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Deterministic generation of single photons via multiplexing repetitive parametric downconversions

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We show through Monte Carlo simulation, under realistic experimental conditions, that a system composed of just a few repetitive spontaneous parametric downconversion processes can approximate an on-demand single-photon source with nearly deterministic single-photon emission.
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Research on single-photon sources remains one of the most active areas in quantum information science, as it is relevant to many quantum information applications, such as linear optical quantum computing[1] and long-distance quantum key distribution.[2–4] An ideal single-photon source is defined by the following properties. It emits individual photons in the form of a single temporal-, single spatial-, and single polarization-mode. On each request, it emits its single photons with a unity probability of success. The single-photon pulses are time-bandwidth limited and the single photons emitted, either by different sources or on different requests from the same source, are indistinguishable.

Schemes for approximating single-photon sources have been implemented in a variety of physical systems and are often categorized as either deterministic or probabilistic (for recent reviews, see Refs. 5–8). Single-photon sources categorized as deterministic usually involve taking some physical system, i.e., atoms, ions, etc., upon request, to a prescribed excited state which nearly instantly transits to a prescribed lower energy state via spontaneous decay radiating a single photon. This type of source, which starts with a deterministic process, becomes probabilistic in practice due to photon loss mechanisms such as photon absorption and inefficient photon collection. In addition, it remains a technical challenge to produce time-bandwidth limited, indistinguishable single photons for many sources of this type. Another approach to single-photon sources relies on spontaneous parametric downconversion (SPDC) and spontaneous four-wave mixing (SFWM) processes, which are probabilistic in nature, and emit photons in pairs. The successful detection of a single photon by, for example, a photon-number-resolving detector (PNRD), heralds the presence of its partner, a heralded single photon, with a probability of success limited to 25% due to the Bose statistics of the process. It is possible though, to get around this limit by multiplexing many probabilistic photon sources into a single, more deterministic single-photon source system.[9–15] Although without photon storage, such schemes not only consume a large amount of resources, but also pose serious technical challenges.

In this paper, we show through Monte Carlo simulations that a system composed of only a few multiplexed SPDC processes pumped by a single pulsed laser can emit a single photon per request with probabilities of success as high as 90%, and can do so at high repetition rates. In addition, the single photons are emitted in the fundamental mode of a cavity, are time-bandwidth limited, and are indistinguishable from pulse to pulse.[16] Such a single-photon source would have advantages for many quantum information science and engineering applications.

Fig. 1 depicts an implementation that multiplexes two repetitive SPDC processes. In each of the SPDC processes, a photon from the pump field (ω) may be consumed to create a pair of daughter photons ω_s (mode *a*) and ω_i (mode *b*), with $\omega = \omega_s + \omega_i$ for energy conservation. In our scheme, a cavity is implemented only for mode *a*. The pump pulse potentially excites a pair of photons in modes *a* and *b* by interacting with the nonlinear crystal. The number of photons generated in mode *b* is detected by a PNRD, while mode *a* photons circulate inside the cavity with circulation time (τ) designed to be equal to the interval between the successive pump laser pulses. The nonlinear gain and bandwidth of the two repetitive SPDC processes are individually adjusted to be identical. When exactly one photon is detected in mode *b* of a given SPDC process, pumping of that process is stopped and the photon in mode *a* is stored. If more than one photon is detected in a mode *b*, that cavity is emptied and pumping continues. When a request for a photon arrives, the cavity tagged with the freshest photon releases its photon, and its pumping resumes. In the ideal case of lossless cavities and perfect detection, system probability of success approaches unity for an appropriate choice of nonlinear gain and number of multiplexed SPDC processes.

It is the goal of this paper to show that a system formed by multiplexing a modest number of repetitive SPDC processes can be used to achieve nearly deterministic single-photon emission given realistic experimental conditions. We model the system for a range of experimental parameters using Monte Carlo techniques. The system just described is clocked with photon requests arriving, for example, every *N* pump pulses (N_{pump}), regardless of whether any of the cavities are occupied with photon(s). We count the number of events that produce single photons, and define the ratio of this number relative to the number of the total Monte Carlo trials, as the probability of success for the system to successfully emit a single photon per request.

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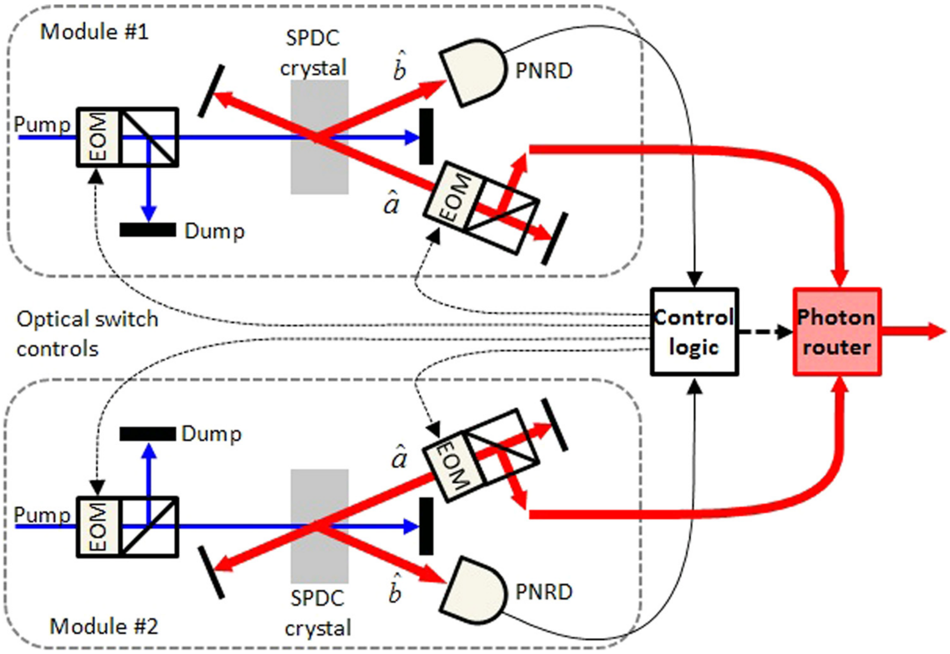

FIG. 1. Schematic of a system for single-photon generation composed of two SPDC modules.

A single pass of a repetitive SPDC process can be effectively described with three operators: the nonlinear squeezing operator $\hat{G}_{a,b} = e^{\chi \hat{a}^+ \hat{b}^+ - \chi^* \hat{a} \hat{b}}$, for SPDC, where χ is the nonlinear gain parameter; the unitary operator $\hat{U}_a = e^{i\theta/2(\hat{a}^+ \hat{c} + \hat{a} \hat{c}^+)}$ which characterizes the single-cycle loss of photons in mode a with $\hat{U}_a \hat{a} \hat{U}_a^+ \to t_a \hat{a} - i r_a \hat{c}$, (the auxiliary mode $\hat{c}$ will be traced out), with transmittance $|t_a|^2 = |\cos(\theta/2)|^2$ and reflectance $|r_a|^2 = |\sin(\theta/2)|^2$; and the simplest nontrivial PNRD detection operator for mode b, which is represented by a set of positive-operator-valued measures (POVM), $\hat{\Pi}_b^{(0)} = \sum_{i=0}^{\infty} (1-\eta)^i |i\rangle_b \langle i|$, $\hat{\Pi}_b^{(1)} = \sum_{i=1}^{\infty} i\eta(1-\eta)^{i-1} |i\rangle_b \langle i|$, and $\hat{\Pi}_b^{(m\geq 2)} = \sum_{m=2}^{\infty} \sum_{i=m}^{\infty} \binom{i}{m} \eta^m (1-\eta)^{i-m} |i\rangle_b \langle i|$, for detecting 0, 1, and greater than 1 photon, with $\hat{\Pi}_b^{(0)} + \hat{\Pi}_b^{(1)} + \hat{\Pi}_b^{(m\geq 2)} = 1$ and $\hat{\Pi}_b^{(m)} \geq 0$, where η includes the optical path and detection efficiency of mode b.

Taking the initial input state for the SPDC process to be vacuum, the state reentering the process after the first pass is given as

$$\rho_m = \mathrm{Tr}_c \lfloor \hat{U}_a^+ \hat{G}_{a,b}^+ \hat{U}_a \hat{\Pi}_b^{(m)} \hat{U}_a^+ \hat{G}_{a,b} \hat{U}_a |0\rangle_{a,b} \langle 0| \rfloor \, |0\rangle_b \langle 0|, \quad (1)$$

with $m = 0, 1, \geq 2$, conditioned on the detection of m-photons by PNRD with the probability of

$$P_b(m) = \mathrm{Tr}_a \lfloor \hat{G}_{a,b}^+ \hat{\Pi}_b^{(m)} \hat{G}_{a,b} |0\rangle_{a,b} \langle 0| \rfloor. \quad (2)$$

Equations (1) and (2) are the basis of our Monte Carlo simulations. In each cycle of a repetitive SPDC process, we determine the probabilities $\{P_b(m)\}$ and the corresponding states $\{\rho_m\}$ for all possible PNRD detection outcomes of mode b. We choose a detection outcome state randomly from $\{\rho_m\}$ based on the probability distribution given by $\{P_b(m)\}$. When either $\rho_{m\geq 2}$ or ρ_0 is chosen, the input state for the next pass is set to be vacuum; if the state ρ_1 is selected (corresponding to detecting exactly one photon in mode b), pumping of that SPDC process is stopped and the photon in mode a circulates inside the cavity. Maximum allowed storage time $n_{\max}\tau$ is determined by a prescribed tolerated loss parameter ξ, $n_{\max} = [\log\xi / \log |t_a|^2]$. After $n_{\max}$ passes, the cavity is emptied (i.e., the state of mode a is reset to vacuum) and pumping is resumed. Upon receiving a request for a photon, the system

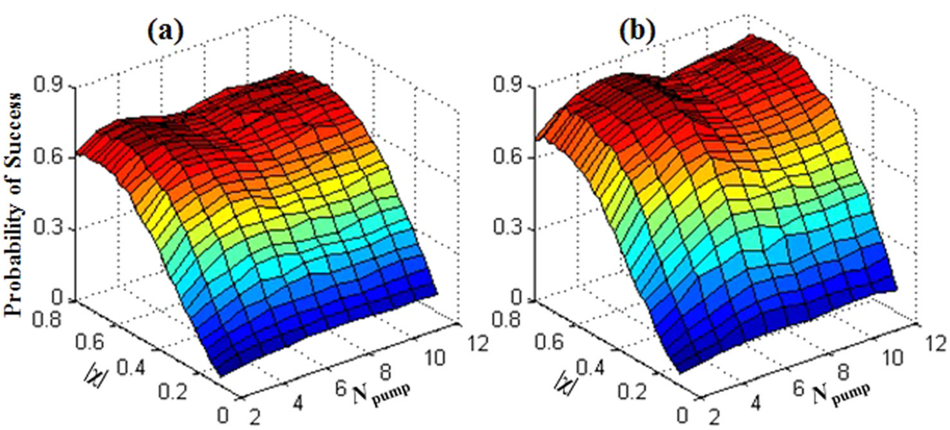

FIG. 2. Probability of emitting a single photon versus $|\chi|$ and N_{pump}, for a two-module system, for (a) medium-performance system ($|t_a|^2 = 0.95$, $\eta = 0.9$) and (b) high-performance system ($|t_a|^2 = 0.99, \eta = 0.95$).

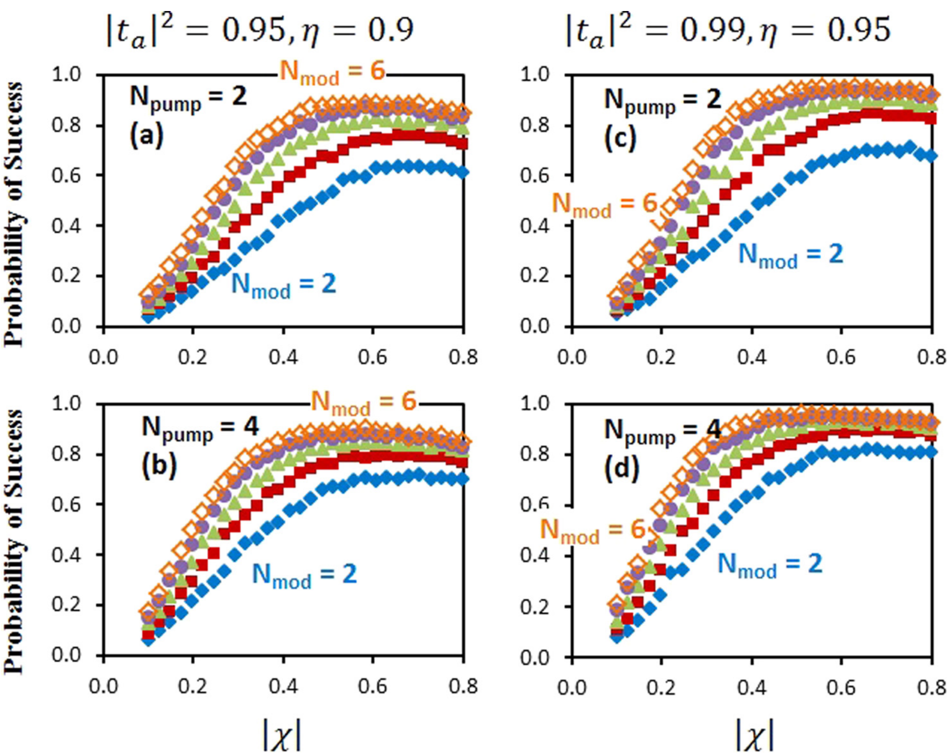

FIG. 3. Probability of success for single-photon generation versus gain and number of SPDC modules (from top to bottom are 6, 5, 4, 3, 2 modules in each sub-figure). (a,b) System with medium-performance; (c,d) high-performance.

will release the stored state from an occupied cavity if one exists. If more than one cavity is occupied, the one with the freshest photon is routed to the output. Note that because of the loss present in the cavity and the monitoring detector's inefficiency, the state ρ_1 may not be a single-photon state. This is modeled by the random selection of a photon-number state from the photon-number-state probability distribution of ρ_1.

Fig. 2 shows the probability of success as a function of $|\chi|$ and N pump for a system of (a) medium-performance with $(|t_a|^2 = 0.95, \eta = 0.9)$ and (b) high-performance with $(|t_a|^2 = 0.99, \eta = 0.95)$, with two SPDC modules. (Reflection loss per optical surface below 0.01% is commercially available, and single-photon detection efficiency above 93% and 95% has been demonstrated.[17–19]) We observe the probability of success, for both medium and high performance systems, increases monotonically with the nonlinear gain; the probability of success increases to a maximum, then droops, and then increases again as N_{pump} increases. This is because the system cannot generate photons quickly enough for small N_{pump}; as N_{pump} increases, all cavities are filled with photons but toward the end of their lifetimes due to the accumulated storage loss; as N_{pump} continuous increasing, the cavities empty the stored photons if the storage time $\geq n_{max}\tau$ and resume pumping to generate new photons. For a system with two SPDC modules, we observe that the probability of success exceeds 60% for a medium-performance system and 80% for a high-performance system for the range of parameters studied.

Fig. 3 shows that the probability of success increases as the number of multiplexed processes increases from 2 to 6, although saturation is seen, with the difference between the probability of success for a 5-process and a 6-process being marginal, for both medium and high performance systems. We observe that the highest probability of success begins to saturate for $|\chi| > 0.3$, with $N_{pump} = 2$ for both medium and high performance systems, saturating at the level of 80% for $|\chi| > 0.4$ for a medium performance system and 90% for a high performance system with $N_{pump} = 4$. For all the cases studied, we observe that the probability of success initially increases as the gain increases. The probability of success reaches the maximum and begins to decrease as the gain continues to increase. This is due to the increased probability of multi-photon pairs relative to that of the single-photon pairs in the parametric downconversion process when the gain gets high. At the low end, we observe that probabilities of success are at the level of 60% for both medium and high performance systems for just 2-multiplexed processes. Since photon detection and switching can be accomplished in less than 100 ns,[15] it is feasible for this system to output a train of time-bandwidth limited single-photon pulses at a few MHz or higher.

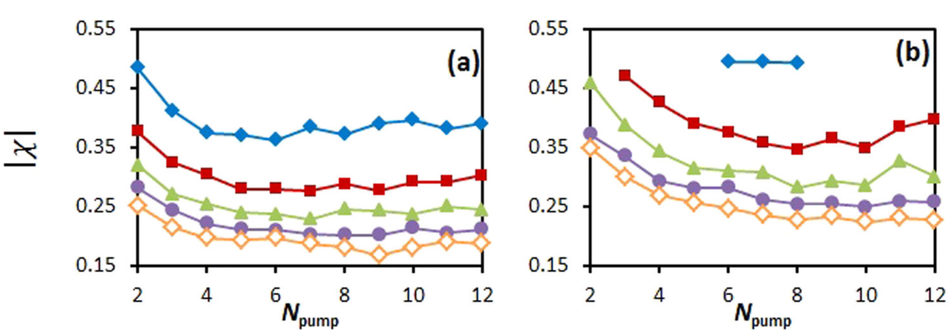

FIG. 4. Contour plots for experimental parameters to achieve or exceed the minimum probability of success of 50% for medium-performance (a) and 80% for high-performance system (b) with different numbers of multiplexed SPDC processes (from top to bottom are 6, 5, 4, 3, 2 multiplexed processes in each sub-figure).

Fig. 4 shows contour plots in the parameter spaces $|\chi|$ and N_{pump}. A system operated with experimental parameters on or above the values shown is able to emit single photons with the minimum probability of success $\geq 50\%$ with medium-performance (a) and $\geq 80\%$ with the high-performance system (b). This wide range of operating parameters offers great flexibility in designing a system to take advantage of this multiplexed approach and points to the feasibility of the scheme.

In conclusion, we have presented a numerical study of a system formed by multiplexing a modest number of repetitive SPDC processes. The cavity-SPDC not only selects the spectral and spatial mode, but also offers storage for the single photon. Such systems can be constructed with currently available techniques and with limited resources; while they can output time-bandwidth limited single-photon pulses with a high probability of success at high repetition rates. In addition, given the flexibility in cavity design and tuning, photons from different sources can be made indistinguishable. We expect such a source, when implemented, shall be a great assistance to advance a broad range of quantum information applications. Furthermore, the proposed scheme can be also applied to the generation of high photon-number Fock state and N00N state. We are conducting a comprehensive study on this subject and the results will be presented elsewhere.

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