2 \frac{\Gamma_k}{2\hbar} [\hat{x}_k, [\hat{x}_k, \rho_c]] dt + \sqrt{\frac{\Gamma_k}{\hbar}} dW_k(t) \mathcal{H}[\hat{x}_k] \rho_c, \quad (14)$$

where the classical control Hamiltonian is defined by

$$H_c = H_0 + H_{\text{grav}} \quad (15)$$

and the conditioning super-operator, $\mathcal{H}$ is defined by

$$\mathcal{H}[\hat{X}] \rho = \hat{X} \rho + \rho \hat{X} - \text{Tr}(\hat{X} \rho + \rho \hat{X}) \rho. \quad (16)$$

The appearance of the conditional average, $\text{Tr}(\hat{x}_k \rho + \rho \hat{x}_k)$ in equation (14) is very similar to the theory of [11] and also [10] although in the latter case the stochastic term is missing and the interpretation is thus very different.

The form of equation (15) defines a direct feedback model of the kind considered in [16]. Using the results there we find that the corresponding unconditional dynamics (that is to say, the dynamics averaged over all measurement records) is given by

$$\begin{aligned} \frac{d\rho}{dt} = & -\frac{i}{\hbar} [H_0, \rho] - \frac{i}{2\hbar} (\chi_2 [\hat{x}_1, \hat{x}_2 \rho + \rho \hat{x}_2] + \chi_1 [\hat{x}_2, \hat{x}_1 \rho + \rho \hat{x}_1]) \\ & - \sum_{k=1}^2 \frac{\Gamma_k}{2\hbar} [\hat{x}_k, [\hat{x}_k, \rho]] - \frac{\chi_1^2}{8\hbar\Gamma_1} [\hat{x}_2, [\hat{x}_2, \rho]] - \frac{\chi_2^2}{8\hbar\Gamma_2} [\hat{x}_1, [\hat{x}_1, \rho]]. \end{aligned} \quad (17)$$

The second term is the systematic effect of the control protocol. It is easy to see that if we fix $\chi_1 = \chi_2 = K$ this reduces to the standard Hamiltonian interaction term given in equation (2). The final two terms represent the effect of feeding back the white noise on the measurement signals to control the dynamics of the other mass.

In the case of highly asymmetric masses, for example the mass of the earth and the mass of a neutron in the experiments on neutron interferometry [21], $m_1 \gg m_2$, so we expect the measurement rates to also be highly asymmetric. In fact for the case of the measurement channel that records the position of the larger mass we expect $\Gamma_1 \gg \Gamma_2$ so that the relative contribution to the noise in the channel from the larger mass to the smaller mass is much smaller than the converse channel. This simply captures the intuition that for large mass the position of the centre of mass should be very classical. Equivalently the rate at which classical information is carried from the larger mass to the smaller mass is much greater than the rate at which information is carried from the smaller mass to the larger mass. This ensures that the decoherence rate of the smaller mass is much less than the decoherence rate of the larger, ‘classical’ mass.

Let us now consider the symmetric case in which $m_1 = m_2$. In that case we expect $\Gamma_1 = \Gamma_2 = \Gamma$. The noise added by measurement and feedback is a minimum at $\Gamma = \chi/2$ linking the decoherence rate due to the continuous measurement to the scale of the gravitational interaction as

$$\Gamma = K/2 \quad (18)$$

so that the rate at which classical information is transmitted by the classical channel is determined entirely by the gradient of the gravitational field. The resulting unconditional dynamics is

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H_0, \rho] - \frac{i}{\hbar} K [\hat{x}_1 \hat{x}_2, \rho] - \frac{K}{2\hbar} \sum_{k=1}^2 [\hat{x}_k, [\hat{x}_k, \rho]]. \quad (19)$$

This is consistent with Diosi’s model [6] which gives the same decoherence rate as obtained here under similar approximations [19]. The form of equation (19) can be generalized to exactly

match the one in reference [8] if we include non-cross terms in the feedback. i.e., in equation (9), we could have terms proportional to $(dJ_1/dt)x_1$ and $(dJ_2/dt)x_2$.

There are similarities between Diosi's approach and the measurement mediated approach described here. Both require that the gravitational interaction between the two degrees of freedom be replaced with a noisy interaction. In the measurement based approach this is incorporated in a way which necessarily preserves positivity as the noise arises from a 'hidden' position measurement of the gravitational centre of mass co-ordinate. In Diosi's approach we need to explicitly constrain the noise to preserve positivity. More recently Diosi [20] has pointed out that a system subject to weak continuous measurement is a natural example of a quantum-classical hybrid dynamics.

Using a recent result of Kafri and Taylor [8], we can show that, in the case when the two systems are Gaussian, the master equation (19) can never entangle them. This is also true for equation (17), assuming $\chi_1 = \chi_2$. Further, the gravitational decoherence in the dynamics is minimal in the sense that, if it were any smaller, evolution under equation (19) would immediately entangle the ground state of the (uncoupled) Hamiltonian, H_0 .

To see this we write the equation (19) in terms of the dimensionless operators $\tilde{x}_k = x_k (m\omega/\hbar)^{-1/2}$,

$$\frac{d\rho}{dt} = -\frac{i}{\hbar} [H, \rho] - ig [\tilde{x}_1 \tilde{x}_2, \rho] - \frac{1}{4} \sum_{k=1}^2 Y_{ij} [\tilde{x}_i, [\tilde{x}_j, \rho]], \quad (20)$$

where $g = \frac{K}{m\omega}$ measures the strength of the gravitational interaction and the matrix $Y_{ij} = \left(\frac{2K}{m\omega}\right) \delta_{ij}$ is the decoherence matrix. Using the result from [8], we note that entanglement is never generated if and only if the matrix $Y - 2ig\sigma$ has no negative eigenvalues, where σ is the 2×2 symplectic matrix

$$\sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (21)$$

Noting that $Y - 2ig\sigma$ has eigenvalues 0 and $4g$ we see that a slightly less noisy matrix $Y_{ij} - \epsilon \delta_{ij}$ produces entanglement for any positive ϵ .

4. An experimental test of gravitational decoherence

We now consider the prospects for an experimental observation of the model proposed here. For simplicity we will assume that the two mechanical resonators have the same mass ($m_1 = m_2 = m$) and frequency ($\omega_1 = \omega_2 = \omega$). The last term in equation (19) is responsible for two complementary effects: it drives a diffusion process in momentum of each of the oscillators at the rate $\hbar K$, which we will call the gravitational heating rate

$$D_{\text{grav}} = \hbar K. \quad (22)$$

The momentum diffusion leads to heating of the mechanical resonators. It is convenient to define this in terms of the rate of change of the phonon number; the average mechanical energy divided by $\hbar\omega$. The heating rate is then given by

$$R_{\text{grav}} = \frac{K}{2 m \omega}. \quad (23)$$

The double commutator term also leads to the decay of off-diagonal coherence in the position basis of each mechanical resonator,

$$\frac{d \langle x'_k | \rho | x_k \rangle}{dt} = (\dots) - \frac{K}{2 \hbar} (x'_k - x_k)^2. \quad (24)$$

This shows that the rate of decay of coherence is more rapid the greater the separation of the superposed states. We can use the natural length scale proceeded by the zero-point position fluctuations in the ground state of each resonator to rewrite the decoherence rate as

$$\Lambda_{\text{grav}} = \frac{K}{2 \hbar} \Delta x_0^2 = \frac{K}{4 m \omega}. \quad (25)$$

Thus the gravitational decoherence rate for position, in natural units, is one half the gravitational heating rate.

These rates can equivalently be expressed in terms of the normal mode splitting when the gravitational interaction is weak, equation (8),

$$R_{\text{grav}} = \frac{\Delta}{2}, \quad (26)$$

$$\Lambda_{\text{grav}} = \frac{\Delta}{4}. \quad (27)$$

We thus see that the key parameter responsible for gravitational decoherence is of the order of the normal mode splitting between the two mechanical resonators due to their gravitational coupling. This has significant consequences for observation.

In order to see gravitational decoherence in this model, we need to arrange for the normal mode splitting to be as large as possible. Writing this in terms of the Newton constant, we see that

$$\Delta = \frac{Gm}{\omega d^3}. \quad (28)$$

In the case of two spheres of radius r , this may be written in terms of the density of the material as

$$\Delta = \frac{4\pi G \rho}{\omega} \left(\frac{r}{d} \right)^3. \quad (29)$$

As $d < 2r$, this quantity is bounded

$$\Delta \leq \frac{\pi G \rho}{6 \omega}. \quad (30)$$

We need to use a material with a large density and a mechanical frequency as small as possible. For example, for depleted uranium spheres and a mechanical frequency of one Hertz, we find that $\Delta \sim 10^{-7} \text{ s}^{-1}$, a value so small that a terrestrial experiment would be challenging.

In a realistic experiment with low frequency mechanical resonators of the kind considered here, thermal noise and frictional damping will be unavoidable. We can estimate the relative size of these effects using the quantum Brownian motion master equation [16],

$$\left. \frac{d\rho}{dt} \right|_{\text{diss}} = \sum_{j=1}^2 -i\gamma_j \left[\hat{x}_j, \left\{ \hat{p}_j, \rho \right\} \right] - 2\gamma_j k_B T m_j \left[\hat{x}_j, \left[\hat{x}_j, \rho \right] \right], \quad (31)$$

where γ_k is the dissipation rate for each of the mechanical resonators assumed to be interacting with a common thermal environment at temperature T . If we compare the form of thermal noise in this equation to the form of gravitational decoherence, for the symmetric case, we see that we can assign an effective temperature to the gravitational decoherence rate given by

$$T_{\text{grav}} = \frac{\hbar K}{2 m \gamma k_B}. \quad (32)$$

If we write this in terms of the quality factor, Q , for the mechanical resonators, it gives an effective thermal energy scale of

$$k_B T_{\text{grav}} = \hbar Q \Delta. \quad (33)$$

In the example discussed in the previous paragraph for the relatively high value of $Q = 10^9$ we find that $T_{\text{grav}} \sim 10^{-9}$ K. One would need an ambient temperature less than this to clearly distinguish gravitational decoherence from environmental effects. Possibly gravitationally coupled Bose–Einstein condensates of atomic gases could reach this regime.

However there is currently a lot of interest in using opto-mechanical systems to look for gravitation decoherence. This is largely due to the ability to use laser cooling techniques to prepare harmonically trapped particles of large mass in the ground state [24, 30]. It is possible that carefully controlled optical levitation [25–28], or magnetic levitation [29, 30] experiments on two gravitationally coupled masses might reach the regime discussed in this paper but it will not be easy. It should be noted in connection with equation (33) that laser cooling also changes both the temperature and the quality factor. There will need to be distinct rounds of cooling and coherent control similar to what is done in ion trap experiments. Perhaps the motivation will come from opto-mechanical attempts to fix a better value for G , the Newton gravitational constant. These efforts will help advance these technologies to the point where the kind of decoherence considered here could be ruled out, thus ruling out the possibility of describing gravity as a classical channel.

5. Conclusion

In this paper we have presented a model in which the force of gravity is mediated by a purely classical channel. The channel is defined by considering a continuous weak measurement of the position of each of two masses and feeding forward the classical stochastic record of measurement results to induce the right gravitational force between the masses. As the weak measurement model is entirely consistent with quantum mechanics, exactly the right amount of noise is introduced to ensure that the resulting master equation, when averaged over all measurement records, is positivity preserving. Using a minimal symmetric argument, we find that the model is equivalent to a gravitational decoherence model first proposed by Diosi. As the systematic effect of the gravitational interaction also fixes the size of the noise, minimizing the noise leads to a model with no free parameters.

It is well known that a consistent quantum–classical hybrid dynamics leads to problems with positivity of classical phase space distributions and quantum density operators [22]. In our

approach these problems are avoided as the classical dynamical component is in fact a quantum measurement signal, explicitly incorporating quantum back-action noise. Another approach to quantum–classical hybrid dynamics has been considered by Hall and Reginato [23].

Our model is a specific example of a general theory of a classically mediated (i.e. non-entangling) force law considered by Kafri and Taylor [8]. Using a result in that paper we find that a classically mediated gravitational channel based on continuous weak measurement can never entangle Gaussian systems. In an experimental setting of two identical gravitationally coupled resonators, this result is manifest as a direct scaling between the normal mode splitting induced by the gravitational force and the gravitational decoherence rate. An experimental test using two gravitationally coupled opto-mechanical resonators would be difficult, but not impossible, with current technology.

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References

- [1] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 *Phys. Rev. Lett.* **91** 130401
- [2] Romero-Isart O, Pflanzner A C, Blaser F, Kaltenbaek R, Kiesel N, Aspelmeyer M and Cirac J I 2011 *Phys. Rev. Lett.* **107** 020405
- Romero-Isart O 2011 *Phys. Rev. A* **84** 052121
- [3] Pikovski I, Vanner M R, Aspelmeyer M, Kim M S and Brukner C 2012 *Nat. Phys.* **8** 393
- [4] Karolyhazy F 1966 *Nuovo Cimento A* **42** 390
- [5] Penrose R 1996 *Gen. Relativ. Gravit.* **28** 581
- [6] Diosi L 2011 The gravity-related decoherence master equation from hybrid dynamics *J. Phys.: Conf. Ser.* **306** 012006
- [7] Baym G and Ozawa T 2009 *Proc. Natl Acad. Sci.* **106** 3035
- [8] Kafri D and Taylor J M 2013 A noise inequality for classical forces arXiv:1311.4558v1
- [9] Diosi L 1990 *Phys. Rev. A* **42** 5086
- [10] Yang H, Miao H, Lee D-s, Helou B and Chen Y 2012 Macroscopic quantum mechanics in a classical spacetime arXiv:1210.0457v2
- [11] Halliwell J J 1998 *Phys. Rev. D* **57** 2337
- [12] Milburn G J, Schneider S and James D F V 2000 *Fortschr. Phys.* **48** 801
- [13] Leibfried D *et al* 2003 *Nature* **422**
- [14] Audenaert K, Eisert J, Plenio M and Werner R 2002 *Phys. Rev. A* **66** 042327
- [15] Milburn G J and Woolley M J 2012 An introduction to quantum optomechanics *Acta Phys. Slov.* **61**
- [16] Wiseman H M and Milburn G J 2010 *Quantum Measurement and Control* (Cambridge: Cambridge University Press)
- [17] Scott A J and Milburn G J 2001 *Phys. Rev. A* **63** 042101
- [18] Milburn G J and Milburn G J 2012 *Proc. R. Soc. A* **370**
- [19] Diosi L 2007 *J. Phys. A: Math. Theor.* **40** 2989
- [20] Diosi L 2014 Hybrid quantum-classical master equations arXiv:1401.0476

- [21] Overhauser A W and Colella B 1974 *Phys. Rev. Lett.* **33** 1237
Colella R, Overhauser A W and Werner S A 1975 *Phys. Rev. Lett.* **34** 1472
Werner S A, Staudenmann J L and Colella B 1979 *Phys. Rev. Lett.* **42** 1103
- [22] Peres A and Terno D 2001 *Phys. Rev. A* **63** 022101
- [23] Hall M J W and Reginatto M 2005 *Phys. Rev. A* **72** 13
- [24] Kiesel N, Blaser F, Delic U, Grass D, Kaltenbaek R and Aspelmeyer M 2013 *Proc. Natl Acad. Sci.* **11** 14180
- [25] Kheifets Tongcang Li S and Raizen M G 2011 *Nat. Phys.* **7** 527
- [26] Asenbaum P, Kuhn S, Nimmrichter S, Sezer U and Arndt M 2013 *Nat. Commun.* **4** 2743
- [27] Guccione G, Hosseini M, Adlong S, Johnsson M T, Hope J, Buchler B C and Lam P K 2013 *Phys. Rev. Lett.* **111** 183001
- [28] Zhang-qi Y, Geraci A A and Tongcang L 2013 *Int. J. Mod. Phys. B* **27** 1330018
- [29] Cirio M, Twamley J and Brennen G K 2012 *Phys. Rev. Lett.* **109** 147206
- [30] Romero-Isart O, Clemente L, Navau C, Sanchez A and Cirac J I 2012 *Phys. Rev. Lett.* **109** 147205