Improvements in the Boltzmann Constant Measurement with Noise Thermometry at NIM

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***Abstract* — Since 2010 we have been developing a quantum-voltage-calibrated Johnson noise thermometer at NIM to measure the Boltzmann constant *k*. With recent improvements in grounding and shielding of the electronics, and matching of the noise sources and transmission lines, the effects of electromagnetic interference and variations of fitting parameters with different bandwidths were greatly reduced. By combining 14 measurements with about 10 hours integrating period for each measurement, the relative standard uncertainties of 6.5×10^{-6} and 2.8×10^{-6} were achieved from statistics and bandwidth fit variations, respectively.**

***Index Terms* — Boltzmann constant, Correlation, Josephson arrays, Noise, Quantization, Thermometry.**

I. Introduction

There has been much interest in developing different methods to measure the Boltzmann constant k at an uncertainty level comparable to that of acoustic-gas-based thermometry that dominates the current CODATA value of k [1]. Since 2010, we have been developing a new quantum-voltage-calibrated Johnson noise thermometer (JNT) at NIM to measure k [2]. Our goal is to demonstrate the reproducibility of the electronic approach developed by NIST [3], and pursue lower uncertainty in order to contribute to the new definition of the Kelvin.

In this paper, we report our improvement in the JNT system and the measurement of k. With improved grounding and shielding of the electronics, the electromagnetic interference (EMI) has been greatly reduced. Both the auto- and cross-correlated noise powers of the thermal and the quantum voltage noise sources are closely matched in the measurement bandwidth. By combining 14 measurement results with about 10 hours integrating period for each measurement, the dominated uncertainties contributed by statistics and variations of fitting results with different bandwidths were reduced to 6.5×10^{-6} and 2.8×10^{-6}, respectively.

II. Improved Measurement

Similar to the determination of the Planck constant by comparing mechanical to electrical power with the Watt Balance, the electronic approach to determine k with a noise thermometer compares the thermal noise power $\langle V_R^2\rangle$ of fluctuating electrons in a sense resistor immersed in a triple point water cell to the quantum accurate electrical power $\langle V_Q^2\rangle$ of a synthesized pseudo-noise waveform generated with a quantum voltage noise source (QVNS). The construction of our JNT system was reported previously [2]. In our experiment, the cross-correlated noise power $\langle V_R^2\rangle$ and $\langle V_Q^2\rangle$ are measured with cross-correlation electronics over a 1 MHz bandwidth with 1 Hz resolution. The resistance R of the sense resistor is measured by a DC resistance bridge. The Boltzmann constant is calculated as

$$k = a_0 \frac{V_{Q-calc}^2}{4RT}, \tag{1}$$

where $V_{\text{Q-calc}}$ is the voltage spectral density of the synthesized quantum voltage noise, T = 273.16 K is the temperature of triple point of water, and

$$a_0 = \left\langle V_R^2 \right\rangle \Big/ \left\langle V_Q^2 \right\rangle \Big|_{f=0} \tag{2}$$

represents the noise power ratio extrapolated to DC that is obtained by fitting the measured noise power ratio from 5 kHz to 650 kHz with a two-parameter formula

$$\left\langle V_R^2 \right\rangle \Big/ \left\langle V_Q^2 \right\rangle = a_0 + a_2 f^2 . \tag{3}$$

This fit accounts for differences in the measured signals due to different transfer functions and mismatched transmission lines.

Important signals that may affect the measurement are electromagnetic interference (EMI) that can overload the preamplifiers and introduce a systematic error [3]. Shielding and grounding are necessary to reduce EMI. In our system, the switching input circuit, each channel's amplifier chain, and their associated lithium-ion battery power supplies are sealed in separate aluminum boxes. The measurement electronics is then placed in a μ-metal box. The transmission lines are also shielded with copper screen and μ-metal. To eliminate ground loops, only the ground of the pulse generator that drives the Josephson junctions of the QVNS through microwave cables is directly connected to the earth ground through a low-resistance connection. The grounds of amplifiers and the sense resistor are connected to that of the Josephson junction chip through the transmission lines. All of the shields are connected to the QVNS probe and then to the pulse generator via the microwave cables. With these improvements, almost no EMI

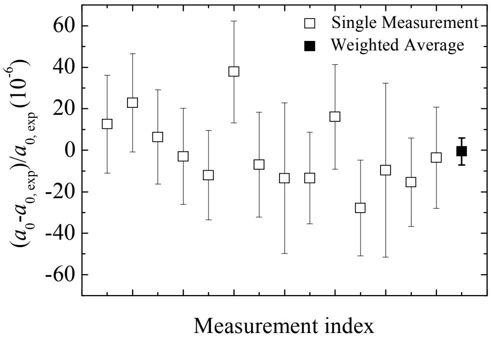

Fig. 1. Relative offsets of the fitting parameters a_0 from the expected value $a_{0,\,exp}$.

signals are observed in the measured spectra for both thermal and quantum voltage noise signals.

To ensure that the nonlinear response of the electronics does not play a role in the measurement, both the cross- and auto-correlation noise power must be closely matched, which requires matching not only the power of the noise sources, but also the transfer function of the transmission lines. In our measurements, the voltage spectral density of the quantum voltage noise $V_{Q\text{-calc}} = 1.2282\ \mathrm{nV/Hz^{1/2}}$ so that the cross-correlated noise power is closely matched to that of the thermal noise of the 100 Ω sense resistor. Furthermore, uncorrelated resistors of resistance about 101.5 Ω were inserted in the QVNS output leads at 4 K, and resistors of about 1.5 Ω were placed in the sense resistor output leads at 273.16 K, the combination of which results in less than 0.1% mismatch of the total powers and output resistance of the noise sources. In addition, the transmission line lengths are carefully trimmed to reach the requirement of closely matched impedances and thus the transfer functions. In a previous paper [2], we demonstrated how flat frequency responses of the noise power ratio and fitting parameters were achieved with well-matched noise sources and transmission lines.

Considering the limitations from the capacity of the batteries for the digitizers and from maintaining the triple point of water cell in the ice bath, we ran the measurement during the night, and each measurement lasted for about 10 hours. In Fig. 1, the relative offset of the fitting parameter a_0 from the expected value $a_{0,\mathrm{exp}}$ for 14 measurements are presented by the empty squares, and the error bars represent the associated standard deviations. Since the integrating periods are not the same for all of the measurements, it is not reasonable to calculate the arithmetic mean as the average result. The weighted average and corresponding standard deviation are calculated with

$$\overline{a_0} = \sum_i \frac{\frac{1}{\sigma_i^2}}{\sum_j \frac{1}{\sigma_j^2}} a_{0,i} \text{ , and } \sigma = \frac{1}{\sqrt{\sum_i \frac{1}{\sigma_i^2}}} \text{ ,} \tag{4}$$

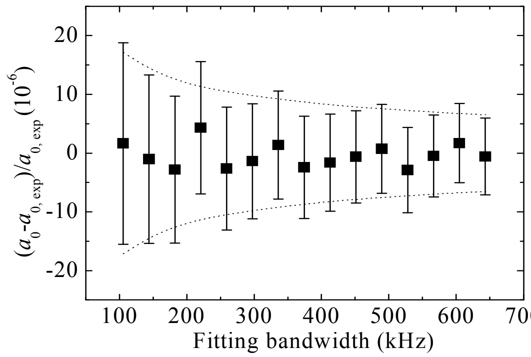

Fig. 2. Weighted averages of relative offset of a_0 for different fitting bandwidths.

where σ_i is the standard deviation of $a_{0,i}$. The calculation results in a relative offset -0.6×10^{-6} with standard deviation 6.5×10^{-6} for a_0, as shown by the solid square in Fig. 1.

In previous measurements, the variations of fitting results over different bandwidths were found, which dominated the measurement uncertainty [3]. In the present measurements, the weighted average of the data fitting over different bandwidths starting from 5 kHz and ending at successively higher frequencies are plotted in Fig. 2. It can be seen that the results are consistent with each other within the standard deviations. The remaining difference contributes a standard uncertainty of approximately 2.8×10^{-6}.

III. Conclusion

In conclusion, with improvements in shielding and grounding, and ensuring well matched noise sources and transmission lines, relative uncertainties from statistics and bandwidth fit variations were reduced to 6.5×10^{-6} and 2.8×10^{-6} in the measurement of k with JNT. With further improvements, we anticipate an electronic measurement of k with relative standard uncertainty of 5×10^{-6} in near future.

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