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Reconstruction of mode structure of faint light sources and its applications

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Abstract
We build upon our newly developed mode reconstruction technique that takes advantage of higher-order photon number statistics. We discuss the application of this technique to practical single-emitters and herald single-photon source analysis. We also examine the importance of this research to fundamental issues of quantum mechanics, proposing new experiments.

Keywords: photon-number statistics, photon-number resolving detection, high order correlation functions, mode reconstruction

(Some figures may appear in colour only in the online journal)

1. Introduction

In physics, generating and detecting single photons is of paramount importance, because a breadth of applications, from foundational tests of nature [1] to precision measurement, rely on single photon technologies [2]. Yet, in quantum optics, the definition of a mode must occur before the definition of a photon, thereby making the task of mode characterization of various sources, both classical and non-classical, fundamental. Multi-mode light appears throughout nature and a range of properties of optical fields depend on the number and structure of the occupied electromagnetic modes [3]. Even if a light source is intended to be single-mode, most of real-life sources emit into more than just one mode. From the practical point of view, the mode structure of a source contains rich information about the underlying physical properties of a source and its immediate surroundings.

Modes of different type result in light with different statistical properties. The resulting statistics for non-interfering modes retain the information about all participating modes when the modes are combined. The importance of studying light statistics was recognized early on, when it became clear that unique quantum properties of some states of light are directly linked to their statistical properties. Particularly, first- and second-order photon-number statistics are used to characterize and distinguish a variety of optical systems including single-photon sources [4–6], photon pair sources [7–9], quantum optical states [10–13], cavity quantum electrodynamics [14, 15], and lasers [16, 17]. However, even though low-order number statistics provide a reliable basic characterization of basic sources, it fails to describe the physics of non-ideal (or multi-mode) sources.

In addition of the generation of non-classical states of light, the detection of low-level light is not a trivial task. Modern state-of-the-art detectors require complex circuitry which may introduce errors resulting in less than faithful detection. The complex behavior of detectors gives rise to other fundamental concerns—in fact there are theories that offer alternatives to quantum mechanics based on the inherent unfaithfulness of single photon detection [18, 19].

Active devices that manipulate single-photon states, such as optically-controlled switches and quantum memories often use strong light fields as control. Interactions of a light field with the components of such a device may also change the mode structure of the output state with respect to the input state.

A method that can separate contributions from different modes of complex light sources and active manipulation devices using a record of photoelectronic detections, therefore, can help with understanding underlying processes that occur within them *in situ*. At the same time, it may be useful in better understanding of the physics behind light detection, given that the sources used for the measurement are independently characterized.

2. Method

The method, first introduced in [20], is based on selecting a combination of modes that may contribute to a mixed state of interest. Then, we derive photon number statistics as a function of each mode's contribution. We illustrate our method by considering mixed states with contributions from one or more modes with thermal statistics, one or more modes with attenuated single-photon statistics, and up to one mode with Poissonian statistics. We stress that the method is general and should work with any combination of modes. The only limitation is that only the total fraction of the power with underlying Poissonian statistics can be determined, because a linear combination of modes with Poissonian statistics retains Poissonian statistics.

Consider a mixed state. It is straightforward to write down the full photon number probability distribution given its mode structure with mean photon numbers μ_i. For thermal and Poissonian statistics $\mu = \langle n \rangle$ is the mean photon number and for attenuated single photon statistics μ is the probability of finding a single photon. For each single mode component, the photon number probability distribution is uniquely described by a probability generating function $G(s)$. Particularly,

$$G_{\text{thermal}}(s) = (1 + \mu(1 - s))^{-1},$$

$$G_{\text{single photon}}(s) = (1 - \mu(1 - s)),$$

and

$$G_{\text{poissonian}}(s) = \mathrm{e}^{-\mu(1-s)}$$

for thermal, single photon and poissonian modes respectively. For a mixed state, $G(s)$ is given by the product of the generating functions for all the underlying modes. It is convenient to translate this formalism into a set of relations between μ_i and the intensity auto-correlation functions, $g^{(k)} = \left\langle : \hat{n}^k : \right\rangle / \left\langle \hat{n} \right\rangle^k$ by taking $g^{(k)} = G^{(k)}(s = 1)/\mu_{\text{total}}^k$ where $G^{(k)}(s = 1)$ denotes the kth derivative of $G(s)$ evaluated at $s = 1$. Because $g^{(1)}$ is always unity, for the first order expression we will use $\langle n \rangle = \mu_{\text{total}} = \sum \mu_i$. It is straightforward to demonstrate that for a mixed state with contributions from N modes, N orders of correlation functions are required to fully determine the mode occupation. For fields of arbitrary (or unknown) mode structure with only a few photons, measuring $g^{(k)}$ requires photon number resolution up to k photons.

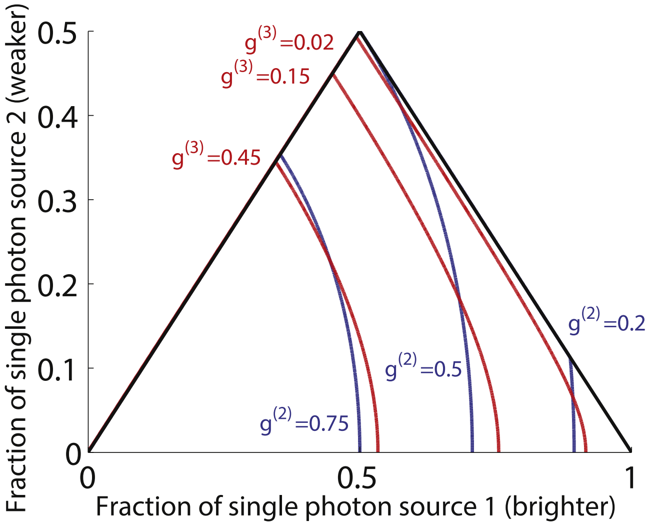

Figure 1. Photon number statistics for a mixed state with two single photon sources and a Poissonian source with a fixed μ_{total}. Lines of constant $g^{(2)}$ (blue), $g^{(3)}$ (red) plotted against the fraction of brighter and weaker single photon sources. The range of allowed values for fractions of single-photon sources is limited by the black solid lines and forms a triangle. The fraction of poisson field changes from $\mu_{\text{poisson}} = 0$ on the top right edge of the triangle to $\mu_{\text{poisson}} = \mu_{\text{total}}$ at the origin.

In this way, a mode structure of a field comprised of a certain number and types of modes can be uniquely expressed through its photon number statistics. We can also solve the reverse problem: that is to resolve mean photon numbers μ_i given the photon number statistics and types of potential modes present in the system. As we will see, our initial selection of mode types does not have to be exact, it can contain modes that are not occupied. The only penalty for considering empty modes is the need to resolve photon number statistics of a higher order to keep the system of equations determined. Overdetermining the system, by using more than N correlation functions to reconstruct N modes, can improve the accuracy of reconstruction, as we will see.

3. Practical single-photon states

3.1. Single-emitter based sources

A typical real-life source of single photons is not ideal: its second-order autocorrelation $g^{(2)}$ is non-zero, even when extra care is taken to minimize a residual $g^{(2)}$, as it is done in [21]. In most cases it is important to find the origins of such behavior. A simple measurement of a $g^{(2)}$, however does not offer any additional information about the source. The origin of a non-zero $g^{(2)}$ for a single-emitter based source may be, for example, residual scattering light from a pump laser, or a second single-photon emitter in a field of view of the collection optics. An application of this method to such a problem can identify the source of the noise and find its strength (relative to the main signal). In this example, for the sake of simplicity we will limit the family of modes to three: two single photon emitters and one Poisson-like background with a fixed μ_{total}. By varying the fraction of power contributed by each mode, such a mixed state can exhibit $g^{(2)}$ between 0 (all single photon) and 1 (all Poisson background). If necessary, more single photon modes can be added to the model, as well as other non-Poissonian sources of light.

A map of the resulting photon number statistics is depicted in figure 1. We observe two families of curves, representing constant values of $g^{(2)}$, $g^{(3)}$. Clearly, measuring just second-order correlation function is not sufficient to determine the mode composition of a given multi-mode input, however, by adding just one more higher-order correlation, we can fully reconstruct the mode composition.

There are several important cases to consider. First, historically [22], a threshold $g^{(2)}(0) < 0.5$ is often used by the experimentalists to differentiate a single photon source from other non-classical sources of light. This threshold is obtained from a simple calculation, were a contribution of just two single photon sources is considered (cf the top right edge of the triangle in figure 1). It turns out that the highest $g^{(2)} = 0.5$ occurs when the two emitters are of equal brightness. Therefore, observation of a $g^{(2)}(0) < 0.5$ ensures that light comes predominately from a single photon emitter. This threshold is very useful in practice, as it has become a defacto standard experimental criterion for a single-photon sources. Lower values of $g^{(2)}(0)$ will be observed if the two single-photon sources have unequal brightnesses. Note, that the only time when this mixture becomes nominally classical, $g^{(2)}(0) \geqslant 1$, is when all light comes from a Poisson mode.

These criteria are based on measuring just $g^{(2)}(0)$, and not the higher-order correlation functions. Because they provide a strict upper bound, they are useful for proof-of-principle experiments. However, the discussion above shown that their use for quantitative source characterization and diagnostics is limited, as they do not provide sufficient information about the state under test. We show that measuring higher-order correlation functions provides information on each component separately.

3.2. Heralded single-photon sources

In a heralded single photon source, photons are generated in pairs in a nonlinear process (such as parametric down-conversion). While each pair is generated at random, the detection of one photon of the pair signals (or *heralds*) the presence of the other with certainty. In most cases, photons of the pair are emitted in separate modes, with one mode used for heralding, and the other is the user output. This source generates nearly-pure single photons, however, by design, the source is limited, as the output always contains an admixture of other modes [23]. First, there is always a chance to emit two pairs at once. Second, if we were to look at a single unheralded mode, it would exhibit thermal statistics, so even a small mode mismatch between a heralding and the heralded signal leads to some thermal background in the output. A real source may also exhibit a Poissonian component in its background, for example, due to the leakage from the strong pump used to excite down-conversion. Therefore, in principle, a real heralded source could produce a mixed state with one or more thermal components in addition to single-photon and a Poissonian ones. As before, to illustrate the method we consider a field that is a mixed state with contributions from one thermal mode, one single-photon mode, one Poissonian mode and a fixed μ_{total}. By varying the fraction of power contributed by each mode, such mixed state can exhibit $0 \leqslant g^{(2)}(0) \leqslant 2$. A zero value of $g^{(2)}(0)$ corresponds to a pure single-photon state, while $g^{(2)}(0) = 2$ indicates a single-mode thermal state.

The map of photon number statistics as a function of fractions of its components is shown in figure 2. As before, a family of mixed states can yield any given $g^{(2)}(0)$. The bold curve is $g^{(2)} = 1$, often used as a nominal threshold for non-classical light, which includes the extreme cases of Poissonian light only and an equal mixture of thermal and single-photon light. For two particular mode distributions between those extremes (points 1 and 2) lines of constant $g^{(3)}$ and $g^{(4)}$ are plotted. See the inset of figure 2 for an example of mode distributions, and observe that mode distributions of cases 1 and 2 yield the same $g^{(2)}(0)$, yet they are significantly different. Note that although $g^{(2)}$ may be identical, the higher-order correlation functions are not, demonstrating how a three-mode distribution can be uniquely identified by μ_{total} plus two orders of the correlation function. For all mixed states that fall above this line a nominal threshold of non-classicality is satisfied, whereas for all states that fall below, it is not. Yet, in all cases there is at least some fraction of non-classical light present. Our method helps detecting non-classical modes of light even when the formal non-classicality criteria for the overall state are not met.

Although an intersection of just $g^{(2)}$ and $g^{(3)}$ gives a one-to-one correspondence between the mode structure and photon number statistics, in practice, the measurements have uncertainties. The significance of these uncertainties is illustrated in figure 2(b). We can see that the same degree of uncertainty in the measurement of the statistics impacts the uncertainty in mode fractions differently in different regions of the mode map. Particularly, it follows from this figure, that the uncertainty in a $g^{(3)}(0)$ value impacts the reconstruction accuracy more in the region of $g^{(2)}(0) > 1.5$. To reduce the impact of uncertainties on mode reconstruction, overdetermined statistics sets should be used. It can be shown that doing so improves reconstruction quality. As it will be seen from the experimental examples of reconstruction, using overdetermined data helps reconstruction algorithms significantly, and we report reconstruction fidelity that exceeds 99% in most cases.

4. Experiment

For the experimental demonstration of this technique, we used a pulsed laser, a pseudo-thermal state (a pulsed laser sent through the rotating ground glass), and a heralded single-photon state. To resolve photon-number statistics up to $g^{(3)}$, we employ a detector tree using four non-photon-number-resolving detectors in a tree configuration, see figure 3. To

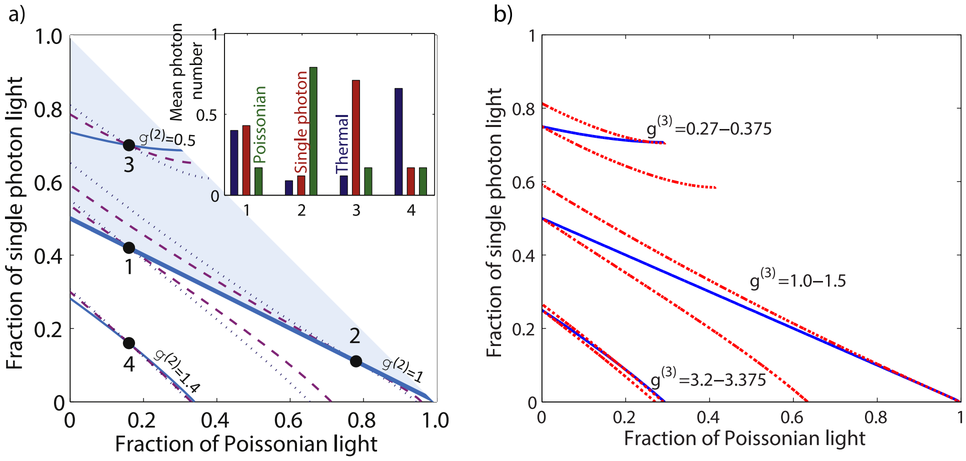

Figure 2. (a) Photon number statistics for a mixed state with an arbitrary contribution of a single photon source, a Poissonian source and a thermal state with a fixed μ_{total}. Lines of constant $g^{(2)}$ (blue solid line), $g^{(3)}$ (red dashed line) and $g^{(4)}$ (blue dotted line) plotted against the fraction of a single photon and a Poisson light. The range of allowed values for fractions of sources forms a triangle. The fraction of thermal field changes from $\mu_{\text{thermal}} = 0$ on the top edge of the triangle to $\mu_{\text{thermal}} = \mu_{\text{total}}$ at the origin. Inset: the mode structure of the four points marked on the main graph. Note that distinctively different mode structures (points 1, 2) yield the same value of $g^{(2)} = 1$. (b) A range of possible $g^{(3)}$ for a field with a given $g^{(2)}$.

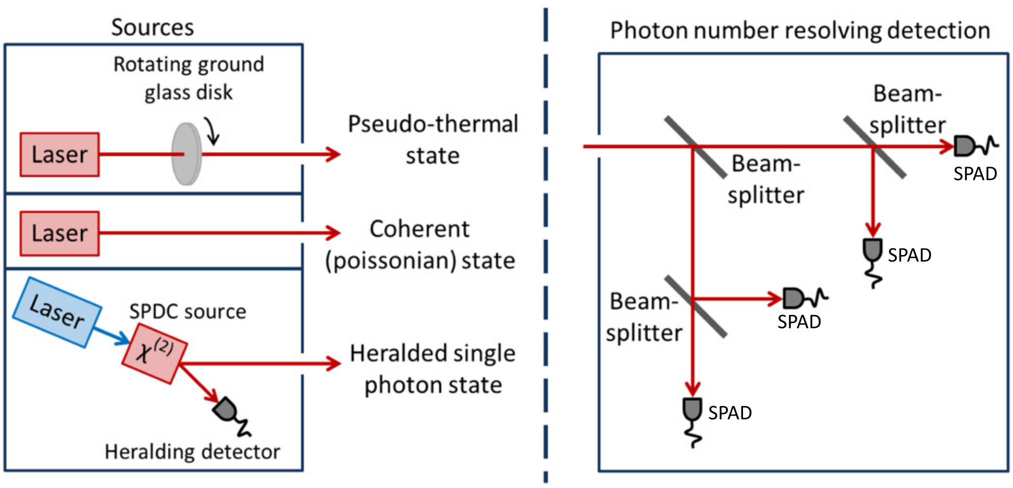

Figure 3. Sources and detector configurations used in a proof of principle experiment. A few sources are incoherently combined on a beamsplitter (not shown), then fed to a photon number resolving detector; here—a tree of click/no-click detectors. SPDC: spontaneous parametric down conversion. SPAD: single-photon avalanche diode.

extract appropriate photon-number statistics from raw data, we perform post-processing of the data because of the positive operator value measure of our detection system [24]. We verify that our pseudo-thermal state has $g^{(2)} > 1.9$ in all cases, i.e. we are close to a true thermal state ($g^{(2)} = 2$). We obtain single photons from a low noise single photon source based on spontaneous parametric down conversion (SPDC) in periodically poled lithium niobate [21], using heralding, described earlier. We verify that the state of the output field, conditioned on detection of a signal photon in a heralding arm, is close to a single photon Fock state, with $g^{(2)} < 0.05$ [21].

We generate an input mixed state in a controllable fashion, by combining up to three modes from our sources, so that we can independently verify the mode structure. Because our pseudo-thermal sources are derived from the same laser, we insert short time delays between the fields to avoid coherent interference between them.

We took data in a variety of regimes. Here we will focus on one example of reconstruction of a mixed state made of just classical modes, and one example involving a single-photon state, heavily polluted with thermal light.

We took data with zero, one, and/or two thermal modes plus up to one Poissonian mode and fit it to two thermal modes plus one Poissonian mode. In figure 4, the rightmost, green, crosshatched bar is the Poissonian mode and the blue bars are the thermal modes. We find excellent agreement for a range of input powers and our method identifies how many modes of which type are present. We stress that the method

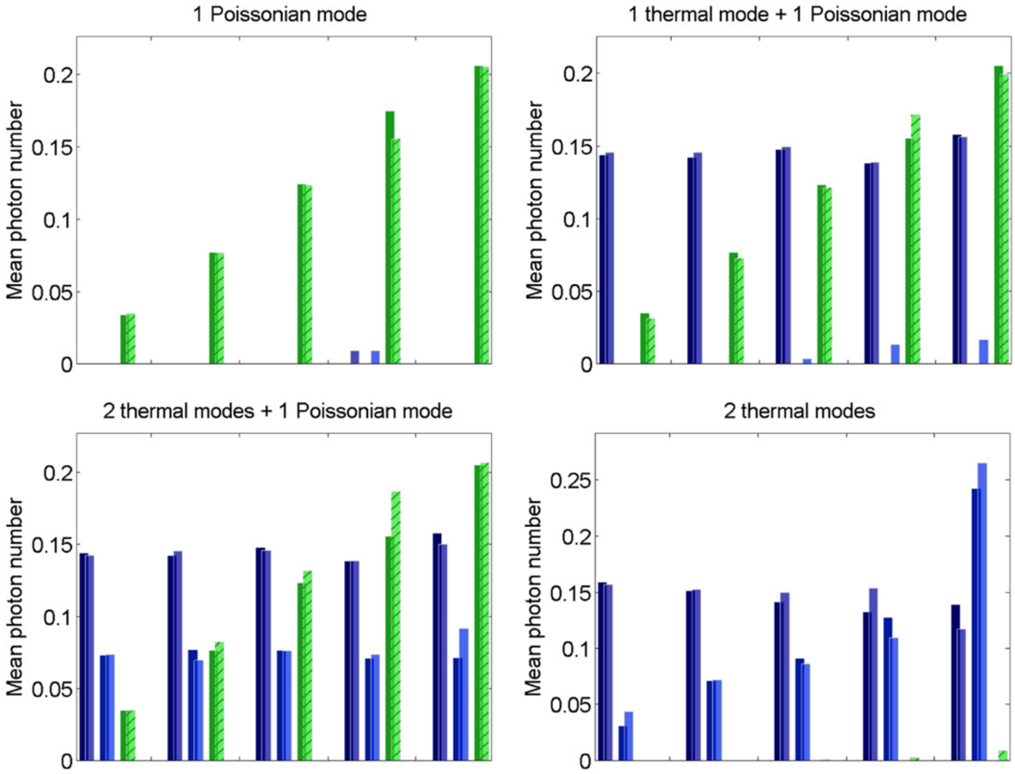

Figure 4. Experimental reconstructure of a mode structure of a classical source. Each group of six bars represents the actual and the reconstructed mean photon number of the two thermal and one poissonian modes. In each group, the four bars on the left (blue) are the thermal modes and the two bars on the right (green, crosshatched) is a poissionian mode. The dark bars in back are the actual mean photon numbers and the translucent bars overlaid are the reconstructed mean photon numbers.

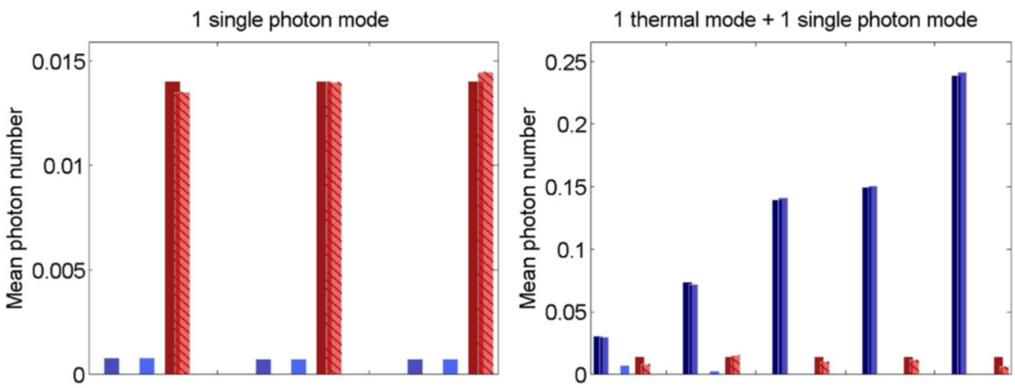

Figure 5. Experimental reconstructure of a mode structure of a nonclassical source. Each group of six bars represents the actual and the reconstructed mean photon number of the two thermal and one single-photon modes. In each group, the four bars on the left (blue) are the thermal modes and the two bars on the right (red, crosshatched) is a single-photon mode. The dark bars in back are the actual mean photon numbers and the translucent bars overlaid are the reconstructed mean photon numbers. Left: single photon source (with residual $g^{(2)} < 0.05$). Right: a mixture of a single photon source with a strong thermal mode ($g^{(2)} > 1$ in all cases)

we employ works well even if the number of modes that are present in the mixed state input is less than the number of modes in the reconstruction model.

Finally, turning to an example with non-classical inputs, we fit the data to two thermal modes plus one single photon mode. We also combined the single photon data with data for a single thermal mode and fit it to two thermal modes plus one single photon mode. In figure 5 the rightmost, red, cross-hatched bar is the single photon mode and the blue bars are the thermal modes.

We first took data for the heralded single-photon source. Mode reconstruction shows that the source is indeed a single-photon source, with a small fraction of light attributed to thermal modes. This is to be expected, because the source has low, but not negligible $g^{(2)}(0)$!

In the second case in figure 5, we simulate and study the non-classical source that is heavily polluted with classical thermal light. The mixing of non-classical and thermal light is common to heralded single photon sources, because the unheralded statistics of such sources are thermal. Note that due to heavy pollution with a classical state, all the mixed states studied here are in the nominally classical regime ($g^{(2)} > 1$). However our method is robust enough to identify the presence of the non-classical component even when it is more than ten times weaker than the classical component.

5. Discussion

The method presented here has significance for practical applications, particularly for development and characterization of non-classical sources. As we have seen, it can be used to find traces of non-classical light in nominally classical mixed states. This is very important for source development, because in many cases the signal obtained from sources in preliminary studies is heavily polluted with light of different origin. The use of our method would help estimate the parameters of the source at hand prior to filtering unwanted light, which in most cases is a difficult exercise. In addition, this method provides information on the type (and therefore, the underlying physics) of unwanted light, allowing for more targeted filtering. On the other hand, we point out that a non-classical source with a heavy background would not be a good source for most practical purposes, thus this method should not be used as a more forgiving substitution to stringent non-classicality criteria.

As we pointed out in the introduction, this method has significance in a foundational discussion, because of the way a photon is defined. We did confirm experimentally that mode structure can be recovered just from a set of photoelectronic records from single-photon detectors. This result is derived in (and thus, is fully compatible with) the framework of quantum mechanics. Any alternative theory must also be able to predict/explain this recovery of mode structure.

Here we are particularly interested in a so-called *prequantum* theory developed by A Khrennikov, which attempts to explain certain quantum effects in terms of classical mechanics and threshold mechanisms in particle detectors. Such an approach increases the importance of the role of detection as compared with traditional quantum theory. The proposed verification experiment [19] is of limited value because it requires an ideal single photon source, and an ideal single photon detector, albeit one with the threshold behavior required by the theory, whereas typical detectors suffer from multiple other issues, see for example [25]. As applied to such a model, this work may lead to the development of more suitable verification tests than that proposed by A Khrennikov [19]. A specific example would be a test using a quantum state compatible with the prequantum theory, but with a nonzero $g^{(2)}(0)$, as any realistic implementation would never achieve $g^{(2)}(0) = 0$.

Such a test requires splitting and recombining the light from a single photon source, so that a beam can be arranged so that it is a mix of two or more single-photon modes originating from the same source. Quantum mechanics prescribes that $0 < g^{(2)}(0) < 1$ for those mixed output states, and that their components can be recovered using our method. On the other hand, these states can also be described by the proposed alternative, the prequantum model. We expect that the ability to recover source mode structure using our experimental method would allow important additional constraints on detector description in those models and potentially provide a falsifiable or distinguishing test.

6. Conclusions

We have presented a method of source characterization based on observation of its photon number statistics. Our proof of principle experiment shows that the method can be successfully used for a broad range of input state types, both classical and non-classical. In addition to its practical value as a characterization tool, this method underlines an important property of light: the retention and the accessibility of the information about its own mode structure.

Acknowledgments

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