Spin-wave propagation in the presence of interfacial Dzyaloshinskii-Moriya interaction

Jung-Hwan Moon,[1] Soo-Man Seo,[1] Kyung-Jin Lee,[1,2,*] Kyoung-Whan Kim,[3,4] Jisu Ryu,[3] Hyun-Woo Lee,[3] R. D. McMichael,[5] and M. D. Stiles[5]

[1]*Department of Materials Science and Engineering, Korea University, Seoul 136-701, Korea*
[2]*KU-KIST Graduate School of Converging Science and Technology, Korea University, Seoul 136-713, Korea*
[3]*PCTP and Department of Physics, Pohang University of Science and Technology, Pohang 790-784, Korea*
[4]*Basic Science Research Institute, Pohang University of Science and Technology, Pohang 790-784, Korea*
[5]*Center for Nanoscale Science and Technology, National Institute of Standards and Technology, Gaithersburg, Maryland 20899, USA*
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In ferromagnetic thin films, broken inversion symmetry and spin-orbit coupling give rise to interfacial Dzyaloshinskii-Moriya interactions. Analytic expressions for spin-wave properties show that the interfacial Dzyaloshinskii-Moriya interaction leads to nonreciprocal spin-wave propagation, i.e., different properties for spin waves propagating in opposite directions. In favorable situations, it can increase the spin-wave attenuation length. Comparing measured spin-wave properties in ferromagnet/normal metal bilayers and other artificial layered structures with these calculations could provide a useful characterization of the interfacial Dzyaloshinskii-Moriya interactions.

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I. INTRODUCTION

Magnetic exchange is the root of magnetism. Intra-atomic exchange stabilizes the magnetic moments and interatomic exchange tends to keep the magnetization spatially uniform. Interatomic exchange is usually symmetric in that the consequences of rotating the magnetization one way or the reverse are equivalent. It loses that symmetry when the system is subject to both spin-orbit coupling and broken inversion symmetry. The antisymmetric component of the exchange interaction, known as the Dzyaloshinskii-Moriya (DM) interaction,[1,2] can give chiral magnetic orders such as spin spirals and skyrmions.[3–11] Understanding chiral magnetic order and its dynamics driven by magnetic fields or currents is currently of significant interest in the field of spintronics.[12–16]

The DM interaction between two atomic spins $\mathbf{S}_i$ and $\mathbf{S}_j$ is

$$\mathcal{H}_{\mathrm{DMI}} = -\mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j), \tag{1}$$

where $\mathbf{D}_{ij}$ is the Dzyaloshinskii-Moriya vector, which is perpendicular to both the asymmetry direction and the vector $\mathbf{r}_{ij}$ between the spins $\mathbf{S}_i$ and $\mathbf{S}_j$. The DM interaction can be classified into two classes depending on the type of inversion symmetry breaking,[17] i.e., bulk and interfacial DM interactions corresponding to lack of inversion symmetry in lattices and at the interface, respectively. The bulk DM interaction has been studied mostly for B20 structures such as MnSi,[5] FeCoSi,[4,8] and FeGe.[9,11] For the bulk DM interaction, $\mathbf{D}_{ij}$ is determined by the detailed symmetry of the lattice structure. On the other hand, the interfacial DM interaction, which is the main focus of this work, occurs at all magnetic interfaces. It can be particularly strong at the interface between a ferromagnet and a normal metal having strong spin-orbit coupling. The DM interaction can be modeled by a 3-site exchange between two atomic spins with a neighboring atom having a spin-orbit coupling.[18] It has been investigated for epitaxial ferromagnet/heavy metal bilayers such as Mn/W,[19,20] Fe/Ir,[10] and Fe/W.[21–23,25]

Recently Chen *et al.* reported that magnetic domain walls in epitaxial Fe/Ni/Cu(001) structures are Néel walls and the domain wall chirality is opposite to that of Ni/Fe/Cu(001) structures.[26] Such behavior is expected for an interfacial DM interaction. The interfacial DM interaction in these structures may be not as large as that of structures having a heavy metal, but is still large enough to affect magnetic textures, which can in turn modify magnetization dynamics substantially.[15] Furthermore, recent experiments on current-driven domain wall motion suggest that the interfacial DM interaction exists in sputtered Pt/CoFe/MgO (Ref. 27) and Pt/Co/Ni (Ref. 28) structures, and plays an important role in domain wall motion. Since sputtered thin films consist of small grains with different lattice orientation, the contributions from the bulk DM interactions tend to cancel and only the interfacial DM interaction contributions remain effective. In this respect, understanding the interfacial DM interaction in sputtered thin films is important not only for the fundamental understanding of topologically protected nanomagnetic structures[29] but also to the development of spintronic devices based on domain walls.[30–32]

Translating the DM interaction in Eq. (1) to a continuum model with magnetization direction $\hat{\mathbf{m}}$ and symmetry breaking in the $\hat{\mathbf{y}}$ direction, the DM energy density is given by

$$E_{\mathrm{DM}} = -D\left[(\hat{\mathbf{x}} \times \hat{\mathbf{y}}) \cdot \left(\hat{\mathbf{m}} \times \frac{\partial \hat{\mathbf{m}}}{\partial x}\right) + (\hat{\mathbf{z}} \times \hat{\mathbf{y}}) \cdot \left(\hat{\mathbf{m}} \times \frac{\partial \hat{\mathbf{m}}}{\partial z}\right)\right]. \tag{2}$$

The sign of D is determined by the details of the system. In this paper, we consider the case where the equilibrium magnetization lies along the $\hat{\mathbf{z}}$ axis, in the plane of the film. We also restrict our attention to the case of spin waves propagating in the x direction so that $\mathbf{m}$ varies only in the x direction, and the second term in Eq. (2) is zero (see Fig. 1).

A net DM interaction is present in any trilayer structure when the first nonmagnetic layer supplies a spin-orbit coupling, the middle layer is a ferromagnet, and the third layer is nonmagnetic, but different from the first layer to break symmetry. Since the observation of efficient domain wall motion in bilayers and trilayers is correlated with the

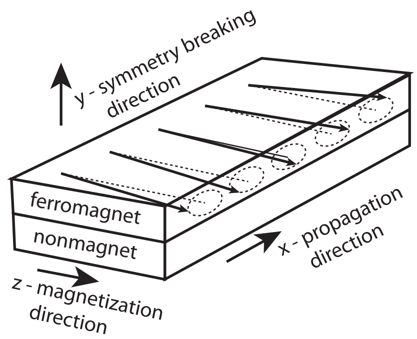

FIG. 1. Geometry under consideration in this paper.

conditions for a strong DM interaction,[27,28] it is useful to study various artificial structures to find large interfacial DM interactions. The spin-wave properties we present below provide a useful probe of the DM interactions in these systems.

In this work, we compute analytical expressions for asymmetric spin-wave propagation induced by the interfacial DM interaction. There have been several related studies on specific systems. Udvardi and Szunyogh predicted that the DM interaction gives rise to asymmetric spin-wave dispersion depending on the sign of the wave vector, based on first-principles calculations for Fe/W(110).[22] Costa *et al.* predicted that the spin-wave frequency, amplitude, and lifetime differ depending on the sign of the wave vector, based on a multiband Hubbard model for Fe/W(110).[24] Zakeri *et al.* reported a series of spin-wave experiments based on the spin-polarized electron-loss spectroscopy for single-crystalline Fe/W(110),[23,25] consistent with these predictions. Cortés-Ortuño and Landeros developed a spin-wave theory for bulk DM interaction,[33] where they demonstrated that the spin-wave dispersion is asymmetric with respect to wave vector inversion.

We focus on the influence of the interfacial DM interaction on spin-wave properties and pay attention to asymmetric spin-wave attenuation and excitation amplitude with respect to wave vector inversion. We provide analytic expressions for asymmetric dispersion, attenuation length, and amplitude of interfacial DM spin waves. In Sec. II, we present spin-wave theory in the presence of the interfacial DM interaction. Section III gives comparisons between analytic expressions and micromagnetic simulations. We summarize our work in Sec. IV.

II. SPIN-WAVE THEORY

A. Quantum spin-wave theory

We begin with a quantum spin-wave theory to find the contribution of the interfacial DM interaction to the dispersion. Quantum spin-wave theory for the symmetric exchange interaction is well established[34] and shows that the exchange interaction results in k^2 dependence of the dispersion for small wave vector k. Here we focus on the interfacial DM interaction in a one-dimensional spin system. The spin chain extends in the x direction and the symmetry breaking is in the y direction, so that the DM vector is in the z direction. The interfacial DM interaction Hamiltonian is given as

$$\begin{aligned}\mathcal{H}_{\mathrm{DM}} &= -2\frac{D_0}{\hbar^2}\sum_j \hat{\mathbf{z}}\cdot(\mathbf{S}_j\times\mathbf{S}_{j+1}) \\ &= \frac{D_0}{i\hbar^2}\sum_j (S_j^+S_{j+1}^- - S_j^-S_{j+1}^+),\end{aligned} \tag{3}$$

where D_0 is the DM energy, and $S_j^+(=S_{jx}+iS_{jy})$ and $S_j^-(=S_{jx}-iS_{jy})$ are the spin raising and lowering operators. We treat the case where the equilibrium magnetization direction is along $\mathbf{D}_{ij}$ because this configuration exhibits the strongest spin-wave asymmetry.[22] For simplicity, we are restricting the calculation to nearest-neighbor exchange. Based on the Holstein-Primakoff transformation[35] and assuming that the total number of flipped spins in the system is small compared to the total number of spins, S_j^+ (S_j^-) can be approximated as $\hbar\sqrt{2s}a_j$ ($\hbar\sqrt{2s}a_j^+$), where s is the total spin on the site, and a_j (a_j^+) is the magnon annihilation (creation) operator. Substituting these approximations into Eq. (3) gives

$$\mathcal{H}_{\mathrm{DM}} = \frac{2sD_0}{i}\sum_j (a_j a_{j+1}^+ - a_j^+ a_{j+1}). \tag{4}$$

Introducing the operators a_k^+ and a_k, which are the Fourier transforms of the a_j's, and summing over j, Eq. (4) becomes

$$\mathcal{H}_{\mathrm{DM}} = \frac{2sD_0}{i}\sum_k (e^{-ika}a_k a_k^+ - e^{ika}a_k^+ a_k), \tag{5}$$

where a is the lattice constant. The contribution to the magnon energy in Eq. (5) is

$$\mathcal{H}_{\mathrm{DM}}^{\mathrm{magnon}} = -4sD_0\sum_k \sin(ka)a_k^+a_k = \sum_k \hbar\omega_k^{\mathrm{DM}}\hat{n}_k, \tag{6}$$

where $\hat{n}_k = a_k^+a_k$ is the number operator for magnons with wave vector k, and the DM interaction contribution to the dispersion is given by

$$\hbar\omega_k^{\mathrm{DM}} = -4sD_0\sin(ka). \tag{7}$$

For small k, Eq. (7) reduces to

$$\hbar\omega_k^{\mathrm{DM}} = -4sD_0ka, \tag{8}$$

a contribution to the dispersion that is linear in k. This antisymmetric contribution to the energy leads to asymmetric spin-wave propagation, i.e., dependent on the direction of k.

B. Classical spin-wave theory

A similar contribution arises in a classical theory of spin waves in thin films with an interfacial DM interaction. We consider small-amplitude spin waves propagating along the x axis in the perturbative limit, where the equilibrium magnetization is in the z direction perpendicular to both the film thickness direction and the spin-wave propagation direction,

$$\hat{\mathbf{m}} = p\hat{\mathbf{z}} + \mathbf{m}_0\exp[i(kx-\omega t)]\exp[-x/\Lambda], \tag{9}$$

where $\mathbf{m}_0 = (m_x, m_y, 0)$, $|\mathbf{m}_0| \ll 1$, $p = \pm 1$, and Λ is the spin-wave attenuation length. The spin-wave dynamics is described by the Landau-Lifshitz-Gilbert (LLG) equation,

$$\frac{\partial \hat{\mathbf{m}}}{\partial t} = -\gamma \hat{\mathbf{m}} \times \mu_0 \mathbf{H}_{\mathrm{eff}} + \alpha \hat{\mathbf{m}} \times \frac{\partial \hat{\mathbf{m}}}{\partial t}, \qquad (10)$$

where γ is the gyromagnetic ratio and α is the damping constant. The effective field $\mathbf{H}_{\mathrm{eff}}$ is given as

$$\mathbf{H}_{\mathrm{eff}} = pH\hat{\mathbf{z}} + J\nabla^2\hat{\mathbf{m}} - D^*\left(\hat{\mathbf{z}} \times \frac{\partial \hat{\mathbf{m}}}{\partial x}\right) + \mathbf{H}_{\mathrm{dipole}}, \qquad (11)$$

where H is the external field, J is $2A/\mu_0 M_{\mathrm{s}}$, D^* is $2D/\mu_0 M_{\mathrm{s}}$, A is the exchange stiffness constant, M_{s} is the saturation magnetization, $\mathbf{H}_{\mathrm{dipole}}$ $[=-\frac{M_{\mathrm{s}}}{4}(1-e^{-2|k|d})m_x\hat{\mathbf{x}} - M_{\mathrm{s}}(1-(1-e^{-2|k|d})/4)m_y\hat{\mathbf{y}}]$[36,37] is the dipolar field, the local demagnetization field along the thickness direction is equal to M_{s}, and d is the film thickness. We note that $\mathbf{H}_{\mathrm{dipole}}$ consists of local contribution (independent of d and k) and nonlocal contribution (dependent on d and k). Inserting Eqs. (9) and (11) into Eq. (10), and neglecting small terms proportional to $1/(k\Lambda)^2$, α^2, and $\alpha/(k\Lambda)$, gives

$$\frac{\omega}{\gamma\mu_0} = \sqrt{(H + M_{\mathrm{s}}/4 + Jk^2)(H + 3M_{\mathrm{s}}/4 + Jk^2) - \frac{e^{-4|k|d}M_{\mathrm{s}}^2}{16}(1 + 2e^{2|k|d})} + pD^*k \qquad (12)$$

and

$$\Lambda_\pm = \frac{1}{\alpha\omega}\left(2\gamma\mu_0 J|k_\pm| + \frac{\gamma\mu_0 M_{\mathrm{s}}^2 d e^{-4|k_\pm|d}(1 + e^{2|k_\pm|d})/8 \pm pD^*(\omega \mp \gamma\mu_0 pD^*|k_\pm|)}{H + M_{\mathrm{s}}/2 + Jk_\pm^2}\right), \qquad (13)$$

where the upper (lower) sign corresponds to the case $k > 0$ ($k < 0$). The dispersion [Eq. (12)] is the sum of the terms in the square root, which is the dispersion in the absence of the DM interaction, and a term linear in k. Therefore, the interfacial DM interaction generates a term linear in k in the dispersion [Eq. (12)] as in the quantum spin-wave theory [Eq. (8)]. As a result, the wave vectors are different for propagation in different directions at a fixed frequency ω. The spin-wave attenuation length also depends on the sign of k when $D \neq 0$ [Eq. (13)].

In the large-k limit (i.e., exchange-DM spin waves), one may neglect the nonlocal magnetostatic contribution so that Eqs. (12) and (13) reduce to

$$\frac{\omega}{\gamma\mu_0} = \sqrt{(H + Jk^2)(H + M_{\mathrm{s}} + Jk^2)} + pD^*k \qquad (14)$$

and

$$\Lambda_\pm = \frac{1}{\alpha\omega}\left(2\gamma\mu_0 J|k_\pm| \pm \frac{pD^*(\omega \mp \gamma\mu_0 pD^*|k_\pm|)}{H + M_{\mathrm{s}}/2 + Jk_\pm^2}\right). \qquad (15)$$

On the other hand, in the small-k limit (i.e., magnetostatic-DM spin waves) that is more relevant to experimental conditions, one may neglect the exchange contribution and assume $|k_\pm|d \ll 1$ so that Eqs. (12) and (13) reduce to

$$\frac{\omega}{\gamma\mu_0} = \sqrt{H(H + M_{\mathrm{s}})} + \frac{M_{\mathrm{s}}^2|k|d}{4\sqrt{H(H + M_{\mathrm{s}})}} + pD^*k \qquad (16)$$

and

$$\Lambda_\pm = \frac{1}{\alpha\omega}\left(\frac{\gamma\mu_0 M_{\mathrm{s}}^2 d/4 \pm pD^*(\omega \mp \gamma\mu_0 pD^*|k_\pm|)}{H + M_{\mathrm{s}}/2}\right). \qquad (17)$$

In this small-k limit, one finds from Eq. (16) that not only the interfacial DM interaction but also the dipolar coupling generates a term linear in k. However, there is an important difference. The interfacial DM interaction contribution changes its sign with respect to the inversion of the wave vector k or the magnetization direction p, whereas the dipolar contribution does not. Due to this feature, one can distinguish the interfacial DM interaction contribution from the dipolar contribution unambiguously.

Not only are the wave vectors, Eq. (12), and the decay lengths, Eq. (13), asymmetric; the amplitudes of the spin wave are different when symmetrically excited. We approximate the ratio of spin-wave amplitudes κ $(=m_y^+/m_y^-)$ by neglecting contributions from nonlocal dipolar coupling, where the plus (minus) sign corresponds to $k > 0$ ($k < 0$). From the susceptibility, one finds

$$m_y = \sqrt{\frac{H_x}{H_y}} \int_0^\infty \frac{dk}{2\pi} \frac{h_k H_y}{2\omega_0 \delta\omega + i\Gamma} \propto \frac{1}{v_g}, \qquad (18)$$

where $H_x = H + Jk^2$, $H_y = H + M_{\mathrm{s}} + Jk^2$, h_k is the Fourier component of the driving field, $\omega_0 = \gamma\mu_0\sqrt{H_x H_y}$, $\delta\omega$ describes the frequency difference from the resonance frequency, Γ describes the damping term, and v_g is the group velocity. Thus, the spin-wave amplitude ratio κ is

$$\kappa = \frac{-pD^* + Jk_- \frac{2H + M_{\mathrm{s}} + 2Jk_-^2}{\sqrt{(H + Jk_-^2)(H + M_{\mathrm{s}} + Jk_-^2)}}}{+pD^* + Jk_+ \frac{2H + M_{\mathrm{s}} + 2Jk_+^2}{\sqrt{(H + Jk_+^2)(H + M_{\mathrm{s}} + Jk_+^2)}}}. \qquad (19)$$

This equation shows that the interfacial DM interaction makes the spin-wave amplitude asymmetric depending on the sign of k or p.

These results only hold when the DM interaction is not strong enough to change the ground state of the magnetic configuration. Setting Eq. (12) to be zero and neglecting nonlocal dipolar coupling, the threshold D^*_{th} is

$$D^*_{\mathrm{th}} = \sqrt{[2H + M_{\mathrm{s}} + 2\sqrt{H(H + M_{\mathrm{s}})}]J}. \qquad (20)$$

When $D^* > D^*_{\mathrm{th}}$, the ground state is not a single domain but rather a chiral magnetic texture and our results may not apply. However, to study a particular interface, one can reduce the thickness-averaged effective D^* below D^*_{th} by simply increasing the thickness of the ferromagnet because the DM interaction in sputtered thin films is an *interface* effect. By doing so, one can study the DM interaction associated with a particular interface in an appropriate layered structure.

III. NUMERICAL RESULTS AND DISCUSSION

In this section, we test the extent to which the analytic formulas derived in the previous section can describe the behavior of thin films. We consider both the short- and long-wavelength limits. We apply a magnetic field along the width (z direction) that is strong enough to make the magnetization uniform and minimize the importance of edge effects—the best case for a successful application of the analytic formulas. The wavelength is determined by the excitation frequency f and the external field $\mu_0 H$. In simulations for the small-k limit, we keep all micromagnetic interactions. On the other hand, in simulations for the large-k limit, we neglect the nonlocal parts of the dipolar coupling but keep the local demagnetization field that acts as an easy-plane anisotropy. This approximation becomes exact in the thin-film geometry and the large-k limit. We use the damping constant $\alpha = 0.01$, the saturation magnetization $M_s = 800$ kA/m, and the exchange constant $A = 1.3 \times 10^{-11}$ J/m. The length of the thin film is 8 μm in the x direction, the width is 20 μm in the z direction, and the thickness is 1 nm in the y direction. In the large-k limit, we discretize along the x direction, the direction in which the magnetization varies, with 2 nm unit cells, but treat the magnetization as uniform in the thickness direction because the film is sufficiently thin, and uniform along the width because the magnetic field is sufficiently strong. In the small-k limit we use a unit cell size of 5 nm along the length. We use the external uniform field $\mu_0 H = 0.1$ T (0.01 T) for the large-k (small-k) limit. To excite spin waves, we apply an ac field (0.1 mT)$\times \cos(2\pi f t)$ to two unit cells at $x = 0$. Therefore, the wave vector k of spin waves for $x > 0$ is positive whereas k for $x < 0$ is negative. For a legitimate comparison between theoretical and numerical results, we include absorbing boundary conditions[38,39] at the system edges to suppress spin-wave reflection.

Figure 2 shows snapshot images of the spin-wave distribution along the x axis. Spin waves are symmetric for $D = 0$ [Fig. 2(a)], whereas the wavelength, amplitude, and attenuation length are all asymmetric depending on the propagation direction for $D \neq 0$ [Fig. 2(b)].

Figure 3 summarizes numerical results obtained in the large-k limit. Numerical results of both spin-wave dispersion and attenuation length are in agreement with analytic expressions [Eqs. (14) and (15)]. In this large-k limit, an interesting observation is that Λ for $D \neq 0$ is longer than Λ for $D = 0$ regardless of the propagation direction. After some algebra with Eq. (15), one finds $\Lambda_{\pm} = v_g^{\pm} F(k_{\pm})/\alpha\omega$, where $v_g^{\pm}$ is the group velocity and $F(k) = \sqrt{(H + M_s + Jk^2)(H + Jk^2)}/(H + M_s/2 + Jk^2)$. Since $F(k)$ is a slowly varying function with the wave vector, the attenuation length is mostly determined by the group velocity. The interfacial DM interaction lowers the frequency gap (i.e., the lowest allowed ω) so that it increases the group velocity at a given ω in the large-k limit, which in turn increases the attenuation length compared to that with $D = 0$. This enhanced spin-wave attenuation length induced by the interfacial DM interaction may be useful for applications based on spin waves. We note however that the damping constant α may also increase with D, because the interfacial DM interaction is usually caused by nonnegligible spin-orbit coupling in the normal-metal layer. In this case, the damping may increase due to spin pumping effects[40] or interfacial Rashba spin-orbit coupling-related spin-motive force.[41,42]

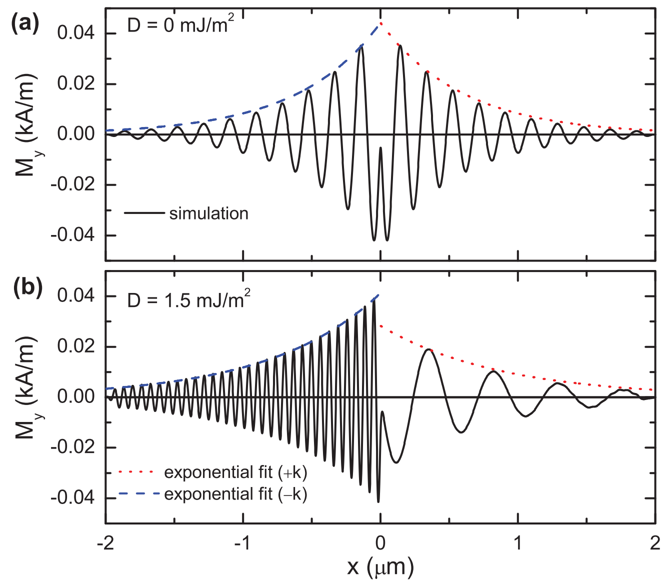

FIG. 2. (Color online) Snapshot images of spin-wave distribution along the x axis for (a) $D = 0$ mJ/m^2 and (b) $D = 1.5$ mJ/m^2. The spin-wave frequency f is 11 GHz.

Figure 3(c) shows numerical results of the amplitude ratio κ as a function of the frequency f, in agreement with the analytic expression [Eq. (19)]. We note that an asymmetry of spin-wave amplitude has been observed when the spin waves are excited

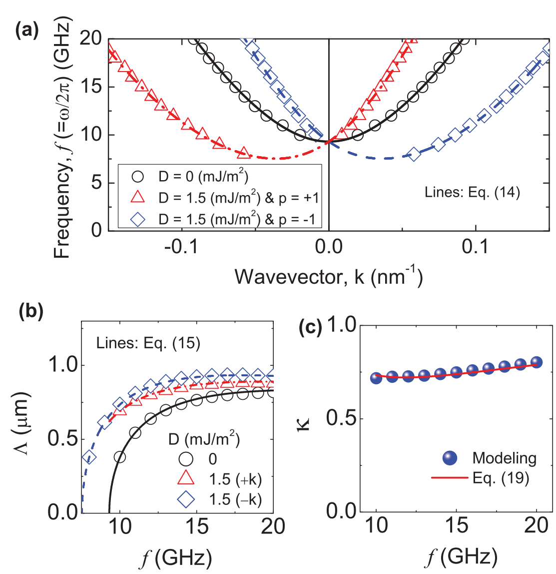

FIG. 3. (Color online) Asymmetric spin-wave propagation induced by an interfacial DM interaction in the large-k limit. (a) Dispersion relation. (b) Attenuation length Λ as a function of the frequency f. (c) Amplitude ratio κ as a function of f. Symbols and lines correspond to numerical and analytic results, respectively.

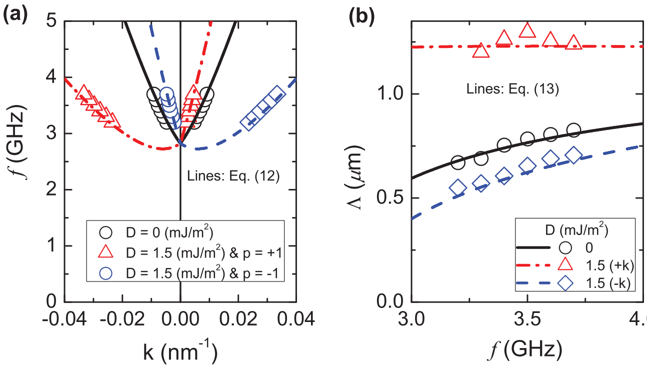

FIG. 4. (Color online) Asymmetric spin-wave propagation induced by an interfacial DM interaction in the small-k limit. (a) Dispersion relation. (b) Attenuation length Λ as a function of the frequency f. Symbols and lines correspond to numerical and analytic results, respectively.

by a magnetic field generated by microwave antennas, and has been called nonreciprocity of spin-waves.[43–47] In these earlier works, the amplitude asymmetry results from a nonreciprocal antenna–spin wave coupling, caused by the spatially nonuniform distribution of the antenna field. However in our results, we use reciprocal coupling in deriving the analytic expressions and performing the numerical simulations, so that the amplitude asymmetry shown in Fig. 3(c) is purely due to the interfacial DM interaction. This interfacial DM interaction induced amplitude asymmetry may find use in spin-wave logic devices as proposed by Zakeri *et al.*[25] In addition, it suggests a reexamination of the interpretation of experiments reporting nonreciprocal antenna–spin wave coupling. These experiments have been done on relatively thin structures, in which interfacial DM interaction may also be an important source of asymmetry.

Figure 4 summarizes numerical results obtained in the small-k limit. Numerical results for both the spin-wave dispersion and attenuation length agree with analytic expressions [Eqs. (16) and (17)]. In Fig. 4(b), one finds a difference in the attenuation length Λ from Fig. 3(b). In the large-k limit, Λ with $D \neq 0$ is larger than Λ with $D = 0$ regardless of the sign of k. In contrast, in the small-k limit with $pD > 0$, Λ with $D \neq 0$ is larger than Λ with $D = 0$ for $k > 0$, whereas it is smaller for $k < 0$. This result is again related to the group velocity. From Eq. (16), one finds $v_g^{\pm} = v_g^0 \pm \gamma\mu_0 p D^*$ where $v_g^0 = \gamma\mu_0 M_s^2 d/4\sqrt{H(H+M_s)}$ is the group velocity with $D = 0$ in the small-k limit. Therefore, for a sign of k, Λ with $D \neq 0$ is larger than Λ with $D = 0$ whereas for the other sign of k, it is smaller.

Since the analytic expression of the dispersion [Eq. (12)] is valid regardless of k, the strength of interfacial DM interaction D can be estimated experimentally by measuring the frequency shift Δf $(=|f_{+k,\pm p} - f_{-k,\pm p}| = |f_{\pm k,+p} - f_{\pm k,-p}|)$, given as

$$\Delta f = \gamma\mu_0 D^* |k|/\pi. \tag{21}$$

With the parameters $M_s = 800$ kA/m, $\gamma = 1.76\times10^{11}$ T^{-1} s^{-1}, and $2\pi/k = 1$ μm, Δf is about 880 MHz for $D = 1$ mJ/m^2, which is smaller than the threshold value [$\approx$3.1 mJ/m^2 for the parameters used in simulations; see Eq. (20)]. We note that propagating spin-wave spectroscopy[48–51] can resolve Δf smaller than 20 MHz.

Another interesting consequence of the interfacial DM interaction is that the dispersion is asymmetric depending not only on the wave vector direction but also on the magnetization direction (i.e., the sign of p). This k- and magnetization-direction-dependent asymmetry in the dispersion is similar to the electron dispersion in a ferromagnet subject to Rashba spin-orbit coupling.[52] We note that this is not an accident because the interfacial DM interaction is directly connected with the Rashba spin-orbit coupling at magnetic interfaces.[53]

IV. SUMMARY

We theoretically study asymmetric spin-wave propagation induced by interfacial DM interactions. We derive analytic expressions of dispersion, attenuation length, and amplitude of interfacial DM spin waves and compare them with numerical results. The frequency shifts induced by the interfacial DM interaction range from MHz to GHz, which should be large enough to be resolved by state-of-the-art experimental tools such as propagating spin-wave spectroscopy. Assuming that the damping does not change with the interfacial DM interaction, the spin-wave attenuation length can increase with increasing interfacial DM interaction. The spin-wave amplitude is asymmetric due to the interfacial DM interaction, even without nonreciprocal coupling between antenna fields and spin waves. This asymmetric spin-wave propagation may be useful to investigate interfacial magnetic properties.

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*kj_lee@korea.ac.kr

[1] I. E. Dzialoshinskii, Sov. Phys. JETP **5**, 1259 (1957).
[2] T. Moriya, Phys. Rev. **120**, 91 (1960).
[3] U. K. Rößler, A. N. Bogdanov, and C. Pfleiderer, Nature (London) **442**, 797 (2006).
[4] M. Uchida, Y. Onose, Y. Matsui, and Y. Tokura, Science **311**, 359 (2006).
[5] S. Mülbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Böni, Science **323**, 915 (2009).
[6] S. D. Yi, S. Onoda, N. Nagaosa, and J. H. Han, Phys. Rev. B **80**, 054416 (2009).
[7] A. B. Butenko, A. A. Leonov, U. K. Rößler, and A. N. Bogdanov, Phys. Rev. B **82**, 052403 (2010).
[8] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Nature (London) **465**, 901 (2010).
[9] X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, Nat. Mater. **10**, 106 (2011).

[10] S. Heinze, K. von Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, and S. Blügel, Nat. Phys. **7**, 713 (2011).
[11] S. X. Huang and C. L. Chien, Phys. Rev. Lett. **108**, 267201 (2012).
[12] F. Jonietz, S. Mülbauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Böni, R. A. Dunie, K. Everschor, M. Garst, and A. Rosch, Science **330**, 1648 (2010).
[13] Y. Tchoe and J. H. Han, Phys. Rev. B **85**, 174416 (2012).
[14] X. Z. Yu, N. Kanazawa, W. Z. Zhang, T. Nagai, T. Hara, K. Kimoto, Y. Matsui, Y. Onose, and Y. Tokura, Nat. Commun. **3**, 988 (2012).
[15] A. Thiaville, S. Rohart, É. Jué, V. Cros, and A. Fert, Europhys. Lett. **100**, 57002 (2012).
[16] J. Iwassaki, M. Mochizuki, and N. Nagaosa, Nat. Commun. **4**, 1463 (2013).
[17] A. Fert, V. Cros, and J. Sampaio, Nat. Nanotechnol. **8**, 152 (2013).
[18] A. Fert and P. M. Levy, Phys. Rev. Lett. **44**, 1538 (1980).
[19] M. Bode, M. Heide, K. von Bergmann, P. Ferriani, S. Heinze, G. Bihlmayer, A. Kubetzka, O. Pietzsch, S. Blügel, and R. Wiesendanger, Nature (London) **447**, 190 (2007).
[20] P. Ferriani, K. von Bergmann, E. Y. Vedmedenko, S. Heinze, M. Bode, M. Heide, G. Bihlmayer, S. Blügel, and R. Wiesendanger, Phys. Rev. Lett. **101**, 027201 (2008).
[21] M. Heide, G. Bihlmayer, and S. Blügel, Phys. Rev. B **78**, 140403(R) (2008).
[22] L. Udvardi and L. Szunyogh, Phys. Rev. Lett. **102**, 207204 (2009).
[23] Kh. Zakeri, Y. Zhang, J. Prokop, T. H. Chuang, N. Sakr, W. X. Tang, and J. Kirschner, Phys. Rev. Lett. **104**, 137203 (2010).
[24] A. T. Costa, R. B. Muniz, S. Lounis, A. B. Klautau, and D. L. Mills, Phys. Rev. B **82**, 014428 (2010).
[25] Kh. Zakeri, Y. Zhang, T.-H. Chuang, and J. Kirschner, Phys. Rev. Lett. **108**, 197205 (2012).
[26] G. Chen, J. Zhu, A. Quesada, J. Li, A. T. N'Diaye, Y. Huo, T. P. Ma, Y. Chen, H. Y. Kwon, C. Won, Z. Q. Qiu, A. K. Schmid, and Y. Z. Wu, Phys. Rev. Lett. **110**, 177204 (2013).
[27] S. Emori, U. Bauer, S.-M. Ahn, E. Martinez, and G. S. D. Beach, Nat. Mater. **12**, 611 (2013).
[28] K.-S. Ryu, L. Thomas, S.-H. Yang, and S. S. P. Parkin, Nat. Nanotechnol. **8**, 527 (2013).
[29] R. Skomski, Z. Li, R. Zhang, R. D. Kirby, A. Enders, D. Schmidt, T. Hofmann, E. Schubert, and D. J. Sellmyer, J. Appl. Phys. **111**, 07E116 (2012).
[30] A. Yamaguchi, T. Ono, S. Nasu, K. Miyake, K. Mibu, and T. Shinjo, Phys. Rev. Lett. **92**, 077205 (2004).
[31] M. Yamanouchi, D. Chiba, F. Matsukura, and H. Ohno, Nature (London) **428**, 539 (2004).
[32] M. Kläui, C. A. F. Vaz, J. A. C. Bland, W. Wernsdorfer, G. Faini, E. Cambril, L. J. Heyderman, F. Nolting, and U. Rudiger, Phys. Rev. Lett. **94**, 106601 (2005).
[33] D. Cortés-Ortuño and P. Landeros, J. Phys.: Condens. Matter **25**, 156001 (2013).
[34] D. D. Stancil and A. Prabhakar, *Spin Waves—Theory and Applications* (Springer, New York, 2009).
[35] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1940).
[36] R. W. Damon and J. R. Eshbach, J. Phys. Chem. Solids **19**, 308 (1961).
[37] R. Arias and D. L. Mills, Phys. Rev. B **60**, 7395 (1999).
[38] D. V. Berkov and N. L. Gorn, J. Appl. Phys. **99**, 08Q701 (2006).
[39] S.-M. Seo, K.-J. Lee, H. Yang, and T. Ono, Phys. Rev. Lett. **102**, 147202 (2009).
[40] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Phys. Rev. Lett. **88**, 117601 (2002).
[41] K.-W. Kim, J.-H. Moon, K.-J. Lee, and H.-W. Lee, Phys. Rev. Lett. **108**, 217202 (2012).
[42] G. Tatara, N. Nakabayashi, and K.-J. Lee, Phys. Rev. B **87**, 054403 (2013).
[43] M. Bailleul, D. Olligs, and C. Fermon, Appl. Phys. Lett. **83**, 927 (2003).
[44] P. K. Amiri, B. Rejaei, M. Vroubel, and Y. Zhuang, Appl. Phys. Lett. **91**, 062502 (2007).
[45] T. Schneider, A. A. Serga, T. Neumann, B. Hillebrands, and M. P. Kostylev, Phys. Rev. B **77**, 214411 (2008).
[46] V. E. Demidov, M. P. Kostylev, K. Rott, P. Krzystechzko, G. Reiss, and S. O. Demokritov, Appl. Phys. Lett. **95**, 112509 (2009).
[47] K. Sekiguchi, K. Yamada, S. M. Seo, K. J. Lee, D. Chiba, K. Kobayashi, and T. Ono, Appl. Phys. Lett. **97**, 022508 (2010).
[48] V. Vlaminck and M. Bailleul, Science **322**, 410 (2008).
[49] M. Zhu, C. L. Dennis, and R. D. McMichael, Phys. Rev. B **81**, 140407(R) (2010).
[50] M. Zhu, B. D. Soe, R. D. McMichael, M. J. Carey, S. Maat, and J. R. Childress, Appl. Phys. Lett. **98**, 072510 (2011).
[51] K. Sekiguchi, K. Yamada, S.-M. Seo, K.-J. Lee, D. Chiba, K. Kobayashi, and T. Ono, Phys. Rev. Lett. **108**, 017203 (2012).
[52] J.-H. Park, C. H. Kim, H.-W. Lee, and J. H. Han, Phys. Rev. B **87**, 041301(R) (2013).
[53] K.-W. Kim, H.-W. Lee, K.-J. Lee, and M. D. Stiles, arXiv:1308.1198.