Controlling the group velocity of colliding atomic Bose-Einstein condensates with Feshbach resonances

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We report on a proposal to change the group velocity of a small Bose Einstein Condensate (BEC) upon collision with another BEC in analogy to slowing of light passing through dispersive media. We make use of ultracold collisions near a magnetic Feshbach resonance, which gives rise to a sharp variation in scattering length with collision energy and thereby changes the group velocity. A generalized Gross-Pitaveskii equation is derived for a small BEC moving through a larger stationary BEC. We denote the two condensates by laser and medium BEC, respectively, to highlight the analogy to a laser pulse travelling through a medium. We derive an expression for the group velocity in a homogeneous medium as well as for the difference in distance, δ, covered by the laser BEC in the presence and absence of a finite-sized medium BEC with a Thomas-Fermi density distribution. For a medium and laser of the same isotopic species, the shift δ has an upper bound of twice the Thomas-Fermi radius of the medium. For typical narrow Feshbach resonances and a medium with number density 10^{15} cm^{-3} up to 85% of the upper bound can be achieved, making the effect experimentally observable. We also derive constraints on the experimental realization of our proposal.

Over the last two decades significant advances have been made to replicate linear and non-linear optical phenomena with matter waves, creating the field of matter-wave optics. For example, atom lasers [1, 2] are sources of coherent ultra-cold atoms generated by extracting atoms from a Bose-Einstein condensate (BEC). The coherence of BECs was demonstrated by interfering two condensates [3]. Atomic mirrors and beam splitters have also been realized [4]. Recently, the matter-wave equivalent of meta-materials (media with negative refractive index) has been proposed [5]. The analog of nonlinear four-wave mixing has been demonstrated using atom lasers [6, 7]. In these experiments three BECs with phase-matched relative momenta generated a fourth beam.

We present a proposal to slow a BEC while propagating through another BEC near a magnetic Feshbach resonance in analogy to slowing of light in dispersive media. Slowing of light occurs when the refractive index of a medium varies sharply with photon frequency. Using electromagnetically induced transparency [8], slow light has been observed with a Bose-Einstein condensate [9, 10] and a hot Rb gas [11] acting as the medium.

Magnetic Feshbach resonances are a tool with which to manipulate the interaction between ultracold atoms [12]. They are used for creating ultracold molecules, molecular condensates , and in the BEC-BCS crossover in Fermi gases [13, 14]. Feshbach resonances play an essential role in condensing ^{133}Cs, ^{85}Rb and ^{39}K [15–17]. Cooling ultracold atoms using Feshbach resonances has been proposed [18]. Collisions can also be tuned using optical Feshbach resonances [19–21], as their width can be dynamically varied with a laser. They, however, tend to suffer from losses due to spontaneous emission.

Figure 1 shows our proposal of slowing a condensate travelling through a large stationary BEC near a magnetic Feshbach resonance. These two BECs play the role of a laser and a medium, respectively. The laser BEC contains atoms of mass m and has a diameter $\ell_{\rm L}$. It propagates with wavevector $\mathbf{k}_0$ of magnitude $k_0 \gg 1/\ell_{\rm L}$. Its average velocity is $\mathbf{v}_0 = \hbar\mathbf{k}_0/m$ and the kinetic energy per particle is $E_0 = \hbar^2 k_0^2/(2m)$. (Here $\hbar$ is the reduced Planck's constant.) It is incident on a stationary BEC of size $\ell_{\rm M}$, such that $\ell_{\rm L} \ll \ell_{\rm M}$ (in all spatial directions). Figures 1b and c show two distinct cases of collisions between condensates. In Fig. 1b elastic scattering out of the two condensates is significant. These losses have limited four-wave mixing experiments [22]. On the other hand, it allows detection of d-wave shape resonances in collisions between BECs [23] and thermal gases [24]. In Fig. 1c the collision takes place at the lossless point, where the scattering cross section is small due to interference of the background and resonance scattering amplitudes. As we will derive the laser BEC then slows down. The slowing can be quantified by δ, the difference in distance travelled by the laser BEC in the presence and absence of the medium.

We first review the two-body physics of a Feshbach resonance. Subsequently, we derive the equation of motion for a laser BEC travelling through a medium. We then derive the group velocity in a homogeneous medium followed by an estimate for δ for an inhomogeneous medium whose density is described by a Thomas-Fermi profile. Finally, restrictions on an experimental realization are given.

Collisions between ultracold atoms are dominated by s-wave scattering and the scattering amplitude $f(E)$ only depends on the relative collision energy, $E = \hbar^2 k^2/(2\mu)$. Then around an isolated magnetic Feshbach resonance [12, 25, 26]

$$f(E) = f_{\rm bg}(E) - e^{2i\delta_{\rm bg}} \frac{\hbar\Gamma(E)/(2k)}{E - E_{\rm res} + i\hbar\Gamma(E)/2}, \quad (1)$$

where μ is the reduced mass and k is the magnitude of relative wavevector. The background scattering am-
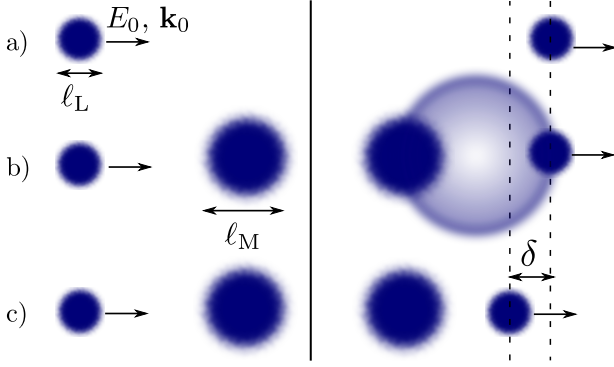
FIG. 1: A schematic of a slow-atom experiment with colliding condensates. The left and right hand side show images of the condensates before and after the collision, respectively. Panel a) simply shows the laser BEC moving in free space with kinetic energy E_0 and wave vector $\mathbf{k}_0$. Panels b) and c) show two cases of a laser BEC propagating through the medium BEC. Panel b) shows a case with large elastic scattering losses indicated by the halo of scattered atoms . Finally, panel c) depicts the collision near a Feshbach resonance where there is negligible scattering loss. This occurs when the scattering amplitude is zero. The distance delay δ of the laser BEC is also indicated.

FIG. 2: (Color online) Real and imaginary part of scattering amplitudes, f and f_{bg}, as a function of collision energy for a narrow ^{87}Rb Feshbach resonance at $B_0 = 68.54$ mT and $\Delta = 0.6$ μT, with $B - B_0$ tuned at 0.05 mT. Also shown is the lossless point where Im $f(E) = 0$. Values obtained from [12], $1a_0 = 0.0529$ nm, and k_{B} is the Boltzmann constant.

plitude $f_{\mathrm{bg}}(E) = e^{i\delta_{\mathrm{bg}}}\sin(\delta_{\mathrm{bg}})/k$ with phase shift δ_{bg}. To a good approximation $f_{\mathrm{bg}}(E) = -a_{\mathrm{bg}}/(1 + ika_{\mathrm{bg}})$, and $\delta_{\mathrm{bg}} = -ka_{\mathrm{bg}}$, where a_{bg} is the background scattering length. The resonance width $\hbar\Gamma(E) = 2ka_{\mathrm{bg}}\Gamma_0$ in the threshold limit $k \to 0$. The energy-independent reduced width $\Gamma_0 = \mu_{\mathrm{res}}\Delta$, where μ_{res} is the difference between the magnetic moments of the resonance state and the asymptotically free atoms and Δ is the magnetic width of the resonance. The resonance energy is $E_{\mathrm{res}} = \mu_{\mathrm{res}}(B - B_0)$, where B is the magnetic field and B_0 is the resonant field. The scattering amplitude $f(E)$ satisfies the optical theorem [25].

Figure 2 shows $f(E)$ near a Feshbach resonance as a function of collision energy E. The resonance occurs at a finite collision energy and $f(E)$ approaches $f_{\mathrm{bg}}(E)$ away from E_{res}. The imaginary part of $f(E)$ is related to the total cross section $\sigma(E)$ and thus to the fraction of scattered atoms. In fact, $\sigma(E) = 4\pi \,\mathrm{Im}\, f(E)/k$ from the optical theorem [25]. On resonance Im(f) is maximal and $\approx 1/k$. More importantly, there exists a collision energy at which $f(E) = 0$ due to an interference between the background and resonance scattering amplitudes. This collision energy, which to good approximation is $E_{\mathrm{res}} - \hbar\Gamma_0$ for positive a_{bg}, is the lossless optimal point mentioned in Fig. 1.

We now describe the many-body physics of colliding BECs, assuming that both BECs contain the same atomic species. Then the collision energy $E \approx E_0/2$. Their dynamics is well described by the time evolution of the order parameter $\Psi(\mathbf{x}, t)$, the expectation value of the annihilation operator $\hat{\Psi}(\mathbf{x}, t)$ in the Heisenberg picture. For a BEC at rest its evolution is well described
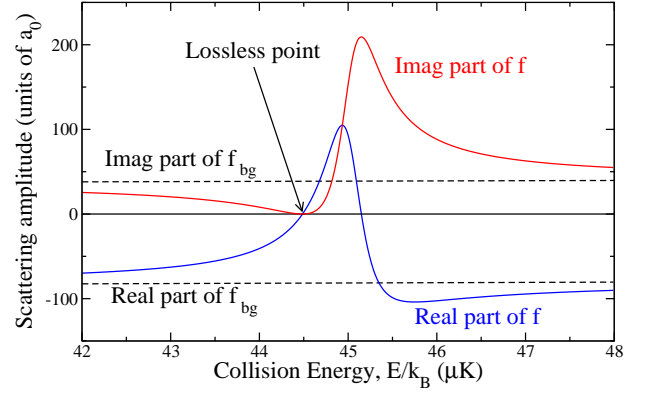
by the Gross-Pitaevskii (GP) equation, derived for an energy-independent and real scattering amplitude. Both assumptions are invalid near a Feshbach resonance at finite energy.

Our starting point is Eq. 38 of Ref. [27] obtained using a cumulant expansion. It includes the time and energy dependence of the two-body scattering and is given by

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{x}, t) = H_{1\mathrm{B}}\Psi(\mathbf{x}, t) + \int \prod_i d\mathbf{y}_i \int_{t_0}^{\infty} dt_1$$
$$\times \Psi(\mathbf{y}_1, t_1)\Psi(\mathbf{y}_2, t_1)\Psi^*(\mathbf{y}_3, t) \quad (2)$$
$$\langle \mathbf{x}, \mathbf{y}_3 | T_{2\mathrm{B}}(t, t_1) | \mathbf{y}_1, \mathbf{y}_2\rangle ,$$

where t_0 is the initial time, $H_{1\mathrm{B}} = -\hbar^2\nabla^2/(2m) + V(\mathbf{x})$, the single-particle Hamiltonian, and $V(\mathbf{x})$ is the external potential. The operator $T_{2\mathrm{B}}$ is the two-body T-matrix in the time domain and the integrals over $\mathbf{y}_i$ for $i = 1, 2, 3$ are in coordinate space.

The momenta of the medium and the laser BEC are non-overlapping. Hence, the wave function $\Psi(\mathbf{x}, t)$ is the sum of orthogonal medium and laser wavefunctions, $\Psi_{\mathrm{M}}(\mathbf{x}, t)$ and $\Psi_{\mathrm{L}}(\mathbf{x}, t)$, respectively. Since the laser BEC is small in size and number density, we linearize Eq. (2) for the laser condensate assuming phase-matching conditions. After rearranging terms, we find

$$i\hbar\frac{\partial}{\partial t}\Psi_{\mathrm{L}}(\mathbf{x}, t) = H_{1B}\Psi_{\mathrm{L}}(\mathbf{x}, t) + 2\int \prod_i d\mathbf{y}_i\Psi_{\mathrm{M}}^*(\mathbf{y}_3, t)$$
$$\times \int_{t_0}^{\infty} dt_1 \langle \mathbf{x}, \mathbf{y}_3 | T_{2\mathrm{B}}(t, t_1) | \mathbf{y}_1, \mathbf{y}_2\rangle$$
$$\times \Psi_{\mathrm{L}}(\mathbf{y}_1, t_1)\Psi_{\mathrm{M}}(\mathbf{y}_2, t_1), \quad (3)$$

which is nonlocal in both space and time and ignores the effect of the laser on the evolution of the medium. Hence, the medium BEC is described by the "energy-independent" GP equation.

A more insightful expression can be obtained when we approximate the integrands in Eq. (3) by power series in derivatives evaluated at $\mathbf{x}$ and t. First, we realize that $T_{2B}(t, t_1)$ only depends on $t - t_1$ and is peaked around $t - t_1 = 0$. Assuming that the wave functions vary slowly in time, the lower limit of the integral over time can be extended to $-\infty$. Next, we note for any pair of functions $g(t)$ and $h(t)$

$$\int_{-\infty}^{\infty} d\tau\, h(\tau) g(t - \tau) = \tilde{h}\left(i\partial/\partial t\right) g(t), \qquad (4)$$

where $\tilde{h}(z) = \int dt\, e^{izt} h(t)$ is the Fourier transform of $h(t)$ and $\tilde{h}(i\partial/\partial t) = \sum_n d^n \tilde{h}/dz^n |_{z=0} (i\partial/\partial t)^n/n!$. Using Eq. (4) with $h(\tau) = \langle.|T_{2B}(\tau)|.\rangle$, the interaction term in Eq. (3) reduces to

$$2 \int \prod_i d\mathbf{y}_i \Psi_M^*(\mathbf{y}_3, t) \qquad (5)$$
$$\times \langle \mathbf{x}, \mathbf{y}_3| T_{2B}\left(i\hbar\partial/\partial t\right) |\mathbf{y}_1, \mathbf{y}_2\rangle \Psi_L(\mathbf{y}_1, t)\Psi_M(\mathbf{y}_2, t),$$

where the T-matrix $T_{2B}(z)$ is now in the energy domain (dropping the $\sim$ notation for simplicity) and the time derivatives only act on $\Psi_L(\mathbf{y}_1, t)\Psi_M(\mathbf{y}_2, t)$.

The T-matrix in coordinate space can be evaluated by transforming to the momentum representation. For s-wave scattering the dependence on the relative momenta can be neglected. That is, to a good approximation the T-matrix in momentum representation is [27]

$$\langle \mathbf{k}_4, \mathbf{k}_3| T_{2B}(z) |\mathbf{k}_1, \mathbf{k}_2\rangle = \delta(\mathbf{k}_4 + \mathbf{k}_3 - \mathbf{k}_2 - \mathbf{k}_1)$$
$$\times - \frac{\hbar^2}{4\pi^2\mu} f\left(z - \hbar^2(\mathbf{k}_1 + \mathbf{k}_2)^2/2M\right), \qquad (6)$$

where the δ-function reflects total momentum conservation, $f(E)$ is the scattering amplitude of Eq. (1), and $M = 2m$.

Inserting the momentum representation of T_{2B} into Eq. (5) and noting formally that $\phi(\mathbf{y}, t) = \exp[-i(\mathbf{y} - \mathbf{x})\cdot i\nabla]\phi(\mathbf{x}, t)$, the Taylor expansion of $\phi(\mathbf{y}, t)$ around position $\mathbf{x}$, the interaction term becomes

$$\frac{2}{(2\pi)^6}\left(-\frac{\hbar^2}{4\pi^2\mu}\right) \iint \prod_{i,j} d\mathbf{y}_i d\mathbf{k}_j \delta(\mathbf{k}_4 + \mathbf{k}_3 - \mathbf{k}_2 - \mathbf{k}_1)$$
$$\times \Psi_M^*(\mathbf{y}_3, t) e^{iQ} f\left(i\hbar\partial/\partial t - \hbar^2(\mathbf{k}_1 + \mathbf{k}_2)^2/2M\right)$$
$$\times \left[e^{-i(\mathbf{y}_1 - \mathbf{x})\cdot i\nabla}\Psi_L(\mathbf{x}, t)\right]\left[e^{-i(\mathbf{y}_2 - \mathbf{x})\cdot i\nabla}\Psi_M(\mathbf{x}, t)\right], \quad (7)$$

where $Q = \mathbf{k}_4 \cdot \mathbf{x} + \mathbf{k}_3 \cdot \mathbf{y}_3 - \mathbf{k}_2 \cdot \mathbf{y}_2 - \mathbf{k}_1 \cdot \mathbf{y}_1$. Performing all integrations, we find a local equation of motion for $\Psi_L(\mathbf{x}, t)$. That is,

$$i\hbar\frac{\partial}{\partial t}\Psi_L(\mathbf{x}, t) = H_{1B}\Psi_L(\mathbf{x}, t) - \frac{4\pi\hbar^2}{\mu}\Psi_M^*(\mathbf{x}, t) \quad (8)$$
$$\times f\left(i\hbar\partial/\partial t + \hbar^2\nabla^2/2M\right)\Psi_L(\mathbf{x}, t)\Psi_M(\mathbf{x}, t).$$

Since $\ell_L \ll \ell_M$ and the spread in the collision energy is much smaller than $\Gamma(E_0/2)$, it is sufficient to expand

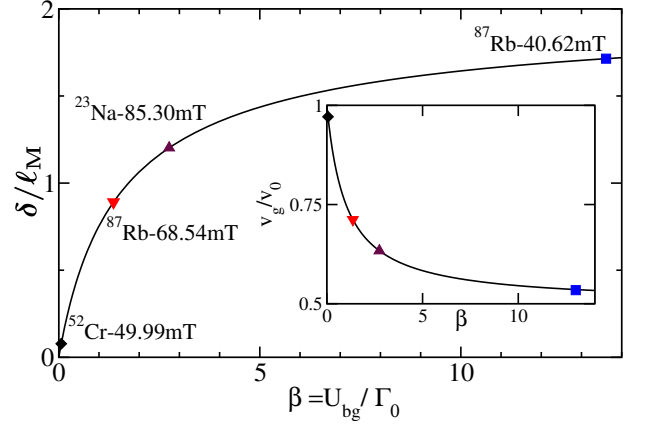FIG. 3: (Color online) Distance delay δ of the laser BEC normalized by the Thomas-Fermi radius ℓ_M of the medium BEC as a function of dimensionless parameter $\beta = U_{bg}/\Gamma_0$, where U_{bg} and Γ_0 are defined in the text. The delay for selected resonances assuming a peak number density of the medium of $n_M = 10^{15}\,\mathrm{cm}^{-3}$ is shown by colored markers. The inset shows the group velocity v_g of the laser BEC in a homogeneous medium BEC as a function of β. Here v_0 is the free space velocity of the laser BEC. Markers indicate v_g for the same selected resonances and n_M as in the main figure.

$f(z)$ to first order around $z = E_0/2$, the average relative collision energy, and derivatives of $\Psi_M(\mathbf{x}, t)$ can be neglected. For our homonuclear system the time evolution of the laser condensate is then given by

$$i\hbar\frac{\partial}{\partial t}\Psi_L(\mathbf{x}, t) = \left[-\frac{\hbar^2}{2m^*(\mathbf{x})}\nabla^2 + V_{mf}(\mathbf{x})\right.$$
$$\left. + V_{deriv}(\mathbf{x})\right]\Psi_L(\mathbf{x}, t), \quad (9)$$

where $m^*(\mathbf{x}) = m[1 + 2\alpha(\mathbf{x})]/[1 + \alpha(\mathbf{x})]$ is the position-dependent effective mass and $\alpha(\mathbf{x}) = (4\pi\hbar^2/m)|\Psi_M(\mathbf{x})|^2 df(z)/dz$ with $df(z)/dz$ evaluated at $z = E_0/2$. The "mean-field" potential $V_{mf}(\mathbf{x}) = [V(\mathbf{x}) - (8\pi\hbar^2/m)|\Psi_M(\mathbf{x})|^2 f(E_0/2)]/[1 + 2\alpha(\mathbf{x})]$ contains the external potential and a potential proportional to the scattering amplitude and medium density. The latter contribution is analogous to the interaction potential in the GP equation except that the scattering amplitude is evaluated at non-zero energy $E_0/2$. Finally, the potential $V_{deriv}(\mathbf{x}) = E_0\alpha(\mathbf{x})/(1 + 2\alpha(\mathbf{x}))$. The factor $1 + 2\alpha(\mathbf{x})$, appearing throughout, results from the $i\hbar\partial/\partial t$ argument of the scattering amplitude.

The operator acting on $\Psi_L(\mathbf{x})$ on the right-hand side of Eq. (9) is not Hermitian as the scattering amplitude is complex valued. In fact, the non-Hermicity leads to atom loss out of both condensates, shown in Fig. 1b as the halo. For a medium number density n_M, the loss rate out of the laser condensate is $n_M v_0 \sigma$. Consequently, at resonance, where $\sigma \approx 8\pi/(k_0/2)^2$, the fraction of atoms remaining in the laser condensate after the collision is $\approx \exp\left(-8\pi n_M \ell_M/(k_0/2)^2\right)$. For typical values of n_M and

ℓ_{M} almost all the laser atoms are lost at resonance.

For our proposal we need to minimize these losses. We can use the lossless point where $f(z) = 0$, indicated in Fig. 2, and the total cross section is zero. The effective mass and the potentials in Eq. (9) are then real. In fact, $m^*(\mathbf{x}) > m$ and $df/dz = a_{\mathrm{bg}}/\Gamma_0$.

At the lossless point the simplest case to analyse is that of a homogeneous medium and $V(\mathbf{x}) = 0$. The potential $V_{\mathrm{mf}}(\mathbf{x})$ vanishes and the effective mass is uniform. Transforming Eq. (9) to momentum space, we find that the propagation or group velocity of the laser BEC is

$$\mathbf{v}_{\mathrm{g}}(\mathbf{k}_0)\Big|_{\mathrm{loss\,less}} = \frac{\hbar \mathbf{k}_0}{m^*} = \mathbf{v}_0 \left[\frac{1+\beta/2}{1+\beta} \right], \qquad (10)$$

where the dimensionless quantity $\beta = U_{\mathrm{bg}}/\Gamma_0 > 0$ and $U_{\mathrm{bg}} = (8\pi\hbar^2/m)a_{\mathrm{bg}}n_{\mathrm{M}}$ is the background mean-field interaction energy. The inset of Fig. 3 shows the group velocity as a function of β. The group velocity lies between $\mathbf{v}_0/2$ and $\mathbf{v}_0$ and approaches $\mathbf{v}_0/2$ when $\beta \to \infty$.

We now turn to propagation through an inhomogeneous medium, but still with $V(\mathbf{x}) = 0$. The assumption $\ell_{\mathrm{L}} \ll \ell_{\mathrm{M}}$ implies that the density variation of the medium orthogonal to the laser propagation direction is negligible and we only need to treat propagation along $\mathbf{k}_0$ passing through the center of the medium BEC.

For simplicity the density profile of the untrapped medium is given by $|\Psi_{\mathrm{M}}(\mathbf{x})|^2 = n_{\mathrm{M}}(1 - x^2/\ell_{\mathrm{M}}^2)$, using the Thomas-Fermi approximation and assume that expansion of the medium can be neglected. Here n_{M} is the peak number density and ℓ_{M} is the Thomas-Fermi radius of the medium. We assume that $V_{\mathrm{deriv}}(\mathbf{x}) \ll E_0/2$ for all $\mathbf{x}$ and, hence, can apply the Wentzel-Kramers-Brillouin (WKB) approximation to estimate δ. We find

$$\delta/\ell_{\mathrm{M}} = 2 \left(1 - \frac{\mathrm{arctanh}\left(\sqrt{\beta/(1+\beta)}\right)}{\sqrt{\beta(1+\beta)}} \right), \qquad (11)$$

and the dimensionless quantity $\beta = U_{\mathrm{bg}}/\Gamma_0$ is evaluated at the peak number density n_{M}. Figure 3 shows δ as a function of β. The maximum δ that can be attained by the laser is $2\ell_{\mathrm{M}}$ for $\beta \to \infty$.

There are several constraints on the realization of the proposal. Firstly, we have $\ell_{\mathrm{L}} \ll \ell_{\mathrm{M}}$. Secondly, scattering is s-wave dominated, so that $k_0 a_{\mathrm{bg}} \ll 1$ or $E_0/2 \ll \hbar^2/(2ma_{\mathrm{bg}}^2) \equiv E_{\mathrm{bg}}$, the Wigner threshold limit. Thirdly, by solving for $\mathrm{Im}\, f(z) = 0$ for positive a_{bg}, we find that the requirement $\Gamma_0 < E_{\mathrm{res}}$ must hold. Fourthly, the energy window around the lossless point, where $\mathrm{Im}\, f$ is small, is on the order of $\Gamma(E_{\mathrm{res}})$. Consequently, the energy width of the laser BEC, $\Delta E_L \approx \hbar^2 k_0/(2m\ell_{\mathrm{L}})$, must satisfy $\Delta E_L \ll \Gamma(E_{\mathrm{res}})$. In other words $\ell_{\mathrm{L}} \gg \hbar^2/(ma_{\mathrm{bg}}\Gamma_0) \equiv \ell_{\mathrm{L}}^{\mathrm{min}}$. Finally, we require resonances for

which δ is comparable or larger than the size of the laser BEC. Since $\ell_{\mathrm{L}} \ll \ell_{\mathrm{M}}$, we have $\beta = U_{\mathrm{bg}}/\Gamma_0$ is at least of order one.

Table I gives a non-exhaustive list of narrow resonances, which satisfy the constraints. For four of these

TABLE I: Resonance parameters, experimental constraints, and spatial delay for nine Feshbach resonances. The first five columns specify the Feshbach resonance. The columns are the atomic species, magnetic resonance position B_0, magnetic width Δ, reduced width Γ_0, and Wigner-threshold limit E_{bg}. The sixth column gives the minimum size, $\ell_{\mathrm{L}}^{\mathrm{min}}$, of the laser BEC. The last column is the shift δ in units of the radius of the medium ℓ_{M}, assuming a peak medium density of $n_{\mathrm{M}} = 10^{15}$ cm^{-3}. Parameters obtained from [12].

Atom	B_0 (mT)	Δ (mT)	Γ_0/k_{B} (μK)	$E_{\mathrm{bg}}/k_{\mathrm{B}}$ (μK)	$\ell_{\mathrm{L}}^{\mathrm{min}}$ (μm)	δ/ℓ_{M}
^{23}Na	119.5	-0.14	14	1900	0.45	0.15
''	90.7	0.10	260	1900	0.025	0.0091
''	85.3	2.5×10^{-4}	0.64	1900	9.8	1.20
^{87}Rb	100.74	0.021	39	200	0.027	0.025
''	91.17	1.3×10^{-4}	0.24	200	4.4	1.25
''	68.54	6×10^{-4}	0.54	200	1.9	0.89
''	40.62	4×10^{-5}	0.054	200	19	1.7
''	9.13	1.5×10^{-3}	2.0	200	0.52	0.38
^{52}Cr	49.99	0.008	22	290	0.076	0.078

resonances the expected δ is shown in Fig. 3 assuming a peak density of $n_{\mathrm{M}} = 10^{15}$ cm^{-3}. If we assume $\ell_{\mathrm{L}}/\ell_{\mathrm{M}} \approx 0.1$ then δ ranges from $0.1\ell_{\mathrm{L}}$ to $20\ell_{\mathrm{L}}$ for the resonances in Table I. For the selected density the chromium resonance is only a marginal candidate for slowing experiments.

In conclusion we have shown that collisions in the presence of a magnetic Feshbach resonance can lead to slowing of a laser BEC as it propagates through a large medium BEC. The slowing is a consequence of the collision-energy dependence of the scattering amplitude near the resonance. Based on a generalized Gross-Pitaevskii equation, we predict a maximal reduction of the group velocity by a factor of two and suggest that the experiment be performed at a magnetic field where the elastic scattering is zero. Such a field always exists near a magnetic Feshbach resonance. For finite-sized condensates slowing can be observed by measuring the spatial delay of the laser BEC, which can not exceed twice the Thomas-Fermi radius of the medium. We show that for narrow resonances this signal is expected to be measurable.

[1] M.-O. Mewes, M. R. Andrews, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **78**, 582 (1997).

[2] E. W. Hagley, L. Deng, M. Kozuma, J. Wen, K. Helmerson, S. L. Rolston, and W. D. Phillips, Science **283**, 1706 (1999).

[3] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **275**, 637 (1997).

[4] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, Rev. Mod. Phys. **81**, 1051 (2009).

[5] J. Baudon, M. Hamamda, J. Grucker, M. Boustimi, F. Perales, G. Dutier, and M. Ducloy, Phys. Rev. Lett. **102**, 140403 (2009).

[6] L. Deng, E. W. Hagley, J. Wen, M. Trippenbach, Y. Band, P. S. Julienne, J. E. Simsarian, K. Helmerson, S. L. Rolston, and W. D. Phillips, Nature **398**, 218 (1999).

[7] K. V. Kheruntsyan, J.-C. Jaskula, P. Deuar, M. Bonneau, G. B. Partridge, J. Ruaudel, R. Lopes, D. Boiron, and C. I. Westbrook, Phys. Rev. Lett. **108**, 260401 (2012).

[8] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).

[9] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature **397**, 594 (1999).

[10] N. S. Ginsberg, S. R. Garner, and L. V. Hau, Nature **445**, 623 (2007).

[11] M. M. Kash, V. A. Sautenkov, A. S. Zibrov, L. Hollberg, G. R. Welch, M. D. Lukin, Y. Rostovtsev, E. S. Fry, and M. O. Scully, Phys. Rev. Lett. **82**, 5229 (1999).

[12] C. Chin, R. Grimm, P. Julienne, and E. Tiesinga, Rev. Mod. Phys. **82**, 1225 (2010).

[13] M. Inguscio, W. Ketterle, and C. Salomon, eds., *Ultracold Fermi Gases* (IOS Press, Amsterdam, 2008), Proceedings of the International School of Physics "Enrico Fermi", Course CLXIV, Varenna, 20-30 June 2006.

[14] S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008).

[15] T. Weber, J. Herbig, M. Mark, H.-C. Nägerl, and R. Grimm, Science **299**, 232 (2003).

[16] S. L. Cornish, N. R. Claussen, J. L. Roberts, E. A. Cornell, and C. E. Wieman, Phys. Rev. Lett. **85**, 1795 (2000).

[17] G. Roati, M. Zaccanti, C. D'Errico, J. Catani, M. Modugno, A. Simoni, M. Inguscio, and G. Modugno, Phys. Rev. Lett. **99**, 010403 (2007).

[18] L. Mathey, E. Tiesinga, P. S. Julienne, and C. W. Clark, Phys. Rev. A **80**, 030702 (2009).

[19] R. Ciuryło, E. Tiesinga, and P. S. Julienne, Phys. Rev. A p. 030701 (2005).

[20] S. Blatt, T. L. Nicholson, B. J. Bloom, J. R. Williams, J. W. Thomsen, P. S. Julienne, and J. Ye, Phys. Rev. Lett. **107**, 073202 (2011).

[21] M. Yan, B. J. DeSalvo, B. Ramachandhran, H. Pu, and T. C. Killian, arXiv:0906.1837 (2009).

[22] A. P. Chikkatur, A. Görlitz, D. M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle, Phys. Rev. Lett. **85**, 483 (2000).

[23] C. Buggle, J. Léonard, W. von Klitzing, and J. T. M. Walraven, Phys. Rev. Lett. **93**, 173202 (2004).

[24] N. R. Thomas, N. Kjærgaard, P. S. Julienne, and A. C. Wilson, Phys. Rev. Lett. **93**, 173201 (2004).

[25] J. R. Taylor, *Scattering Theory: The Quantum Theory of Nonrelativistic Collisions* (Dover Publications, 2006).

[26] T. Köhler, K. Góral, and P. S. Julienne, Rev. Mod. Phys. **78**, 1311 (2006).

[27] T. Köhler and K. Burnett, Phys. Rev. A **65**, 033601 (2002).