FUNCTIONAL CONSTRAINTS AND THE DESIGN OF A NEW WATT BALANCE

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ABSTRACT

The functional constraints driving the design of the new permanent-magnet driven watt balance (NIST-4) outline the variables and compromises worthy of addressing. Construction according to these design parameters will demonstrate the high precision capabilities of a large complex mass measurement system with overall uncertainties on the order of 3 parts in 10^8.

INTRODUCTION

Redefinition of the SI system is a formidable global undertaking. Of the seven SI units, the unit of mass is the only one still defined via an artifact, a platinum-iridium alloy cylinder casted in 1879 known as the *international prototype kilogram (IPK)* housed in the BIPM in Paris. The two major shortcomings of any artifact standard are replication and stability.

The watt balance, conceived in 1975 by Bryan Kibble, is a weight measuring apparatus that virtually compares electrical power to mechanical power, hence the name watt balance. Recent advancements in quantum physics have helped evolve the watt balance into an instrument capable of realizing the unit of mass.

HOW IT WORKS

The NIST-4 watt balance contains only a few key components. A copper wire coil that is suspended from one side of a diamond turned aluminum wheel balanced on a knife edge along the axle of the wheel (Figure 1). This hanging coil is immersed in a 0.55 Tesla magnetic field (some 10 000 times greater than Earth's magnetic field) generated by a one ton SmCo permanent magnet system. A measurement mass and auxiliary mass hang concentric to the coil on the main mass stirrup (Figure 6). The aluminum ELF (Extremely Large Flexure) is used to precisely locate the moving coil in the X and Y directions.

Theoretically, the instrument operates by linking the unit of mass to Planck's constant, a fundamental constant of nature as demanded by quantum physics. Functionally, the instrument achieves this linkage to quantum physics via indirectly comparing electrical power to mechanical power by toggling between two modes during measurement (Velocity Mode and Force Mode).

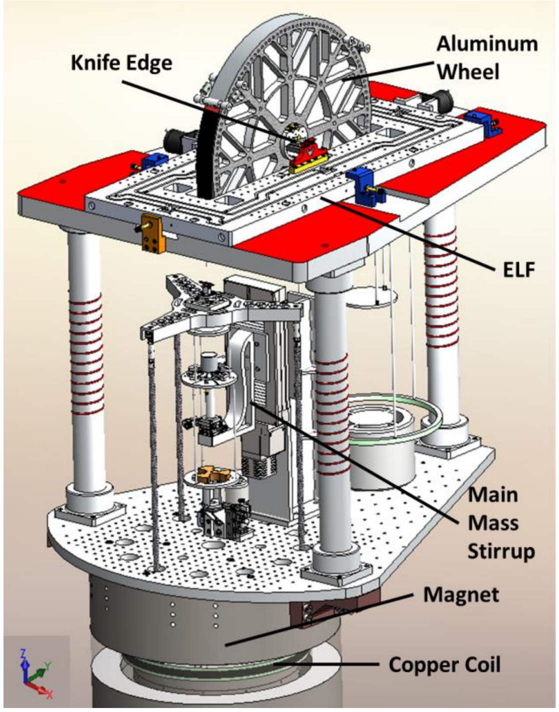
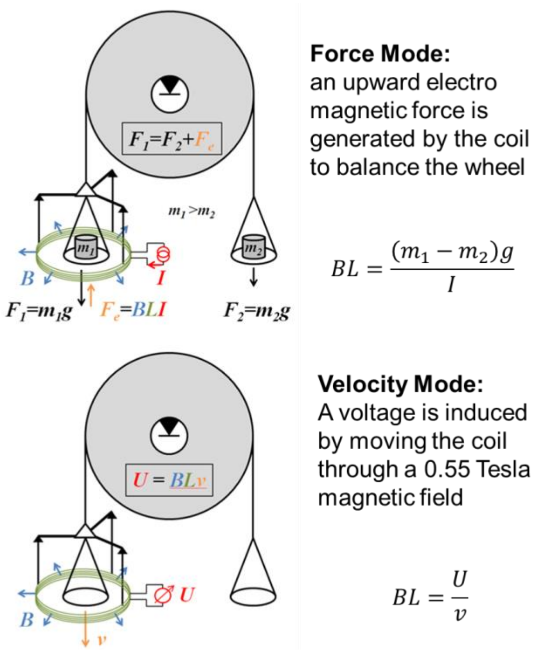


FIGURE 1. Current design phase of the NIST-4 watt balance. The coil and main mass hang concentrically and tangent to the wheel while vertically aligned to gravity.

In Velocity Mode, the coil (wire length L) is moved at a vertical speed, v, through the magnetic field (flux density B) so that a voltage, U, is induced. The voltage is then measured precisely in terms of Planck's constant by comparison to a quantum voltage standard [1].

In Force Mode, the gravitational force of the countermass, m_2g, offsets the weight of the coil and main mass, m_1g. To achieve a balanced state, an upward (or downward if the main mass is unloaded) electromagnetic force, generated by sending an electric current, I, through the coil, levitates the main mass stirrup. Measuring the main mass on the same side as the coil ensures equality in the torque generated by a common lever arm, as long as their centers of mass are vertically aligned to gravity. The current, I, is measured in terms of Planck's constant by monitoring the voltage drop across a known resistor, this time making use of both quantum voltage and resistance standards.



Force Mode: an upward electro magnetic force is generated by the coil to balance the wheel

$$BL = \frac{(m_1 - m_2)g}{I}$$

Velocity Mode: A voltage is induced by moving the coil through a 0.55 Tesla magnetic field

$$BL = \frac{U}{v}$$

FIGURE 2: Force and Velocity Modes

Cancelling out the BL factor common to both equations and rearranging the variables, power is revealed and mass is solved for:

$$BL = \frac{U}{v} = \frac{mg}{I} \quad (scalar\ version)$$

$$Therefore\colon\ UI = mgv = m\boldsymbol{g}\cdot\boldsymbol{v}$$
$$\therefore\ m = UI/\boldsymbol{g}\cdot\boldsymbol{v}$$

Note that $\boldsymbol{g}$ and $\boldsymbol{v}$ must be parallel vectors (vertical), hinting at the significance of physical alignment during measurement.

Now, $P_{elec} = UI\ and\ P_{mech} = m\boldsymbol{g}\cdot\boldsymbol{v}$, implying the balance of mechanical power and electrical power. However, it is important to recognize that both types of power are "virtual" because they are not directly measured in either mode.

The advantage of a wheel balance over a traditional beam balance becomes apparent when attempting to design for this "virtual" functionality. A watt balance must independently meet two criteria: (1) balance opposing lever arms about the knife edge and maintain equilibrium in the nominal position, and (2) translate the coil in a purely vertical fashion aligned to gravity. Although both balance designs meet criteria (1), the wheel allows pure vertical motion of the coil as it rotates whereas the beam incorporates a parasitic horizontal motion as it pivots.

THE MOVING COIL

The moving coil has six degrees of freedom of motion given by three components (v_x, v_y, v_z) of its velocity $\boldsymbol{v}$ and three components $(\omega_x, \omega_y, \omega_z)$ of its angular velocity $\boldsymbol{\Omega}$ about its center of mass [2]. Additionally, the coil can generate a force $\boldsymbol{F}$ with components (F_x, F_y, F_z) and a torque $\boldsymbol{\Gamma}$ with components $(\Gamma_x, \Gamma_y, \Gamma_z)$. Of the twelve variables that contribute to the virtual mechanical power, F_z and v_z are the only desired components. Note:

$$UI = \boldsymbol{F}\cdot\boldsymbol{v} + \boldsymbol{\Gamma}\cdot\boldsymbol{\Omega}$$

or

$$UI = F_xv_x + F_yv_y + F_zv_z + \Gamma_x\omega_x + \Gamma_y\omega_y + \Gamma_z\omega_z$$

To achieve the precision necessary for realization of mass, the ratio of each off-axis component to F_zv_z must be minimized to 3 x 10^{-9} (ppb). There are essentially three ways to accomplish this: (1) diminish the five off-axis force components, (2) diminish the five off-axis velocity components, or (3) reduce both off-axis forces and velocities to diminish the off-axis product Fv. For example, if

$$\frac{F_x}{F_z} = 10^{-4} \quad and \quad \frac{v_x}{v_z} = 10^{-5}$$

then,

$$\frac{F_xv_x}{F_zv_z} = 10^{-9}\,.$$

The off-axis x and y force and torque components are minimized by concentrically aligning the electrical center of the coil to the center of the radial magnetic field. By doing this, the magnetic flux gradients are balanced on opposing sides of the coil. Compliancy in the stirrup system, accomplished by small flexure cubes, allows for monitoring of the parasitic forces and torques during Force Mode. However, compliant systems result in amplified parasitic motions.

HIGH PRECISION ALIGNMENT

In order to achieve the required ppb level of uncertainty for mass redefinition, the physical alignment of the moving coil's electrical center to the field's magnetic center is important. These two points must be aligned to sub-micron levels. However, the coil's location traces back to the location of the knife edge whose 35 kg load must be precisely positioned with a device that is both vacuum compatible and nonmagnetic.

An Extremely Large Flexure (ELF) was designed and machined from a monolithic block of 6061 T6 aluminum to adjust the XY position of the knife edge and everything attached. The ELF weighs 33 kg, is 97 cm x 46 cm x 3.8 cm, and is mirror symmetric about the x and y axis. The system contains 24 individual flexures measuring 1.8 mm wide and 8 mm long. These flexures allow the movement of the middle plate with respect to the outer ring.

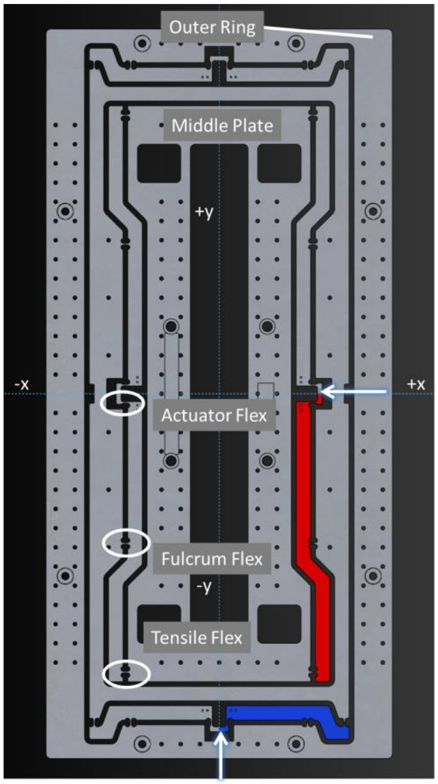


FIGURE 3. Top view of the ELF. Pushing on the tabs indicated by the arrows result in X and Y displacements of the middle plate. Desired displacements = ±0.5 mm in X and Y.

Note the symmetric design and the collinear flexures are significant in removing off axis motions, preserving pure linear motion in X and Y. Focusing only on the shaded vertical beam, one sees three collinear flexures. Pushing on the top actuation flexure in the –x direction pivots the whole beam about the fulcrum flexure, resulting in a pulling force applied to the tensile flexure and displacement of the middle plate in the +x direction. The same is conducted on the bottom shaded beam for displacements in the y direction.

Because maximum actuation force is a design constraint, the flexures were originally designed to be extremely thin, minimizing the spring constant. However, the thinner the flexures, the more sag the middle plate experiences, especially under a 35 kg external load. Optimization of flexure thickness (vertical stiffness) and actuation force can be determined by varying the location of the fulcrum. Longer lever arms amplify the actuation force, allowing for stiffer flexures. In the current design, finite element analysis (FEA) indicates the middle plate should sag approximately 1 mm under a 100 kg load (safety factor = 2.9) while a 50 N X force and 100 N Y force will translate the middle plate by 1 mm.

The force to push the actuator flexures are generated via two stepper motors driving a 1/4-80 adjustment screw. A second benefit of distancing the fulcrum from the actuator flexure is that it increases the resolution of the screw and stepper motor. If the fulcrum is exactly in the middle between the actuator and tensile flexures, the displacement ratio of the actuator flexure to tensile flexure is 1:1. If the fulcrum is shifted such that the new ratio is 10:1, then the actuator resolution also increases by an order of magnitude. A stepper motor with a step size of 1.8 degrees coupled with a 1/4-80 lead screw has a resolution of 1.5 µm. A 10:1 lever arm can magnify the resolution to 0.15 µm. With microstepping techniques, an even finer resolution can be achieved.

CHARACTERIZATION OF ELF

To verify the static loading calculations, the ELF was loaded with 31 kg at the origin. Due to the limited dynamic range of the dial indicator used in this experiment, it was zeroed in between each incremental loading of 5.5 kg. The total amount measured by the dial indicator summed to 490 µm.

To verify this summed displacement, a flat was bolted to the outermost frame of the ELF and spanned all the way to the middle plate. After loading, the separation displacement between the flat and the middle plate was measured with feeler stock, indicating 508 µm (3.5% disparity between two measurement types).

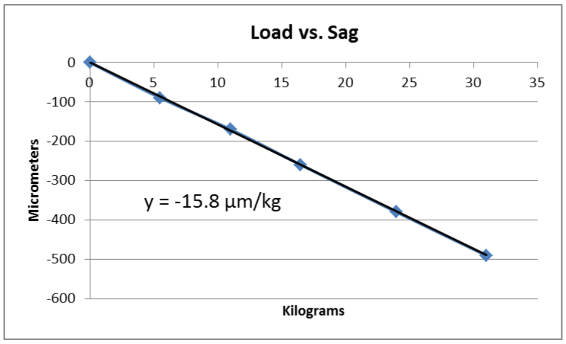


FIGURE 5. A total of 31 kg (incremental masses of 5.5 and 7.3 kg bricks) of load was placed on top of the ELF. This resulted in 490 µm of sag and shows the expected linear trend. Error bars not shown for readability, but commensurate with indicated scatter.

The parasitic motions of roll (rotation about x axis) and pitch (rotation about y axis) during x and y translation were also of concern. Two precision bubble levels were placed on the surface of the middle plate, both with a resolution of 5 arcseconds per division. Each translation direction was independently driven 1.2 mm in 150 µm increments. Under no external load, the pitch had no measurable variation while the roll had a deviation of ± 1.5 arcseconds. Under the full external load, the pitch and roll both had a variation of ± 2 arcseconds.

AUXILIARY MASS

When switching between Velocity Mode and Weighing Mode in the existing watt balance, a ½ kg counter mass is loaded and unloaded from the mass pan hanging from the far side of the wheel. A design flaw is exposed during this process: the knife edge experiences a 1 kg heavier load in Weighing Mode than in Velocity Mode, drastically affecting the hysteresis in the knife edge.

To remedy this issue for NIST-4, the counter mass was replaced with a ½ kg fixed tare mass. A ½ kg auxiliary mass was introduced to the main mass side to serve as the new counter mass. Using its on/off system, the auxiliary mass is removed from the system during Weighing Mode and reinserted during Velocity Mode as a novel way to balance the wheel.

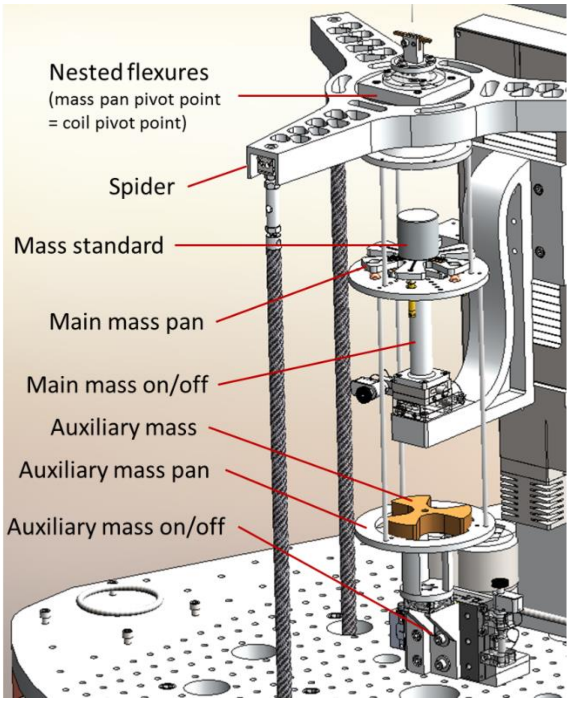


FIGURE 6. Main mass side hanging assembly. The aux mass hangs below the main mass and has its own mass on/off mechanism. It is only loaded on its mass pan during Velocity Mode.

Existing Watt Balance

Mode	**F_{coil}**	**M_{main}**	**M_{aux}**	**=**	**$M_{counter}$**	**Knife Load**
Velocity	0	0	0	=	0	M_{tare} + 0
Weigh (M_{off})	½	0	0	=	½	M_{tare} + 1
Weigh (M_{on})	-½	1	0	=	½	M_{tare} + 1

NIST-4

Mode	**F_{coil}**	**M_{main}**	**M_{aux}**	**=**	**$M_{counter}$**	**Knife Load**
Velocity	0	0	½	=	½	M_{tare} + 1
Weigh (M_{off})	½	0	0	=	½	M_{tare} + 1
Weigh (M_{on})	-½	1	0	=	½	M_{tare} + 1

FIGURE 7. The NIST-4 knife edge load is equal between both modes after introducing the aux mass to the system. This reduces the hysteresis in the knife edge.

When the NIST-4 watt balance is undergoing velocity sweeps, the aux mass is loaded to counterbalance the ½ kg fixed tare mass. Compared to the traditional method of removing the ½ kg counter mass completely, the NIST-4 Velocity Mode has a 1 kg heavier load. However, this 1 kg extra load on the knife edge is now consistent with that of Weighing Mode and should significantly reduce the mechanical hysteresis in the knife edge. Current tests are underway to determine the validity of the auxiliary mass mechanism.

CONCLUSION

NIST-4 will be used to realize the unit of mass once the redefinition of the SI has occurred with uncertainties of 3 parts in 10^8. To disseminate mass with similar uncertainties demands high precision mechanical components capable of sub-micron repeatability and accuracy. The NIST-4 watt balance continues to prove its merit as a great precision engineering experiment.

REFERENCES

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[3] Robinson I A. Alignment of the NPL Mark II watt balance. IOP Publishing, Meas. Sci. Technol. 2012; 23: 17pp.