

Liquid Piston Prover Uncertainty Analysis Spreadsheet

Introduction: The spreadsheet, "Liquid Piston Prover Uncertainty Analysis" was written by John Wright, Jodie Pope, and Aaron Johnson, all of the NIST Fluid Metrology Group. Contact john.wright@nist.gov, jodie.pope@nist.gov, or aaron.johnson@nist.gov to obtain a copy of the spreadsheet, for explanations, to make comments, or to suggest improvements.

This uncertainty analysis follows the paper by Johnson, A. N., Crowley, C. J., and Yeh T. T., *Uncertainty Analysis of NIST's 20 Liter Hydrocarbon Liquid Flow Standard*, FLOMEKO, Taipei, 2010 which is reproduced in Pope, J. G., Wright, J. D., Johnson, A. N., and Crowley, C. J., *Liquid Flow Meter Calibrations with the 0.1 L/s and 2.5 L/s Piston Provers*, NIST Special Publication 250-1039r1, December, 2013. The Pope et al. 2013 publication will be referred to herein as SP-250-1039r1. The Johnson et al. 2010 and SP 250-1039r1 publications can be obtained from www.nist.gov/publication-portal.cfm and searching on the author's name. Other useful references are: 1) England, W. B., *An Uncertainty Analysis for a Positive Displacement Liquid Flow Calibrator Using the Water Draw Technique*, Cal Lab, Jan-Feb-Mar, 2013, 2) Page, D. W. and Hope, S., *Liquid Flow Standards at the TMDE Support Center Corpus Christi Army Depot*, NCSLI Proceedings, 2007. For general instruction on uncertainty analysis see 1) *Evaluation of Measurement Data - Guide to the Expression of Uncertainty in Measurement* (GUM), JCGM 100:2008, (GUM 1995 with minor corrections), http://www.bipm.org/utils/common/documents/jcgm/JCGM_100_2008_E.pdf, and 2) Coleman, H. W. and Steele, W. G., *Experimentation, Validation, and Uncertainty Analysis for Engineers*, A. John Wiley and Sons, 3rd edition, 2009.

The variable names used in this spreadsheet generally follow the names used in SP 250-1039r1. Written descriptions of the variables are given in column A of the "Flow" worksheet and most of the variables are shown in the diagram to the right. Note that the quantity called V_{cv} is the fixed or constant portion of the connecting volume: the unswept portion of the cylinder at the end of a collection (V_{cyl}^f) is also part of the connecting volume, but it is not called by the name "connecting volume" in this analysis.

Equations 9a and 9b in SP 250-1039r1 are the basis equations for this uncertainty analysis. The nominal volumetric flow ($K_v N_{en} / \Delta t$) is multiplied by a long list of factors: R 's that account for reference condition corrections, S 's that account for storage effects, and G 's that account for density differences between the cylinder and the MUT. (Note that the list of corrections is different for volumetric and mass flow).

Cells that are not used for data entry are generally locked and the sheet is protected so that users cannot accidentally alter the formulas, names of variables, etc. The locked cells can be altered if the user chooses "unprotect sheet" on the "review" menu. There is no password. However, there are a few cells that have conditional formatting and can be used for data entry by the user OR for calculations depending upon the users choice (e.g. K_v on the "Flow" worksheet). These are not locked and formulas can be destroyed if the user is not careful. If a user does make an improvement to this spreadsheet, and feels it might be useful to others, please send a copy of the spreadsheet and an explanation of the improvement to john.wright@nist.gov.

The main output of this spreadsheet is the uncertainty of the mass and volume flows found in cells F5 and F6 on the Flow worksheet. The "Dimensional Kv" and "Waterdraw Kv" can calculate the uncertainty of the prover K-factor too based on values acquired during the liquid piston prover calibration process.

This uncertainty analysis assumes that all components are uncorrelated and this is more conservative than taking correlated uncertainties into account. For instance, the initial and final temperatures of the liquid in the cylinder are correlated to some degree: the measurements are made with the same sensors that share any calibration bias. The S_1 and S_3 correction terms use the difference in these two temperatures and taking into account the shared bias (correlation) would reduce the uncertainty of the flow measurement. Because correlations are not taken into account and doing so would reduce the uncertainty, this spreadsheet is taking a conservative approach. Correlation is not taken into account because of the difficulty of estimating the required correlation coefficients.

Not everyone makes all of the corrections in SP 250-1039r1, but users of this spreadsheet *must* enter estimated values of the inputs needed for the correction factors to the "Flow" worksheet and *must* enter values for their uncertainties to calculate their flow uncertainty (whether you actually measure them with a sensor or not). In most cases, the corrections are small and using large uncertainty values will not lead to large flow uncertainties. In cases where conservative estimates based on experience yield unacceptably large flow uncertainty, the user should conduct exploratory experiments (for example, temporary set-ups with extra temperature sensors) to obtain better uncertainty values to enter into the spreadsheet.

If a system is applying one of the corrections, a sensor is necessary to measure the inputs to the correction. In this case, the uncertainty of the inputs to the correction will be lower, and the uncertainty of the flow measurement will be smaller than if no correction is made. Using R_1 for encoder temperature as an example, if a lab does not measure the encoder temperature (and therefore cannot make the correction), the user of this spreadsheet should enter the typical room temperature as the encoder temperature and enter an uncertainty of the encoder temperature based on the temperature variations that occur in the room (perhaps several degrees). If the lab does measure the encoder temperature and make the R_1 correction, the uncertainty of the encoder temperature will be smaller (perhaps a fraction of a degree).

Instructions: Start on the "Flow" worksheet and enter values for your prover in the blue highlighted cells. Blue highlighted cells are for user data entry, red cells are outputs and black font is for calculated values (usually locked cells). Note that flow is NOT an entered quantity: rather it is calculated using the number of encoder pulses (N_{en}), the time of a collection (Δt), and the volumetric prover K-factor (K_v). The mass and volume flows are calculated from the user inputs and are displayed in red font at the top of the "Flow" worksheet. Users should enter the number of encoder pulses and collection times actually used during calibrations in order to see the uncertainty for their particular operating practices. Also, users should use the "Flow" worksheet repeatedly to enter N_{en} and Δt values that represent the range of flows that they calibrate on the piston prover: the uncertainty will vary with the flow and the operating practices.

There are comments given on many cells to assist the user in making good data entries or to explain what the quantities are (look for red triangles in the corner of commented cells, like the one on this cell).

When the user finds that the uncertainty of the piston prover varies over the flow operating range, the lab should quote: 1) the largest uncertainty over the operating range or 2) give a formula that gives the uncertainty as a function of flow, or 3) give a table of uncertainties for a variety of flows over the operating range.

One of the blue highlighted cells (cell C2) on the "Flow" worksheet has a pull down list where the user can select between 1) the "Dimensional Kv" worksheet, 2) "Waterdraw Kv" worksheet, or 3) "Enter Values" as the source of K_v and its uncertainty. To use the pull down menu, place the mouse cursor on the cell and click once so that a down arrow appears to the right, click on the down arrow to show the pull down menu choices, and finally click on one of the choices in the displayed pull down menu. To alter K_v and its uncertainty, go to the worksheets "Dimensional Kv" or "Waterdraw Kv" and enter the blue highlighted input values. Alternatively, choose "Enter Values" and enter K_v and its uncertainty in the appropriate cells on the "Flow" worksheet.

Nominal values for the cylinder diameter, shaft diameter, and the maximum possible length of the piston stroke are entered on the "Flow" worksheet in all cases since these values are used to estimate the unswept portion of the cylinder volume V_{cyl}^f as described now. The volume of fluid in the cylinder when the piston is at the left-most position (V_{cyl}^i) is calculated from the diameter and stroke length inputs. We then use the number of encoder pulses input by the user for the collection and K_v to calculate a nominal volume of the collection and the difference is taken to be V_{cyl}^f (see Figure 4 in SP 250-1039r1 also shown in the comment to this cell). In many provers, the flow collection does not always start at one end of the cylinder, but the calculation described here will yield the largest value of V_{cyl}^f and hence the most conservative uncertainty.

The blue-highlighted cells in columns C and D are used to enter the values of necessary inputs to the uncertainty analysis. The units in column D sometimes have pull down lists that show what units can be used to enter input values. These are generally English units or Centimeters-Grams-Seconds (CGS) units. Do not enter values in other units: they must be converted to one of the units in the pull down list or they will not be converted correctly.

Uncertainty values can be entered in columns E or F. Column E accepts dimensional uncertainty values while column F accepts relative uncertainty in percent of reading. Guidance on how to estimate the values of uncertainty inputs is given in comments on the cells. See also SP 250-1039r1 for discussion of how to determine the magnitude of uncertainty components. Note that if heating or cooling of the fluid is used to change fluid properties, greater temperature non-uniformity through the prover and larger temperature uncertainties should be assumed.

Column G allows the user to select the probability distribution function and the confidence level of the uncertainty values entered in columns E or F. Available uncertainty types are: G1=Gaussian (normal) distribution, standard uncertainty (i.e., $k=1$, approximately 68% confidence level), G2= Gaussian with $k=2$ or approximately 95 % confidence level uncertainty, or R=rectangular distribution. Note that the the uncertainty value for a rectangular distribution should be "a", not "2a" (see GUM Figure 1, page 17). For example, if the variable is estimated to never fall outside of +/-5 of the input value, the user should enter 5 as the uncertainty value. If nothing is entered for uncertainty type, G1 is assumed.

Column J converts the uncertainty information provided by the user in columns E, F, and G to calculate a $k=1$ or 68 % confidence level relative uncertainty in percent.

Column L calculates the normalized sensitivity coefficient (S) which quantifies the relationship between the input variable uncertainty and the output variable uncertainty (for example, volumetric flow). It can be calculated numerically or analytically. In this case, the basis equation for flow (equation 9 in SP 250-1039r1) has been differentiated with respect to each input variable (and multiplied by the input variable and divided by the flow) to obtain the analytic expression for the normalized sensitivity coefficients. All analytical expressions for the sensitivity have been checked numerically.

Columns M and N calculate the product of each uncertainty component and its normalized sensitivity coefficient, square them, totalize them, multiply by a coverage factor of 2 (to obtain an approximate 95 % confidence level uncertainty).

Columns Q, R, S, and T perform the uncertainty calculations for mass flow. It is necessary to calculate volume and mass flow uncertainties separately since different corrections apply for volumetric and mass flow (equations 9a and 9b).

The "Waterdraw Kv" and "Dimensional Kv" worksheets accept data from the calibration of the liquid piston prover as inputs to the uncertainty analysis.

The "Waterdraw Kv" worksheet has a pull down list that allows the user to select between a "volume" or "mass" waterdraw. Different inputs are required for the two methods and the blue-highlighted cells for user data entry will change depending on which type of waterdraw is selected. Also, the font on calculations for the method not chosen will turn light grey to become less visible. The volume waterdraw uses a calibrated graduated cylinder or test measure to quantify the liquid displaced by the piston over a given number of encoder pulses while the mass waterdraw weighs the displaced liquid in a collection tank. All of the corrections listed in SP 250-1039r1 are available in the Waterdraw Kv worksheet, so, this worksheet can be helpful in processing waterdraw data.

The "Dimensional Kv" worksheet allows the user to enter the appropriate dimensions to calculate K_v geometrically. (These same dimensional values must be entered on the "Flow" worksheet even if the Waterdraw method is being used: nominal values of K_v used in the uncertainty analysis are calculated from the dimensions entered on the "Flow" worksheet.) Important corrections for temperature are not applied on the "Dimensional Kv" worksheet, so the user must enter dimensions that have been corrected to the reference temperature and must include uncertainty components like out-of-roundness, diameter uniformity, temperature, etc. in their uncertainty for the dimensional values. See Aguilera, J. J., Yeh, T. T., and Wright, J. D., Inside Diameter Measurements for the Cylinder of a 20 L Piston Prover, Proceedings of the International Symposium on Fluid Flow Measurement, Queretaro, Mexico, 2006, http://www.nist.gov/manuscript-publication-search.cfm?pub_id=830966.

We give advice about uncertainty components to consider and how to estimate them in comments on data input cells. More ideas on how to estimate uncertainty components can be found in SP 250-1039r1 and the other references.

The meter under test (MUT) temperature and pressure are time averages during the collection and therefore the user does not enter initial and final time values for these quantities.

