Polarization-selective Kerr-phase-shift method for fast, all-optical polarization switching in a cold atomic medium

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We examine an all-optical atomic-polarization-gate scheme using a polarization-selective Kerr phase-shift technique. Using a Kerr π-phase-shift technique, we selectively write a π phase shift to one of the circularly polarized components of a linearly polarized signal field while leaving the other component unchanged. Upon recombination, the signal field acquires a 90° linear-polarization rotation, completing the critical polarization-gate operation. We demonstrate with numerical simulations that a special phase-control light-field detuning can be obtained which results in a complete linear-polarization rotation with a phase-control light.

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Quantum computers and quantum information processing hold the promise to revolutionize information science [1–5]. The realization of a quantum computer requires highly efficient quantum-state manipulation protocols where operations of controlling qubits with qubits form the core architecture of the system. One of the preferred and also widely discussed schemes to achieve such controlling-qubits-with-qubits operation is the polarization-encoded gate operation in all-optical atomic systems [6–11]. While there are many proposals [12–16] based on various phenomena including a magnetic field, a light-field-induced shift, and nonlinear effects such as Kerr cross-phase modulations, an experimental demonstration of the effective manipulation of a polarization-encoded light field has proved to be difficult. The fast, all-optical zero-to-π Kerr phase gate demonstrated recently [17] brings the real possibility of achieving manipulation of a polarization-encoded light field in an all-optical atomic system. The simple Kerr phase gate, however, depends on the intensity of the phase-control field [18–21], making it a scalar gate. Polarization gates, on the other hand, are vector gates where two or more independent photonic states are present simultaneously. Furthermore, these entangled states (photonic or atomic) are also vector states that involve the mixing of several quantum states of the system and cannot be expressed as simple products of states. Thus, for quantum communications and quantum computing, it is critically important to be able to execute vector-state manipulations.

Manipulation of photonic quantum states is traditionally achieved using linear optical elements such as beam splitters and polarizers. However, there has not been a demonstration of a polarization-gate operation using a scalar operation such as the nonlinear Kerr effect in atomic media. Recently, we developed a polarization-selective Kerr-phase-shift (PSKPS) technique which permits fast and complete polarization-gate operations using a weak phase-control light field [22]. The PSKPS method demonstrated experimentally employed an active Raman gain (ARG) scheme for a signal field, and a complete orthogonal polarization rotation has been achieved with a phase-control light intensity as low as $2\,\mathrm{mW/cm^2}$ [23].

While theoretically it is possible to operate a polarization gate in such a gain medium, the spontaneous and stimulated Raman gains introduce complications at a weak light level. In this study we examine a polarization-gate operation using the combination of PSKPS and electromagnetically induced transparency (EIT). While at a low phase-control light level the ARG scheme is much more efficient and robust, the EIT scheme has several advantages that cannot be matched by the ARG medium at a weak light level. For instance, the absorptive nature of the EIT process avoids the ambiguity associated with generation of multiple stimulated signal photons due to the gain process. We show that the combination of PSKPS and EIT can achieve a controlled-NOT (CNOT) polarized-gate operation at weak light levels without the requirement of focusing the gate-operation field to the diffraction limit [15]. It must be noted, however, that the applicable regime of the present study is only weak-light optical telecommunications (i.e., not the quantum information regime) since we consider the level of optical control field with more than 1000 photons.

We consider an ensemble of lifetime-broadened ^{87}Rb atoms to demonstrate the principle of high-fidelity all-optical atomic polarization-gate operations. The physical principle of the polarization-gate operations using the PSKPS method may be best illustrated with the four-level atomic medium under EIT conditions shown in Fig. 1(a). Consider the case where all atoms are initially in the $|0\rangle \equiv |5S_{1/2},F=2,m_F=0\rangle$ state. A very weak linear-polarization-encoded signal light field resonantly couples $|0\rangle = |5S_{1/2},F=2,m_F=0\rangle \rightarrow |5P_{1/2},F'=2,m_{F'}=0\rangle$ transition. To reduce the loss of the signal field due to the strong one-photon resonance, we introduce a strong, linearly polarized coupling field E_c that drives transitions from the $|5P_{1/2},F=2\rangle$ manifold to the $|5S_{1/2},F=1\rangle$ manifold. The key consideration of this scheme is that the atomic transition selection rules dictate that the direct $|0\rangle = |5S_{1/2},F=2,m_F=0\rangle \rightarrow |5P_{1/2},F'=2,m_{F'}=0\rangle$ transition by the linearly polarized signal field is forbidden. Thus, the two circularly polarized components of the signal field separately couple the $|0\rangle \rightarrow |L\rangle = |5P_{1/2},F'=2,m_{F'}=-1\rangle$ and the $|0\rangle \rightarrow |R\rangle = |5P_{1/2},F'=2,m_{F'}=+1\rangle$ transitions, whereas the field E_c simultaneously excites the $|L\rangle \rightarrow |a\rangle = |5S_{1/2},F=1,m_F=-1\rangle$ and $|R\rangle \rightarrow |b\rangle = |5S_{1/2},F=1,m_F=+1\rangle$ transitions [Fig. 1(a)].

The PSKPS strategy to achieve polarization-gate operations requires the introduction of a different Kerr phase to different signal components using a weak phase-control field $E_{\rm ph}$. This key step allows one to use a scalar gate such as a Kerr phase gate [17] to construct a vector-gate operation such as

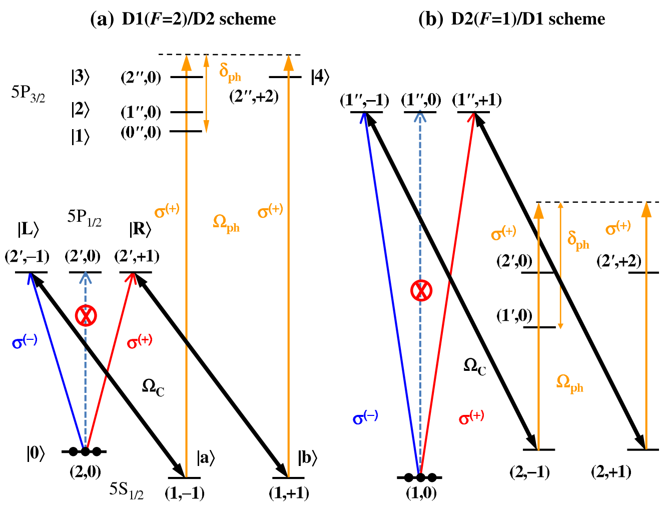

FIG. 1. (Color online) Energy diagram of ^{87}Rb and laser couplings for an all-optical atomic polarization gate using electromagnetically induced transparency and a Kerr phase gate. (a) D_1(EIT)/D_2(Kerr) scheme with initial state $|5S_{1/2},F=2,m_F=0\rangle$ and (b) D_2(EIT)/D_1(Kerr) scheme with initial state $|5S_{1/2},F=1,m_F=0\rangle$. Magic detuning $\delta_{\rm magic}$ refers to a specific phase-control-field detuning $\delta_{\rm ph}$ that can result in a complete orthogonal switch of the signal-field linear polarization with a very low phase-control field.

a polarization-gate operation. In our calculation, the energy levels in the $5P_{3/2}$ manifold interacting with the phase-control field are labeled as $|j\rangle \equiv |5P_{3/2},F''=j-1;m_{F''}=0\rangle$ ($j=1,2,3$) and $|4\rangle \equiv |5P_{3/2},F''=2,m_{F''}=+2\rangle$, respectively.

Referencing Fig. 1(a), we write the total electric-field vector as $\mathbf{E}(\mathbf{r},t)=\sum_\beta E_\beta(t)\mathbf{e}_\beta \exp i(k_\beta z-\omega_\beta t)+\text{c.c.}$ where $\mathbf{e}_\beta$ is the unit vector denoting the polarization direction and E_β ($\beta = S,c,\text{ph}$) are the envelope functions of the signal, coupling, and phase-control fields, respectively. The linearly polarized signal field is expressed with its two circular components as $\hat{\mathbf{E}}_S=\mathbf{e}_+\hat{E}_{S+}+\mathbf{e}_-\hat{E}_{S-}$.

In the interaction picture and within the electric-dipole and rotating-wave approximations, the system Hamiltonian operator reads $\hat{H}_{\rm int}=\hbar\sum_{j=1}^4\delta_j|j\rangle\langle j|+\hat{H}_1$, with

$$\begin{aligned}\hat{H}_1 = &-E_{S+}D_{R0}|R\rangle\langle 0|-E_{S-}D_{L0}|L\rangle\langle 0|\\ &-E_cD_{La}|L\rangle\langle a|-E_cD_{Rb}|R\rangle\langle b|\\ &-E_{\rm ph}D_{4b}|4\rangle\langle b|-E_{\rm ph}\sum_{j=1}^{3}D_{ja}|j\rangle\langle a|+\text{c.c.},\end{aligned}\tag{1}$$

where δ_j is the phase-control-field detuning above the energy level $|j\rangle$. If we define δ_1 in reference to the $j=1$ level of ^{87}Rb, then $\delta_2=\delta_1-\Delta_{21}$, $\delta_3=\delta_1-\Delta_{21}-\Delta_{32}$, and $\delta_4=\delta_1-\Delta_{21}-\Delta_{32}-\Delta_{43}$, where Δ_{kl} represents the hyperfine splitting between levels $|k\rangle$ and $|l\rangle$. D_{kl} is the dipole matrix element for the $k\to l$ transition.

The atomic response is described by the optical Bloch equations [24]

$$\frac{\partial}{\partial t}\hat{\sigma}=-\frac{i}{\hbar}[\hat{H}_{\rm int},\hat{\sigma}]+\hat{\sigma}_{\rm rel},\tag{2}$$

where $\hat{\sigma}$ is the atomic density matrix operator and $\hat{\sigma}_{\rm rel}$ describes relaxation processes such as spontaneous emission and dephasing due to the collision. We note that in the present work we consider the case with more than 100 photons in both signal and phase-control fields. Consequently, the Langevin noise operator, which must be included in a single-photon treatment [25], can be neglected. Indeed, at the typical optical telecommunication level, which is the concern of the present work, such single-photon level fluctuation is negligible.

The Maxwell equations for the positive-frequency part of the two circularly polarized components of the weak signal field of linear polarization are [see Fig. 1(a)]

$$\frac{\partial E_{S\pm}^{(+)}}{\partial z}+\frac{1}{c}\frac{\partial E_{S\pm}^{(+)}}{\partial t}=\frac{4\pi}{c^2}P_{S\pm}^{(+)},\tag{3}$$

where the polarization operators for the signal-field polarization modes are given by

$$P_{S+}^{(+)}=\mathcal{N}D_{R0}\sigma_{R0},\qquad P_{S-}^{(+)}=\mathcal{N}D_{L0}\sigma_{L0},\tag{4}$$

with $\mathcal{N}$ the atom number density and σ_{R0} and σ_{L0} the relevant atomic density matrix elements. Under the adiabatic approximation the linear phase and loss ($\phi_{\rm L}^{(\pm)}$, $\alpha_{\rm L}^{(\pm)}$) and the nonlinear phase and loss ($\phi_{\rm NL}^{(\pm)}$, $\alpha_{\rm NL}^{(\pm)}$) per unit length of both polarization components can be analytically calculated using Eqs. (1)–(4) [26], yielding

$$\phi_{\rm L}^{(+)}=\phi_{\rm L}^{(-)}=0,\quad \alpha_{\rm L}^{(+)}=\alpha_{\rm L}^{(-)}=\frac{\kappa_0\Gamma_0}{\Gamma_0\gamma+|\Omega_c|^2},\tag{5a}$$

$$\phi_{\rm NL}^{(+)}=-S_0\delta_4\frac{|\Omega_{4b}^{(\rm ph)}|^2}{\delta_4^2+\Gamma_4^2},\quad \alpha_{\rm NL}^{(+)}=S_0\Gamma_4\frac{|\Omega_{4b}^{(\rm ph)}|^2}{\delta_4^2+\Gamma_4^2},\tag{5b}$$

$$\phi_{\rm NL}^{(-)}=-S_0\sum_{j=1}^{3}S_j\delta_j,\quad \alpha_{\rm NL}^{(-)}=S_0\sum_{j=1}^{3}S_j\Gamma_j,\tag{5c}$$

with $S_0=\kappa_0|\Omega_c|^2/(\Gamma_0\gamma+|\Omega_c|^2)^2$, $S_j=|\Omega_{ja}^{(\rm ph)}|^2/(\delta_j^2+\Gamma_j^2)$, $\kappa_0=\omega_S\mathcal{N}L|D_0|^2/(2c\hbar\epsilon_0)$ (defining $|D_{L0}|=|D_{R0}|=|D_0|$ and $|D_{La}|=|D_{Rb}|=|D_1|$), $2|\Omega_c|=|\Omega_{La}|=|\Omega_{Rb}|=|D_1|E_c/\hbar$, $\Gamma_0=\Gamma_L=\Gamma_R$, and $\gamma=\gamma_a=\gamma_b$. Γ_j ($j=1,\ldots,4$) is the resonance linewidth of the jth upper phase-control state and $2\Omega_{mn}^{(\rm ph)}=D_{mn}E_{\rm ph}/\hbar$ is the Rabi frequency of the phase-control field $E_{\rm ph}$ for the relevant transition.

Our goal is to induce, with appropriate choices of phase-control-field detuning and intensity, phase changes $\phi^{(+)}=\phi_{\rm NL}^{(+)}=\pi$ and $\phi^{(-)}=\phi_{\rm NL}^{(-)}=0$ in the $\sigma^{(\pm)}$ components. Since initially the amplitudes of the circular-polarization components of the input signal field are exactly the same, if the nonlinear losses of the two circular-polarization modes can be made equal, i.e., $\alpha_{\rm NL}^{(+)}=\alpha_{\rm NL}^{(-)}$ (note that the linear loss is the same for both polarization modes), then at the exit of the medium the two components will form a linearly polarized light field again but with its polarization *orthogonal* to that of the input signal field. This completes the polarization-gate operation that can be described by the phase transformation [also see Fig. 2(a)]

$$\mathbf{V}=\begin{pmatrix}\sigma^{(+)}\\ \sigma^{(-)}\end{pmatrix}\Rightarrow\begin{pmatrix}e^{i\pi}\sigma^{(+)}\\ \sigma^{(-)}\end{pmatrix}=\begin{pmatrix}-\sigma^{(+)}\\ \sigma^{(-)}\end{pmatrix}=\mathbf{H},$$

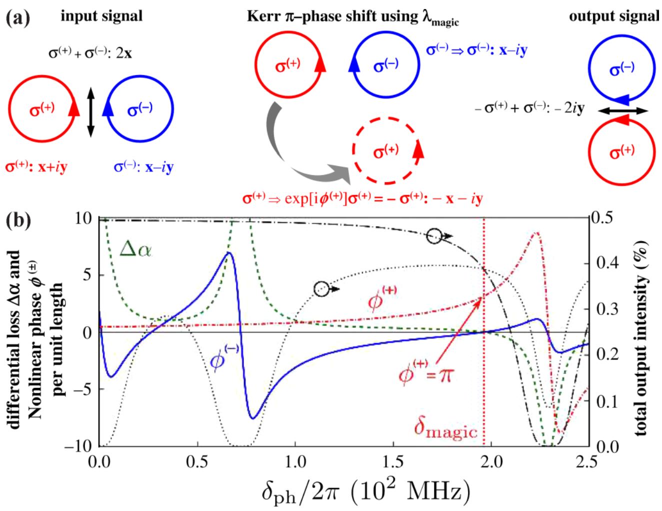

FIG. 2. (Color online) (a) Conceptual illustration of the fundamental principle of an all-optical atomic polarization gate (CNOT gate) using the PSKPS technique. Left: Two circularly polarized components of the linearly polarized input signal field. Middle: PSKPS results in a π phase shift to the $\sigma^{(+)}$ component only. Right: The polarization of the combined output signal field. (b) Plot of differential loss $\Delta\alpha = \alpha^{(+)} - \alpha^{(-)}$ (short-dashed curve) and Kerr phase shifts $\phi^{(\pm)}$ of the two circular components (dot–short-dashed and solid curves) as functions of the phase control light detuning $\delta_{\rm ph}$. Also shown are the individual losses of each polarization component (dot–long-dashed and dotted curves, right axis), indicating the merit of the process. A phase-control light detuning $\delta_{\rm ph}/2\pi = \delta_{\rm magic}/2\pi$ near 195 MHz where $\Delta\alpha = 0, \phi^{(+)} = \pi$, and $\phi^{(-)} = 0$ are simultaneously satisfied is shown with the vertical dashed red line. The coupling-field amplitude used for this simulation is $E_c = 75$ V/m.

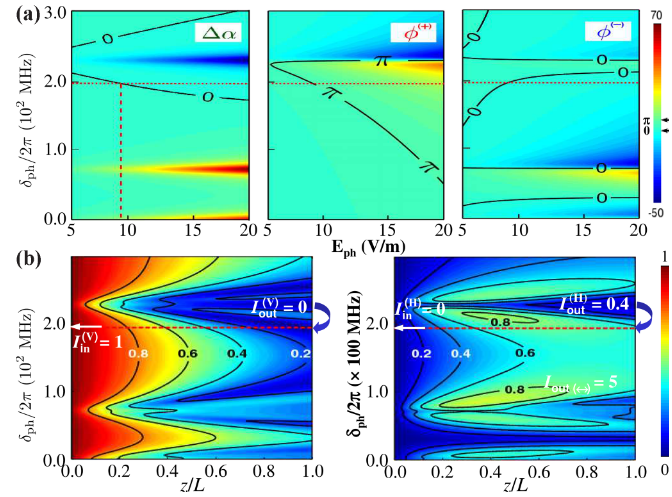

FIG. 3. (Color online) (a) Contour plots of $\Delta\alpha$ and $\phi^{(\pm)}$ as functions of the phase-control-field amplitude and detuning. The equal-altitude lines (zero for $\Delta\alpha$ and $\phi^{(-)}$ and π for $\phi^{(+)}$) guide the selection of a magic phase-control-field detuning (the horizontal dashed line). The vertical line indicates that about 100 photons are required to achieve the complete polarization-gate operation. (b) Contour plots of $I_{\rm in}^{(\mathbf{V})}(z;\delta_{ph})$ (left panel) and $I_{\rm out}^{(\mathbf{H})}(z;\delta_{\rm ph})$ (right panel) as functions of $\delta_{\rm ph}$ and the normalized propagation distance z/L. The dashed line indicates a magic phase-control-field detuning with which the polarization of the signal field changes from **V** at the entrance ($z/L = 0$, left plot: $I_{\rm in}^{(\mathbf{V})} = 1$ and $I_{\rm in}^{(\mathbf{H})} = 0$) to **H** at the exit ($z/L = 1$, right plot: $I_{\rm out}^{(\mathbf{V})} = 0$, $I_{\rm out}^{(\mathbf{H})} = 0.4$) of the medium. $E_c = 75$ V/m. Plots are normalized with respect to $I_{\rm in}^{(\mathbf{V})}(z = 0)$.

where **V** and **H** stand for vertical and horizontal polarizations. Mathematically, the above-described phase transformation requires the following conditions to be met simultaneously (N and M are integers):

$$\alpha^{(+)} = \alpha^{(-)}, \quad \phi^{(+)} = (2N+1)\pi, \quad \phi^{(-)} = 2M\pi, \quad (6)$$

where $\alpha^{(\pm)} = \alpha_{\rm L}^{(\pm)} + \alpha_{\rm NL}^{(\pm)}$ are the total losses of the two circular polarization modes.

Achieving Eq. (6) with minimum phase-control-field intensity at weak light levels requires the phase-control light field to interact with different upper states for the different circular components at a specific detuning which is hereby termed a "magic" detuning $\delta_{\rm ph} = \delta_{\rm magic}$. In the case of Fig. 1(a), such a phase-control-field detuning can be found at weak light levels by solving Eqs. (5) and (6) repeatedly, successively reducing the phase-control light intensity while keeping the loss low. Alternatively, one can preset the phase-control light at the single-photon level and seek such a magic detuning by varying the EIT coupling field to minimize the signal-field loss. This preferred operation point is indicated by the crossing point of the two black curves in Fig. 2(b) and the corresponding phase-control-light detuning is about $\delta_{\rm ph}/2\pi = \delta_{\rm magic}/2\pi \approx$ 195 MHz.

In Fig. 3 we explore possible conditions for operation with a single phase-control photon numerically using Eqs. (3)–(6). In Fig. 3(a) we show contour plots of the differential loss $\Delta\alpha = \alpha^{(+)} - \alpha^{(-)}$ and nonlinear Kerr phase shifts $\phi^{(\pm)}$ for the $\sigma^{(\pm)}$ modes of the signal field as functions of phase-control-field amplitude and detuning. The medium used in calculation is a lifetime-broadened ^{87}Rb cloud with a density of 3.5×10^{12} cm^{-3} and length of 1 mm. The equal-altitude contour lines represent the zero amplitudes for $\Delta\alpha$ and $\phi^{(-)}$ and π amplitude for $\phi^{(+)}$. These equal-altitude lines give a guide for selection of $\delta_{\rm ph}$ for the phase-control field so that Eq. (6) can be satisfied. For instance, with respect to the scheme given in Fig. 1(a), we found that at $\delta_{\rm ph}/2\pi = \delta_{\rm magic}/2\pi \approx 195$ MHz the photonic-polarization-gate operation can be achieved with only about 100 photons in the phase-control field ($E_{\rm ph} \approx 10$ V/m; vertical line in the left panel).

In Fig. 3(b) we show the normalized signal intensities $I_{\rm in}^{(\mathbf{V})} \propto |\hat{E}_{S+} + \hat{E}_{S-}|^2$ (left panel) and $I_{\rm out}^{(\mathbf{H})} \propto |\hat{E}_{S+} - \hat{E}_{S-}|^2$ (right panel) as functions of phase-control-light detuning and the propagation distance. The dashed line indicates the phase-control-light detuning with which the polarization of the signal field rotates from vertical and linear at the entrance of the medium ($z/L = 0$, left) to horizontal and linear at the exit of the medium ($z/L = 1$, right). The corresponding loss of the signal-field intensity is about 60%.

While the scheme shown in Fig. 1(a) has exhibited a promising future as described in Figs. 2 and 3, other excitation combinations of the lowest S and P states of the ^{87}Rb atom do not perform well. Figure 4 shows the results obtained using a D_2(EIT)/D_1(Kerr) with the initial state being $|5S_{1/2}, F = 1, m_F = 0\rangle$ [see Fig. 1(b)]. For this scheme a reasonable

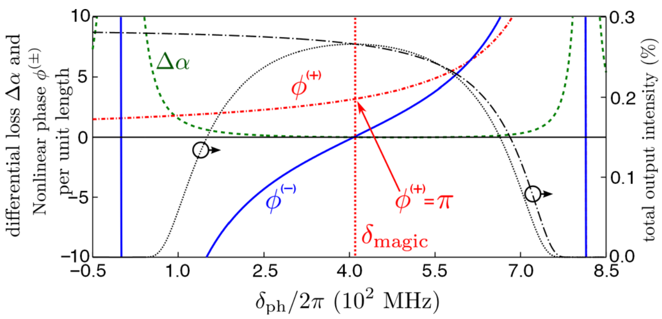

FIG. 4. (Color online) Differential loss $\Delta\alpha$ and Kerr phase shifts $\phi^{(\pm)}$ of the two circular components as functions of the phase-control-field detuning for the scheme depicted in Fig. 1(b). In this scheme, the magic detuning can be found only with more than 3000 phase-control photons. The loss of the signal field is about 75% even with $E_c = 150$ V/m.

solution that satisfies Eq. (6) can be obtained only with more than 3000 phase-control photons ($E_{ph} \approx 58$ V/m). However, the total loss of the signal field is >70% even though the coupling field $E_c = 150$ V/m is twice that used in Figs. 2 and 3. These calculations show the importance of choosing the right energy levels and laser excitation configurations.

We note that without the phase-control-light field the light shifts to states $|a\rangle$ and $|b\rangle$ by the EIT control field are identical, and therefore do not introduce a differential group velocity. Indeed, as far as transition rates are concerned there is no difference between the two channels (they are completely symmetric). The phase-control light can introduce a differential group velocity between the two channels because of the difference in couplings. However, the phase-control light is on the order of 100 photons and its contribution is third order, and thus the group-velocity modification by the phase-control field is negligible. The dominant contribution to the group velocities of the two probe-field components is the EIT control field and the state coupling it acts on, which are symmetric in our case.

The schemes studied here are based on alkali-metal elements that have one or more Kerr-phase-generating states that may have a nonvanishing transition back to the ground state. This leads to unwanted complications such as leakage by four-wave-mixing (FWM) channels. While these channels are very weak, ideally they should be avoided completely in few-photon operations. The ideal system will have, in addition to the above-described properties that allow efficient PSKPS-based gate operation, the property that the successive transitions always lead to $\Delta J = +1$, resulting in a total $\Delta J = 3$ angular momentum change for the three-photon process which blocks any possible FWM leakage back to the ground state. Such systems may be found in noble gases and elements in the second column of the periodic table. In contrast to the ARG system, it is difficult to achieve polarized-gate operation in hot atomic systems with EIT configuration. The reason is attributed to the Doppler broadening of each state, which not only enhances the loss of the signal field but also causes the overlap of the excited states coupled by the phase field. Due to these effects, both circularly polarized components of the signal field acquire the same phase shift and experience a significant loss during their propagation.

In conclusion, we have investigated an EIT-based PSKPS scheme for all-optical atomic-polarization-gate operation. By selectively introducing different Kerr phase shifts to different polarization components of a linearly polarized signal field, we show that it is possible to find a magic wavelength for a phase-control light field so that a complete vertical-to-horizontal linear polarization rotation occurs in atomic media. Our numerical calculations have shown that the schemes and methods studied can indeed lead to a polarization-gate operation at the field level that is comparable to current optical telecommunication devices.

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[23] To put this control-light intensity in perspective, a single phase-control photon with wavelength of 780 nm and 10 ns pulse length in a photonic hollow fiber or ridge waveguide of mode diameter 5 μm yields a peak intensity of $I_0 \simeq 0.1\ \mathrm{mW/cm^2}$. The $2\ \mathrm{mW/cm^2}$ intensity would be equivalent to only 20 photons.

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