# The Impact of Turbulence on High Accuracy Time-Frequency Transfer across Free Space

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**Abstract:** Atmospheric optical path-length variations are measured across a 2-km optical link through a frequency comb-based system with femtosecond-level precision. Without mitigation, the turbulent piston effect will severely restrict time-frequency transfer from optical clocks.

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Optically based time-frequency transfer (TFT) is necessary, as the accuracy and precision of the next generation of optical atomic clocks will far exceed the capabilities of microwave-based systems to faithfully transfer the clock signals across free-space. Because optical systems operate at much higher carrier frequencies, they are fundamentally better suited to transfer high-accuracy time-frequency signals than microwave-based systems. However, just as for free-space optical communications, atmospheric turbulence can limit optically based TFT. While the impact of turbulence on free-space laser communication is well studied [1-3], its impact on optical TFT is not. Many of the same issues arise, such as scintillation and beam wander, which cause signal fading. However, in addition, TFT is sensitive to time-of-flight variations. Time-of-flight variations originate from the coupling of the spatial variation in phase via the wind (frozen turbulence) as well as, on long timescales, changes in atmospheric conditions such as temperature and pressure. The so-called “piston” effect, which is the variation in the optical phase averaged across the receive aperture, is expected to dominate the time-of-flight variations at short timescales. This effect is closely related to angular jitter at the receive aperture, which arises from the variation in the slope of the optical phase across the aperture. For the purpose of TFT, we translate these variations in the optical phase to their effective time-of-flight variation, measured in seconds.

Here we report measurements of the time-of-flight variations across a 2 km free-space link from timescales of milliseconds to hours. The measurements were made with a dual frequency-comb system that had femtosecond-level accuracy (corresponding to 300 nm fluctuations in the optical path length) and that was capable of making coherent measurements despite signal fading [4]. Our results are in reasonable agreement with predictions. Specifically, at very low Fourier frequencies (corresponding to timescales of tens of minutes to hours), the time of flight will vary with the overall air temperature and pressure. At higher frequencies, the measured variations follow the $f^{-8/3}$ dependence expected from the Kolmogorov spectrum for the piston effect [5]. The measured timing fluctuations greatly exceed the uncertainty of atomic optical clocks, which can reach $10^{-18}$ fractional uncertainty [6]. Therefore some form of two-way transfer is critical to the free-space transfer of high-accuracy time-frequency signals across free space, as demonstrated in Ref. [4].

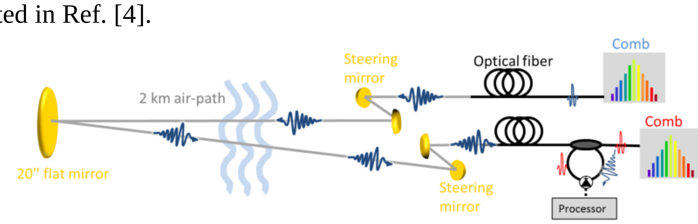


**Figure 1**: Experimental setup for the measurements. A coherent pulse train from a frequency comb (blue) propagates through 500 m of optical fiber and a 2 km air-path before being sampled by a second frequency comb (red), providing femtosecond –level timing information.

In order to achieve femtosecond-level timing information, a dual comb measurement technique was used as described in Ref [4]. A coherent pulse train (frequency comb) was generated by phase-locking a femtosecond Er-fiber laser to an optical oscillator. This pulse train was transmitted across the link and then sampled with a second frequency comb locked to the same oscillator but with a repetition rate offset by $\Delta f_r$ . The result is a series of interferograms, or cross correlations, that occur every $1/\Delta f_r$. A variation in the time-of-flight appears as a variation in the spacing of the interferograms. This method can have femtosecond-level sensitivity and is robust against interruptions of the signal. Consequently phase fluctuations can be measured for turbulence conditions that would prevent a continuous link. Environmental variables including temperature, humidity, pressure, and wind speed were also measured at two points in the link. Fast steering mirrors on both sides of the link compensated for beam wander via a dither lock with a ~ 200 Hz bandwidth. The mirror correction signal was digitized and recorded for comparison with the direct phase measurement. A simplified schematic is shown in Figure 1.

Figure 2a shows the power spectral density (PSD) of the measured time-of-flight variations across the 2 km link (dark blue line). The increase in noise at Fourier frequencies greater than a few Hertz is due to noise from the 500 meters of fiber optic cable that connects the combs to the free-space link. At lower frequencies, the time-of-flight variations are a result of atmospheric effects. The expected turbulence-induced piston effect yields a temporal PSD for a Kolmogorov spectrum of $S_P(f) = 0.016 c^{-2} C_n^2 L V^{5/3} f^{-8/3}$ $s^2$/Hz over $V/L_0 < f < 0.3V/D$, where *c* is the speed of light, $L$ ~ 2 km is the air-path length, $V$ is the wind speed, $C_n^2$ is the turbulence structure function, $D$ ~ 5 cm is the aperture diameter, and $L_0$ ~ 100 m is the outer scale [5]. As shown, we find good agreement between the measured PSD and this formula both in terms of the $f^{8/3}$ dependence and the absolute values assuming $V = 0.7$ m/s, and $C_n^2 = 1.2\text{x}10^{-14}$ $m^{-2/3}$, a reasonable value near the ground surface for Boulder, Colorado [7]. At frequencies below 5 mHz, the measured PSD begins to deviate from the calculated spectrum as the frequency no longer exceeds $V/L_0$. This deviation yields an approximate value of $L_0$ =140 m for the outer scale. Finally, the measured piston PSD can be compared to the PSD of the angle-of-arrival fluctuations, which are effectively measured through the correction signals applied to the fast steering mirror. Theoretically, the PSD for the angle-of-arrival fluctuations is related to the piston PSD by $S_\alpha(f) = \left(2\pi c / V n_{gr}\right)^2 f^2 S_P(f)$, where $n_{gr}$ is the group index [5]. As shown in Fig. 2a, we find good agreement between the measured piston PSD and the scaled PSD of the angle-of-arrival fluctuations over the inertial region of $V/L_0 < f < 0.3V/D$.

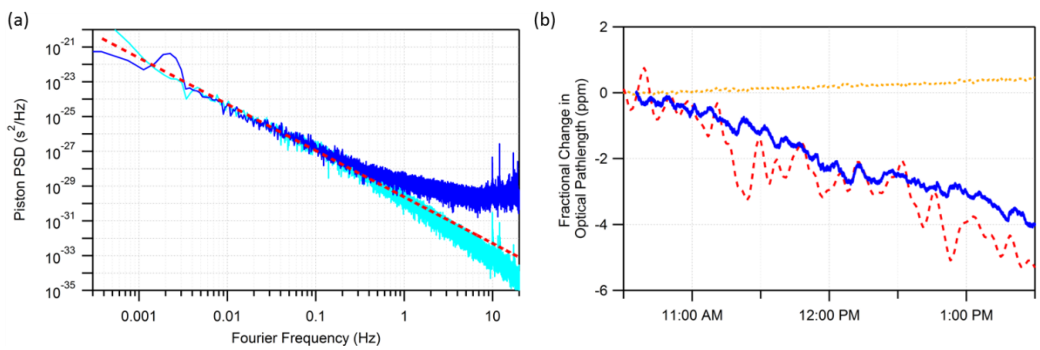


**Figure 2**: (a) PSD of the time-of-flight fluctuations measured across the 2-km free space link based on a 2.3-hour data set (dark blue, solid). The predicted piston PSD for V=0.7 m/s, L=2 km, and $C_n^2 = 1.2\text{x}10^{-14}$ $m^{-2/3}$ is also shown (red dashed), and an appropriately scaled PSD of the measured angle-of-arrival PSD (light blue, solid) . (b) Fractional change in optical path-length over a 3-hour measurement. (To convert to time, this value is multiplied by the 6.7 μs time-of-flight over the 2 km link.) Also shown is the expected variation from the measured temperature variation (red, dashed) and pressure variation (orange, dashed) over the 3-hour measurement. Temperature variation accounts for nearly all of the optical pathlength changes.

The coherence of the frequency combs enabled measurements over the course of hours, exceeding the timescale set by $L_0/V$, i.e., tens to hundreds of seconds. On very long timescales, we anticipate that the fluctuations in optical pathlength are due to changes in the environment, as it is expected to change by -0.75 ppm/C with temperature and by 2.7 ppb/Pa with pressure. Figure 2b shows measurements of the fractional change in optical pathlength as well as the expected fractional change from the recorded temperature and pressure values. The slight quantitative disagreement arises from the non-uniformity of the optical path and the point measurement of the temperature.

We have shown that measurement of a coherent train of pulses via frequency comb is a very sensitive probe of the piston effect across a wide range of timescales. The structure constant, $C_n^2$, can be extracted from the measurement of the piston PSD with knowledge of the wind speed. Measurements of the piston-effect spectrum as well as the angle-of-arrival jitter show the expected theoretical frequency dependence over the range for which the Kolmogorov spectrum is valid. Understanding of the piston mode over a range of timescales can guide future optical transfer of time or frequency information as well as interferometry.


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