

Two-port microwave calibration at millikelvin temperatures

Leonardo Ranzani, Lafe Spietz, Zoya Popovic, and José Aumentado

Citation: [Review of Scientific Instruments](#) **84**, 034704 (2013); doi: 10.1063/1.4794910

View online: <http://dx.doi.org/10.1063/1.4794910>

View Table of Contents: <http://scitation.aip.org/content/aip/journal/rsi/84/3?ver=pdfcov>

Published by the [AIP Publishing](#)

Articles you may be interested in

[An interferometric scanning microwave microscope and calibration method for sub-fF microwave measurements](#)
Rev. Sci. Instrum. **84**, 123705 (2013); 10.1063/1.4848995

[In situ broadband cryogenic calibration for two-port superconducting microwave resonators](#)
Rev. Sci. Instrum. **84**, 034706 (2013); 10.1063/1.4797461

[Calibration of shielded microwave probes using bulk dielectrics](#)
Appl. Phys. Lett. **93**, 123105 (2008); 10.1063/1.2990638

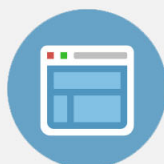
[Calibration of a helical resonator for microwave dielectric and conductivity measurements of metals](#)
Rev. Sci. Instrum. **72**, 1760 (2001); 10.1063/1.1347379

[Broadband calibration of long lossy microwave transmission lines at cryogenic temperatures using nichrome films](#)
Rev. Sci. Instrum. **71**, 4596 (2000); 10.1063/1.1322577



Re-register for Table of Content Alerts

Create a profile.



Sign up today!



Two-port microwave calibration at millikelvin temperatures

Leonardo Ranzani,^{1,a)} Lafe Spietz,¹ Zoya Popovic,² and José Aumentado^{1,b)}

¹National Institute of Standards and Technology, Boulder, Colorado 80305, USA

²University of Colorado at Boulder, Boulder, Colorado 80309, USA

(Received 18 January 2013; accepted 25 February 2013; published online 15 March 2013)

In this work we introduce a system for 2-port microwave calibration at millikelvin temperatures operating at the coldest stage of a dilution refrigerator by use of an adapted thru-reflect-line algorithm. We show that this can be an effective tool for characterizing common 50 Ω microwave components with better than 0.1 dB accuracy at temperatures that are relevant to many current experiments in superconducting quantum information. © 2013 American Institute of Physics. [<http://dx.doi.org/10.1063/1.4794910>]

I. INTRODUCTION

In the field of superconducting quantum information, microwave quantum circuits are measured at millikelvin temperatures, usually placed at the coldest stage of a helium dilution refrigerator.^{1–8} Although these devices operate only at such low temperatures, all the ancillary microwave components (directional couplers, circulators, isolators, amplifiers, attenuators, etc.) are *always* designed for use at higher temperatures, typically 10–300 K. Usually, one is forced to rely on room temperature characterization data to infer the component low-temperature behavior. When a component is cooled to millikelvin temperatures, however, its scattering parameters (S-parameters) are likely to change,⁹ and for accurate measurements and error correction, the cryogenic S-parameters should be determined. If we wish to accurately characterize a component at these temperatures we must implement a calibration scheme that places the reference plane of the measurement at the DUT (Device Under Test), thus *de-embedding* the intervening passive and active components up to the ports of the room temperature vector network analyzer (VNA). In this work, we describe a setup that allows accurate calibration at both room and cryogenic temperatures by use of the Thru-Reflect-Line (TRL) calibration procedure^{10–13} and demonstrate its use by calibrating the measurement of 50 Ω passive microwave components.

Several calibration procedures^{10,14,15} that employ a set of well-known standards can be used to characterize the microwave response of the elements between the VNA and the DUT from the measurement, thus allowing an accurate measurement of the DUT alone. In the simplest one-port calibration procedure (“Short-Open-Load,” or SOL), three well-known calibration impedance standards are needed. In a two-port calibration, Short-Open-Load-Thru (SOLT) and TRL, multiple standards are required, resulting in an overdetermined set of equations for the error coefficients. Since multiple standards are needed, it has always been difficult to perform an accurate microwave calibration in research cryogenic systems operating at *millikelvin* temperatures, which have traditionally had limited space and multiple

cascaded microwave components. Compared to simpler room temperature measurements that can usually employ short transmission lines, these cryogenic measurements necessitate the integration of several stages of cryogenic attenuators, couplers, isolators and amplifiers, as well as long transmission lines at both room temperature and low temperature. We contrast this with other cryogenic microwave measurements at higher temperatures $\gtrsim 4$ K where the room temperature-to-low temperature circuit is usually much simpler.^{9,16} Because of the unique complications of measuring in a dilution refrigerator, it has been difficult to characterize devices whose behavior is especially interesting at millikelvin temperatures, i.e., SQUID (Superconducting Quantum Interference Device) amplifiers, parametric amplifiers, electrical and mechanical resonators, etc., and calibration has usually been limited to one-port calibrations^{4,17} or to a simple, but less accurate, “response” calibration.^{3,13,18} The latter refers the transmission scattering parameter S_{21} of the DUT to a reference transmission line (which is easily implemented by use of a transfer switch). Although it is possible to perform a multiple-standard calibration in these systems by swapping out the standards at the cold stage, it can mean a tedious thermal cycling of the entire system to swap out standards one at a time—a process that can take many days (depending on the complexity of the cryogenic system) during which the measurement chain may drift unpredictably. Nowadays, however, there is ample room at the coldest stages of many of the newer “dry” dilution refrigerators which utilize pulse-tube cooling in place of liquid cryogens and it is now possible to place relatively large microwave components at millikelvin temperatures. This in turn enables improved calibration techniques, since microwave switches can be placed inside the coldest stage of the refrigerator to select various standards *in situ*. To the best of our knowledge, this paper demonstrates for the first time an accurate 1–14 GHz full 2-port microwave measurement system at millikelvin temperatures, enabled by multiple switched cryogenic standards.

II. MEASUREMENT SETUP

The experimental setup is shown in Figure 1. The dilution refrigerator is divided into 4 stages at decreasing temperatures (~ 300 K, ~ 4 K, ~ 1 K, and ≤ 20 mK). In this scheme

a)leonardo.ranzani@colorado.edu.

b)jose.aumentado@nist.gov.

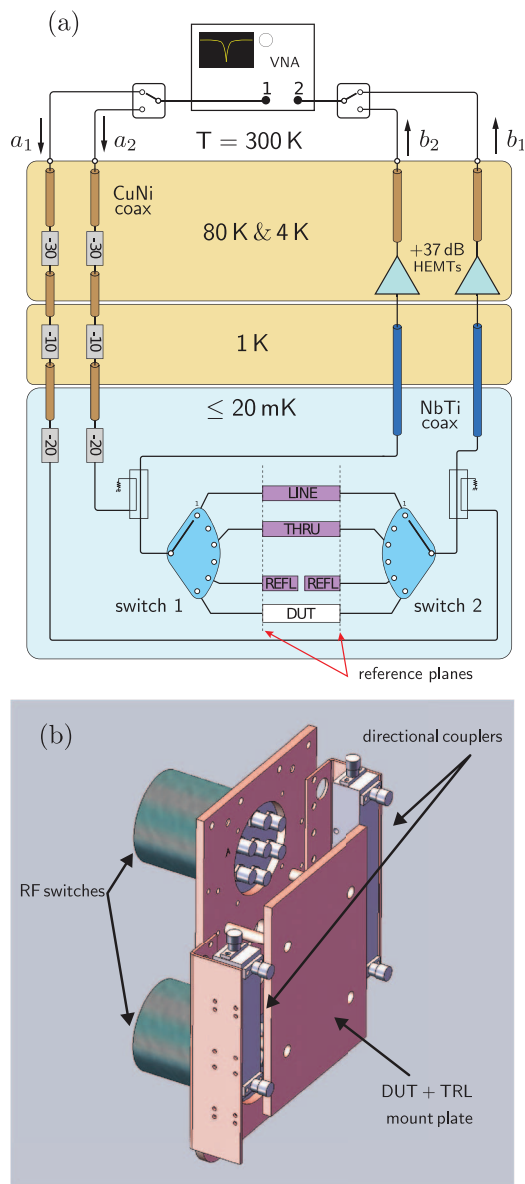


FIG. 1. (a) Schematic of the cryogenic S-parameter measurement circuit. The signals a_1 and a_2 travel down to the RF switches A and B, located at the mixing chamber ($T \leq 20$ mK) where they are then routed to a set of calibration standards and DUTs. The reflected and transmitted signals, b_1 and b_2 are routed through a HEMT amplifier at 4 K and a subsequent post-amplifier mounted at room temperature. A calibration algorithm is then used to de-embed the entire measurement setup and extract the DUT scattering parameters. (b) Detailed view of the microwave switch plate assembly.

we measure all four S-parameters via two input and two output paths which connect the DUT at the mixing chamber ($T \lesssim 20$ mK, i.e., the coldest stage in our system). The total loss of each input path was about 70 dB at 5 GHz from the top of the refrigerator (room temperature) to the mixing chamber. This number includes cable loss and attenuators which are typically used to thermalize black body radiation to the intervening cold stages. The input signal is coupled into the RF switches via 20 dB couplers with 22 dB directivity from 2 to 26 GHz. The output microwave signals from the couplers are connected to wideband (2 to 15 GHz) 37-dB gain high electron mobility transistor (HEMT) amplifiers anchored to the 4 K stage. These amplifiers are followed by room tempera-

ture amplifiers with 36 dB gain. The total output gain, including cable loss, was about 67 dB. The RF 6-port switches (dc to 18 GHz bandwidth) are commercial mechanical pulse-latched switches, driven by 125 mA current pulses of 10–15 ms duration. We actuated the switch solenoids using a commercial switch controller that is directed over GPIB (General Purpose Interface Bus).

The two input paths and the two output paths are connected to a two-port commercial vector network analyzer (VNA) through two SPDT (single pole, double throw) switches operating at room temperature Figure 1(a). A detailed view of the switch assembly is shown in Figure 1(b). The calibration standards and devices under test are placed on a copper mounting plate and connected to the RF switches through a set of short (6 in.) commercial SMA coaxial cables in an effort to make each switch position as identical as possible up to the chosen reference plane. The measurement bandwidth is limited by the cryogenic HEMTs to 1.5 GHz $< f < 15$ GHz. Although the circuit shown in Figure 1(a) does not include isolators, they can be inserted without any modification to the calibration algorithm, and we have operated this circuit equally well with and without isolators inserted into the output amplification chains. This is an important detail for measurements of quantum circuits where excess noise propagating back from the cryogenic HEMT amplifiers can be an issue and several stages of isolation can be necessary.

Practical switch details. The solenoids inside the RF switches were comprised of fine copper wire with a small, but finite, resistance and therefore generated Joule heating when driven even for the few milliseconds required to latch the switch positions. In our cryogenic system, the cooling power is only $10 \mu\text{W}$ at 20 mK, so any heating from switching must be minimized to keep the dilution refrigerator operation stable and the overall calibration procedure relatively short. Since the solenoid resistances dropped by over a factor of 10 to just a few ohms (when cooling from room temperature to low temperature), this heating was small enough that the initial heat pulse during a switch would raise the mixing chamber temperature to ~ 40 mK and cool back below 20 mK in ~ 15 min. In order to switch between standards, a total of 4 control current pulses must be sent to the switches (1 open/close cycle for each switch). Therefore, the typical measurement period for the calibrations was of order 4 h when limiting the mixing chamber temperature rise to < 40 mK.¹⁹ Because only three positions are required on each switch to perform the calibration, there were three additional ports that could be used for multiple DUTs that can all be de-embedded within the same cooldown. We note that there are even SP10T switches available that use the same pulse-latched mechanism and, in principle, provide up to 7 DUT positions available for full 2-port S-parameter calibrations.

A. Calibration algorithm

The experimental setup that we show in Fig. 1(a) can be used to perform a calibrated 2-port microwave measurement with any desired set of standards and is limited only by the number of available low temperature switch ports. However, we selected the TRL calibration procedure for a few

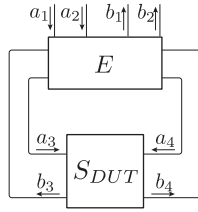


FIG. 2. Error model of the measurement system. The $b_{1,2}$ and $a_{1,2}$ waves (going in and out of the VNA) correspond to the labelling in our schematic circuit in Figure 1(a). The error matrix E encompasses the effects of intervening cryogenic circuitry which is de-embedded in the calibration.

particular reasons. First the TRL calibration does not require a well-known load and is therefore easier to implement at low temperatures where it may be difficult to provide a reliable load impedance standard. Second, one can produce custom on-chip calibration standards that can be co-fabricated with the device under test, effectively moving the calibration reference plane to the DUT, which may be a small component embedded within more complicated on-chip matching circuitry. Our measurement scheme employs two input paths, a_1 and a_2 (from VNA port 1) and two output paths, b_1 and b_2 (returning into VNA port 2). We assume that both the input to output paths are unidirectional. In our setup, this is ensured by the attenuators in the input paths and the unilateral nature of the amplifiers in the output paths. Under this assumption, we can describe the transformation of the input and output waves into the waves scattered by the DUT as (see Figure 2),

$$\begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix} = E \begin{bmatrix} b_3 \\ b_4 \\ a_3 \\ a_4 \end{bmatrix}, \quad (1)$$

where E is the matrix of error coefficients and $a_{3,4}$ and $b_{3,4}$ correspond to the input/output waves scattered directly at the DUT.

If we neglect leakage from the input to the output networks, E can be simplified to

$$E = \begin{bmatrix} E_{11} & 0 & E_{13} & 0 \\ 0 & E_{22} & 0 & E_{24} \\ E_{31} & 0 & E_{33} & 0 \\ 0 & E_{42} & 0 & E_{44} \end{bmatrix}. \quad (2)$$

The 8 unknown elements of E constitute an 8-term error model that has to be determined from the measurements of the calibration standards.²⁰ In any general calibration procedure, a sufficient number of calibration standards N must be measured to fix the values of these error terms. In this case, the unknown elements of E are not independent and can be determined by N (usually much less than 8) standards. The standards will yield a set of measured S-parameter matrices:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = S_m^i \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad \text{for } i = 1, \dots, N. \quad (3)$$

Knowing the standard S-parameter actual equivalent matrices, S_a^i , and the measured data, S_m^i , the error model unknown parameters can be calculated. The TRL calibration

procedure is a good choice for cryogenic applications because it has the ability to correct for imperfections in the (sometimes custom) calibration standards¹⁰ in addition to being accurate. In the case of TRL, the *actual* S-parameters for the thru, reflect, and line standards are:

$$\begin{aligned} S_a^T &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ S_a^L &= \begin{bmatrix} 0 & e^{-\theta} \\ e^{-\theta} & 0 \end{bmatrix} \\ S_a^R &= \begin{bmatrix} \Gamma & 0 \\ 0 & \Gamma \end{bmatrix}. \end{aligned} \quad (4)$$

The line phase delay θ and the reflect standard reflection coefficient Γ are known only approximately and may depend on the temperature, dielectric constant, and dispersion characteristics. The reflection coefficient Γ , ideally equal to 1 for an open standard, depends on the parasitic capacitance and radiation from the termination of the reflect standard. An advantage of this calibration procedure is that θ and Γ do not need to be known *a priori*, although they are precisely determined in the calibration process. The 8-term error model and the two unknown parameters θ and Γ comprise a set of ten unknowns. We applied a weighted least squares error algorithm to solve this problem. This strategy has already been demonstrated to be both robust and accurate.^{12,20} First, from the measurements $S_m^{T,R,L}$, we can use the relations in Eqs. (2)–(4) to build a linear system of equations in the eight-element error-term basis, i.e.,

$$A(\Gamma, \theta) \mathbf{t} = \mathbf{0}, \quad (5)$$

where the vector $\mathbf{t}$ is defined as

$$\mathbf{t} \equiv \begin{bmatrix} E_{11} \\ E_{22} \\ E_{13} \\ E_{24} \\ E_{31} \\ E_{42} \\ E_{33} \\ E_{44} \end{bmatrix}. \quad (6)$$

The system matrix $A(\Gamma, \theta)$ depends on both the measured and actual S-parameters, $S_m^{T,R,L}$ and $S_a^{T,R,L}$, encapsulating the behavior of the unknown Γ and θ (the detailed expression²⁰ is reported in the Appendix), while the vector $\mathbf{t}$ (or, equivalently, the matrix E) captures the scattering of the elements that we would like to de-embed. Taken together, Eq. (5) constitutes a system of 10 equations, which depend linearly on $\mathbf{t}$ and non-linearly on Γ and θ . The solution to the calibration problem is therefore one that minimizes the following error function:

$$\epsilon = \mathbf{t}^\dagger A^\dagger C^{-1} A \mathbf{t}. \quad (7)$$

Here $A^\dagger$ indicates the Hermitian conjugate of A , while $C = \text{Cov}(A\mathbf{t})$ is the covariance matrix of the residuals of (5). The matrix C can be computed²⁰ knowing the covariance

matrix of the data $S_{meas}^{T,R,L}$. Here we assume that $\text{Cov}(S_{meas})$ is diagonal and $\langle S_{meas}^2 \rangle = \langle S_{meas} \rangle^2$. This means that the measured data are uncorrelated and characterized by the same signal-to-noise ratio. To solve the calibration problem, we searched for the minimum of the error function ϵ with respect to Γ , θ , and $\mathbf{t}$.

ϵ is quadratic in the elements of $\mathbf{t}$; therefore the problem can be split into two parts. First, with Γ and θ fixed, we searched for the minimum of ϵ with respect to $\mathbf{t}$ by performing a QR decomposition of a reduced system matrix T_R (see the Appendix). We then proceeded by iteration over Γ and θ using a constrained interior-point algorithm.²¹ The covariance matrix C is estimated at each step n by using the result from the previous step $\mathbf{t}_{n-1}$. We constrained the optimal solution for Γ and θ to $-1 < \text{Im}\Gamma < 1$ and $0 < \text{Re}\Gamma < 1$ to assure the passivity of the reflect standard and guarantee uniqueness of the solution.¹⁰ The above constraints are valid if the reflect standard is an open. One can also use a short standard, in which case, the constraint should be changed to $-1 < \text{Re}\Gamma < 0$. The initial conditions for the algorithm were set to $\Gamma_0 = 1$, θ_0 equal to the design value for the line standard and the covariance matrix to $C_0 = I_{10}$ (the 10×10 identity matrix). Once the error coefficients are determined, the actual S-parameters of the DUT can be calculated by inverting Eq. (1). This procedure is called de-embedding and it is detailed in the Appendix. It is worth noting that the algorithm we chose to employ is general enough to be applied to different calibration procedures other than TRL, or extended by adding more calibration standards for better accuracy. This can be done by modifying the actual standard S-parameter matrices in Eq. (4) and following the same procedure outlined above.

B. Calibration error sources

The calibration algorithm described above is based on the assumption that the calibration standards and DUT fixtures are identical up to the calibration reference plane. The cold RF switch ports and the cables connecting the switches to the standards and devices need to be as identical as possible. Any variation between different switch ports or connections is equivalent to an error in the standards or device fixtures and cannot be fully corrected. The rest of the measurement system (from the room temperature VNA down to the low temperature RF switches) is accounted for in the error model (Figure 2) and can be calibrated. However, random drifts in the measurement system (before and after the RF switches) can also contribute to the total measurement error over extended measurement periods and need to be considered carefully. In summary our, calibration errors are due to four main contributions:

1. Variations in the microwave path connecting the low temperature RF switches and the devices (cables, connectors, device fixtures). These paths are assumed to be identical by the calibration algorithm described here.
2. Variations between the switch ports (different transmission or return loss intrinsic to the switches).
3. Drifts due to changes in the measurement system during the calibration process or after the calibration is performed.

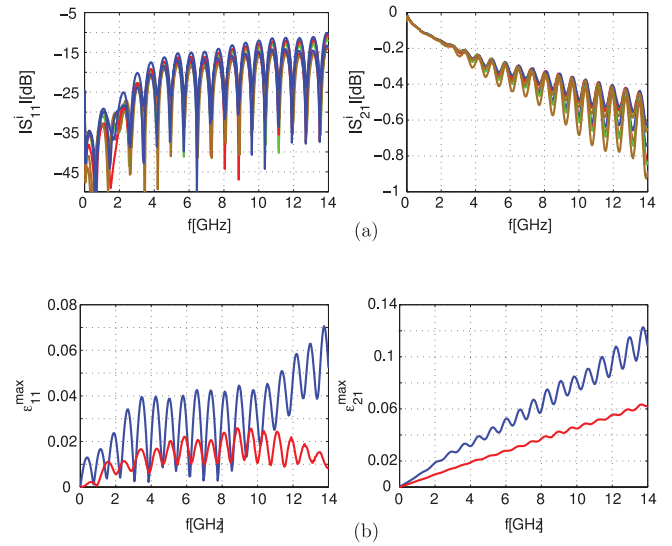


FIG. 3. 6-port switch cable symmetry. We show the (a) measured return and transmission loss for 6 nominally identical short (6 in.) commercial M/M (male/male) subminiature microwave cables (SMA) attached to one switch and (b) the maximum variation between the curves in (a). The variations in (b) are defined as $\epsilon_{hk}^{\max} = \max_{i,j=1\dots 6} (|S_{hk}^i - S_{hk}^j|)$, where i varies across the different cables and is plotted on a linear scale. The red and blue curves correspond to the separate switches.

4. Electronic noise of the VNA and the amplifying stages.

To reduce the impact of 1, we chose cables connecting the RF switches to the devices to be as identical as possible, in order to achieve small and uniform return and transmission losses. The cables were 6 in.-long standard hand-formable SMA cables covered by an aluminum jacket. The measured data for the set of cables used are plotted in Figure 3(a). The cables have similar characteristics and, as expected, the standing wave in Figure 3 corresponds to the length of the cables. The maximum return loss was 10 dB at 20 GHz with a relative variation of 1.5 dB at most. The red and blue lines in Figure 3(b) represent the maximum variation across the 6 cables attached to the first and second 6-port switch, respectively. The impact of the cable variations on the calibration depends on the affected path (DUT or standards). The worst case is when only the DUT path is affected, because the errors affect the calibration coefficients directly. In this case, to first order¹³ we can expect the following correction to the DUT S-parameters:

$$\Delta S_{21}^{\text{DUT}} = S_{21}^{\text{DUT}} (1 \pm \epsilon_{21}^{\max}), \quad (8)$$

$$\Delta S_{11}^{\text{DUT}} = S_{11}^{\text{DUT}} \pm \epsilon_{11}^{\max}. \quad (9)$$

Therefore the return loss error $\epsilon_{11}^{\max} = \max_{i,j=1\dots 6} (|S_{11}^i - S_{11}^j|)$ approximately introduces an uncalibrated extra reflection in the DUT, while the transmission error $\epsilon_{21}^{\max}$ introduces an extra loss (or gain) to the DUT transmission coefficient. For example, using the data in Figure 3(a), a DUT having 20 dB return loss at 8 GHz would result in a measured, de-embedded value between 9 and 11 dB (0.32 ± 0.04). Variations along the standard paths

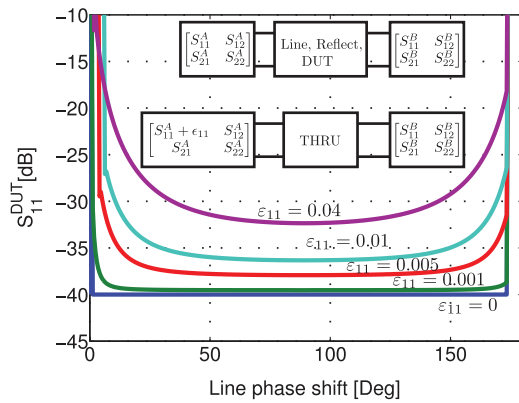


FIG. 4. Simulation of DUT input return loss error caused by a variation in the thru calibration standard path as a function of the phase difference between the line and the thru standards. In the simulation the thru had zero length. The DUT is a 20 dB attenuator with 40 dB return loss. The input and output error matrices had unit transmission and 20 dB return loss. The input return loss of the error matrix in front of the thru standard was modified by ϵ_{11} to simulate a systematic error in the thru input path.

have a smaller impact in general, because they are partially compensated by the fact that the error is convolved among the three standards. Return loss errors have the highest impact on amplitude measurements, while transmission errors impact the phase and the accuracy of the calibration reference plane.²² An example of the impact of a uniform error in the thru standard transmission on a typical device measurement is shown in Figure 4, as obtained from numerical simulation. In the simulation we assume that the error matrix in front of the standards and DUT is characterized by a 20 dB input return loss and 0 dB transmission loss. The DUT is an attenuator with 20 dB transmission loss and 40 dB input and output return loss. We modified the input return loss of the error matrix in front of the thru standard only, to simulate a systematic error in the cable connecting one cold RF switch to the thru standard. The de-embedded DUT input return loss is plotted for different systematic errors in the thru path as shown. We see from Figure 4 and Eq. (8) that a 3 dB error in the return loss (which is now 23 dB) results in a minimum 7.7 dB error in the de-embedded DUT return loss (from 40 dB to 32.3 dB). The same error in the DUT path would have given a de-embedded return loss of 19 dB (or 21 dB error) because the extra reflections would have been attributed entirely to the DUT. The impact of the error grows when the difference in phase between the line and thru standards approaches 0° and 180° . A common rule of thumb is to limit the operating frequencies so that the phase difference is between 20° and 160° .¹³

It is important to remember that the *variation* between the S-parameters of different electrical paths, *not their absolute value*, limits the calibration accuracy. Nevertheless, cables characterized by low return and transmission loss are typically characterized by better uniformity and are therefore preferable. Variations in the cable length also affect the measurement accuracy. The measured maximum phase error is shown in Figure 5. The red and blue curves correspond to the maximum phase error across the cables attached to the first and second 6-port switches in Figure 1. This phase

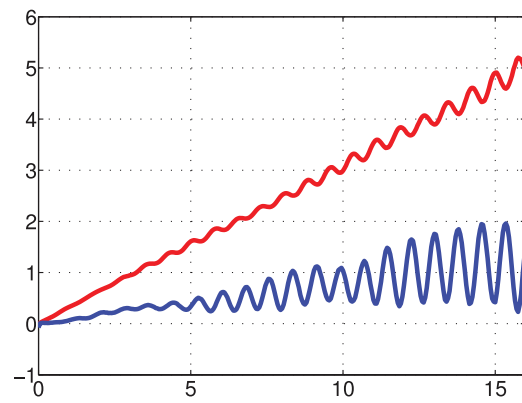


FIG. 5. Measured maximum phase error between the cables connecting the two 6-port switches to the device and calibration standards. The maximum phase error is plotted separately for each switch in red and blue.

error corresponds to ~ 0.2 mm of length deviation for the cables connect to the first switch (red line). In the worst case, when the phase errors in Figure 5 affect only the DUT path, the de-embedded DUT transmission phase will be affected by the sum of the two phase errors and the reflection phase by the phase error at the corresponding port.

The second source of uncertainty in our analysis is created by variations between switch positions within each switch, however these are negligible compared to the variations in the cables connecting the standards and DUT in our setup (see below). The return loss of multiple positions/paths through the common port is plotted for one of the microwave switches in Figure 6. Since it is already ~ 10 dB lower than the cable return loss, we neglect its effect for our calibration error analysis.

An important caveat for the measurement of the switch and test cable symmetry is the fact that these measurements are performed at room temperature and not at the cryogenic temperatures at which we will measure our DUT and standards. In order to simplify the estimate, we make the assumption that these room temperature values are not significantly different when the switches and test cables are cold.

Our third source of uncertainty is given by drift in the measurement chain. By using switches at the coldest stage where the DUT and standards reside, we avoided the need for multiple cool-downs to cycle through the TRL standards

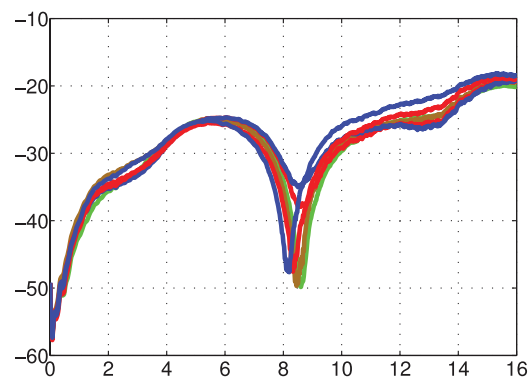


FIG. 6. Return loss measured at the 6-port common port, when switched to each of the 6 positions of a single RF switch.

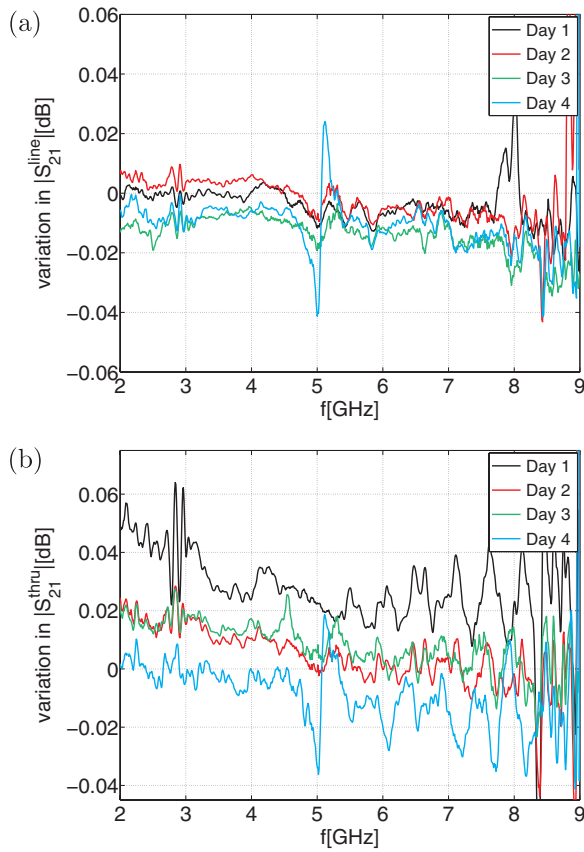


FIG. 7. Time dependence of transmission of the measurement chain and standards. Measured variation in input return loss for the reflect calibration standard (a) and transmission loss for the thru calibration standard (b) over multiple days. The values shown are expressed as a ratio of the corresponding day's S-parameters to those measured on day 0.

and, therefore, any corresponding changes in the measurement chain due to thermal cycling to room temperature. However, there can still be variations in the measurement chain due to small changes in the temperature of the different cold stages, as well as temperature variations in the room since the measurement circuit includes post-amplifiers, cables and the vector network analyzer at room temperature. To assess the stability of the entire measurement circuit we compared the measurements of the standards $S_m^{T,R,L}$ between several calibration runs extending over a multiple days. The difference over one day was typically less than 0.04 dB in magnitude (Figure 7) and up to 1° at 9 GHz in phase (Figure 8). The difference over 5 days was the same in amplitude, but it had a larger phase drift up to 3° at 9 GHz.

Finally the amplifier noise added by the measurement chain can be a challenge, especially when one needs to use a weak signal to probe nonlinear quantum circuits. However, the power necessary to measure the standards can be up to -40 dBm at the input of the standards and, with state-of-the-art HEMT noise temperatures, $T_N \sim 10$ K, the necessary integration period for each standard can be just a few seconds. For these measurements we used a relatively small probe power, equivalent to ~ -65 dBm at the TRL standards. In this system, the time for heating/cooling when pulsing the switch solenoids dominated the calibration cycle period.

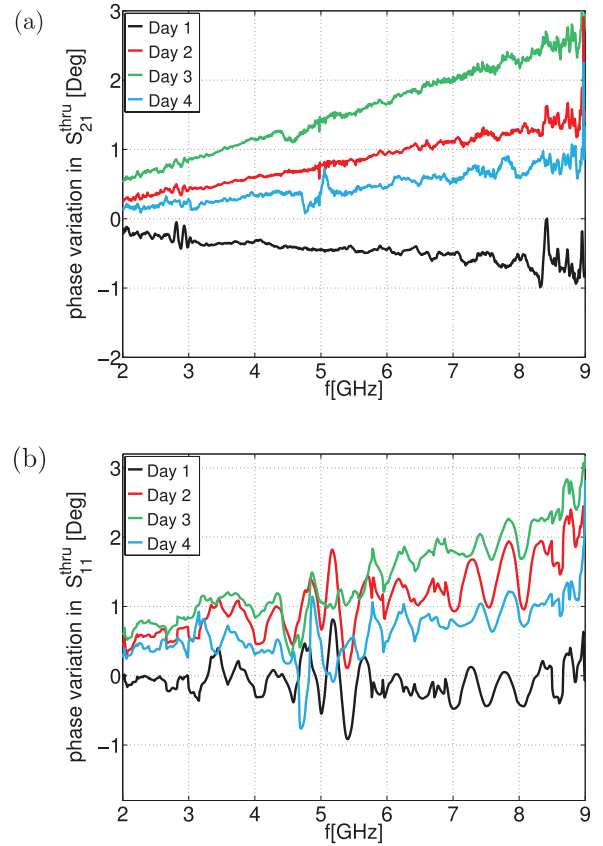


FIG. 8. Time dependence of the phase drift in the measurement chain and standards. Measured phase stability of the thru standard over multiple days. Transmission coefficient (a) and reflection coefficient (b).

III. APPLICATIONS

We tested the calibration system by measuring some devices commonly employed in cryogenic and quantum information experiments. We use a set of $50\ \Omega$ coaxial TRL standards for calibration. The thru and the line standards are sub-miniature (SMA) DC-18 GHz coaxial female/female “barrel” connectors of two different lengths (15 mm and 20 mm), commonly used in microwave labs. For the open standard we use a short barrel terminated into an open cap. Given the choice of calibration standards, the calibration reference plane is set at the output of the through standard (15 mm barrel), where the open is situated. The calibration bandwidth is limited by the range of frequencies where the phase difference between the line and through standards is between 20° and 160° . The line is 5 mm longer than the through and the coaxial cable dielectric is PTFE (polytetrafluoroethylene, $\epsilon_r = 2.1$), and therefore the calibration theoretically works in the 2.3 GHz to 18 GHz band. The upper frequency limit is practically set by the limit of the measurement system (15 GHz, set by the HEMT amplifiers). The reference impedance of the calibration is set by the characteristic impedance of the standards (the thru and the line) and is approximately equal to $50\ \Omega$. The exact value depends on manufacturing tolerances of the barrels and on the temperature. The measured S-parameters could be renormalized to $50\ \Omega$ through an independent comparison with a known load,⁹ although we did not do so here.

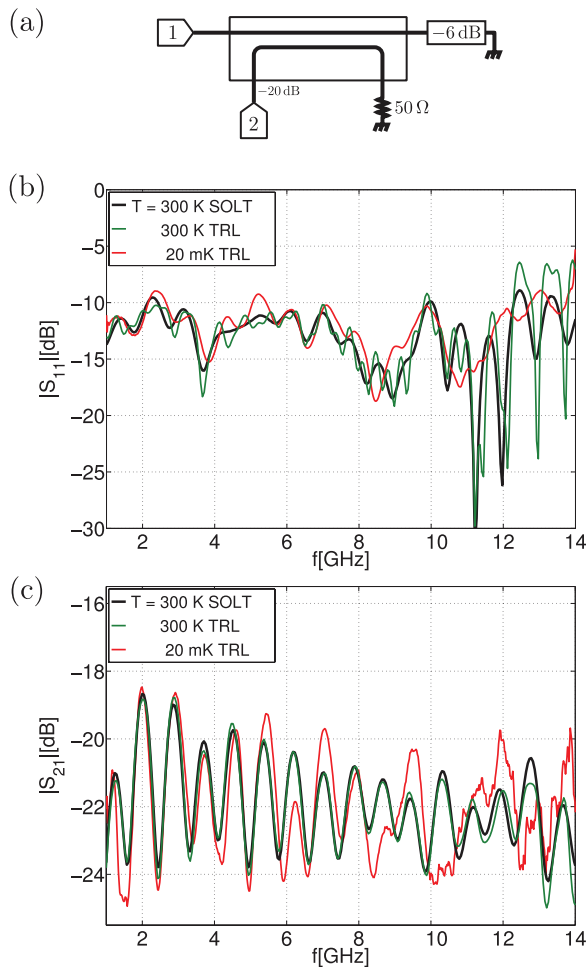


FIG. 9. (a) Directional coupler DUT used to test calibration system. (b) De-embedded return loss and (c) transmission loss. Measurements taken at 20 mK (red) and 300 K (green) using our TRL system mounted on the dilution refrigerator (schematic shown in Figure 1) are compared with standard room temperature automated SOLT calibrated measurements taken at the bench top with short test cables at room temperature (black).

As a first test we measure a simple 20 dB directional coupler (Figure 9(a)) to test the performance of the system. These are often used in reflectometer circuits in quantum information experiments, but are often hand-tuned for optimal room temperature operation²³ and not, of course, for low temperature applications. In this setup, we terminated the isolated port with a shorted 6 dB attenuator. This unusual choice ensured that the return loss was at least 12 dB, above the limit that we could de-embed in our measurement (as discussed in Sec. II B). Otherwise, this type of configuration is typically employed in microwave setups for experiments at millikelvin temperatures (for instance, in our own calibration circuit, Figure 1). We first characterized the device at room temperature with a commercial VNA and an automated SOLT calibration kit from the same manufacturer. The results were later compared with measurements at ~ 20 mK and at 300 K performed using our calibration system as shown in Figure 9. The results at room temperature agree within 0.5 dB from 1 GHz to 12 GHz. These errors are mostly due to variations in the switch connection cables and are within the limits shown in Figure 3(b). By comparing the 300 K and the 20 mK data using our measurement system we can see that at low

temperature the peaks in the DUT transmission and reflection coefficients shift slightly, up to ± 1 dB in S_{21} . Presumably this change corresponds to some physical property of the coupler which is sensitive to the temperature, possibly differential thermal contraction in the materials.

As a second test, we measured a commercial isolator which is explicitly specified as a cryogenic device. These and other related circulators and isolators (which are intended for cryogenic operation at the outset) are often tested only at 77 K and rarely at 4 K (temperatures easily accessible with liquid nitrogen or helium and commercial cryocoolers), whereas these are often operated at much lower temperatures (≤ 50 mK) in dilution refrigerators. Many of these isolators

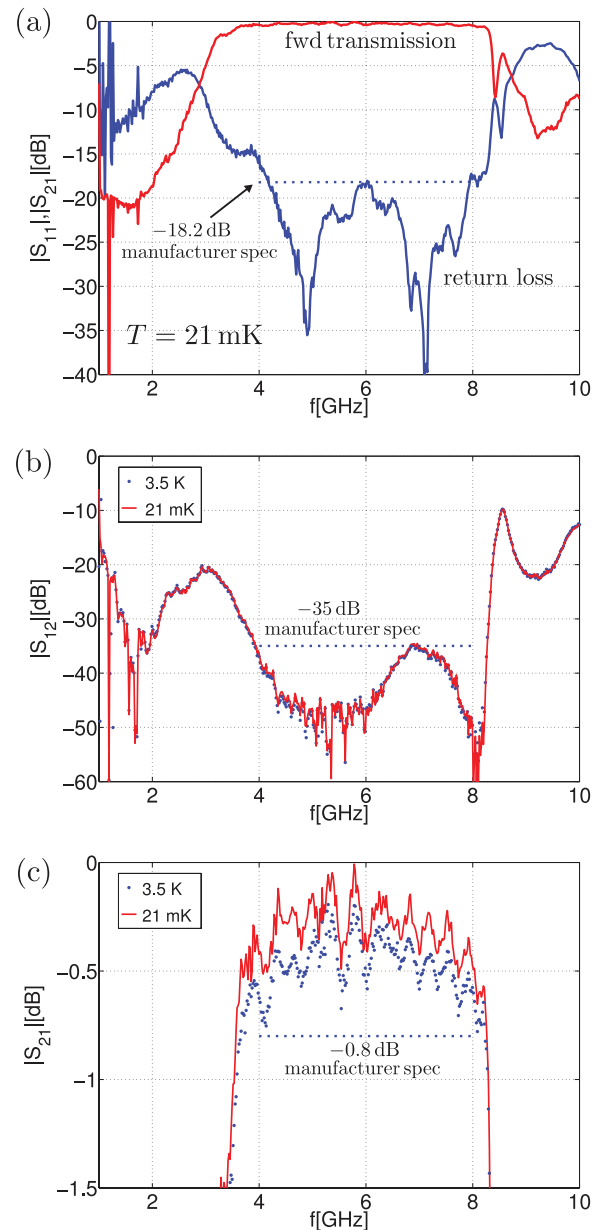


FIG. 10. Calibrated cryogenic measurements of a commercial double-circulator. (a) Calibrated forward transmission (red) and return loss (blue) of a double-circulator configured as an isolator, measured at $T = 21$ mK in our dilution refrigerator setup (Figure 1). We can compare the performance at $T = 3.5$ K and 21 mK for the (b) reverse isolation and (c) forward transmission. This device was specified for performance over the 4–8 GHz band, but only measured by the manufacturer at 77 K.

and circulators, including the one measured here, are encased in easily machinable aluminum alloy (such as 6061) and become superconducting below 1 K. Since these devices require large magnetic field biasing, several rare-earth magnets are integrated into the case and it is not clear whether the Meissner effect will shift the field bias on the ferrite in a way that degrades the performance (e.g., reverse isolation, return loss, operating band, etc.).

This particular circulator is specified to have better than 35 dB isolation and 18 dB return loss in the 4–8 GHz range, when operating at 77 K. We test the isolator in our experimental setup at 21 mK. The results agree with the manufacturer specifications, as shown in Figure 10. These data confirm that the device still operates within its specifications in a low temperature environment. In fact, the transmission loss is better, possibly due to temperature-dependent reduced metal losses which improve at lower temperatures. However, one can see some small difference at 3.5 K (above the aluminum superconducting critical temperature), but it is relatively small (~ 0.2 dB in transmission, Figure 10) indicating that any field modification due to the superconducting state of the case has a minimal impact for quantum information measurements.

IV. CONCLUSIONS

To conclude, we have presented a full 2-port microwave TRL calibration system integrated into a dry dilution refrigerator and discussed the systematic error sources that limit the dynamic range and accuracy. In this system, the dominant source of error is the variation in the test cables connecting the standards and DUT to the switches. Even with these sources of error we have been able to de-embed the intervening cryogenic microwave circuit connecting devices at very low temperatures ($T < 30$ mK) with up to 30 dB return loss and better than 0.1 dB accuracy in transmission measurements up to 10 GHz. In addition, we demonstrated calibrated measurements of a directional coupler and isolator at these low temperatures. While these measurements were performed with a nominal 50 Ω reference impedance, in principle, the calibration can be done with much different impedances with appropriate corresponding standards.²⁴ Finally, although the technique presented here has been applied within a specific cryogenic context, this approach may be valuable for other measurements in similarly extreme environments.

ACKNOWLEDGMENTS

The authors would like to extend thanks to James Booth at NIST Boulder, for his valuable and insightful feedback.

APPENDIX: DETAILS OF THE CALIBRATION PROCEDURE

The TRL calibration algorithm consists of two main parts. First we use measurements of the standards ($S_m^{T,R,L}$) to fit for the unknown parameters in Eq. (5), $\mathbf{t}$ (alternatively E), θ and Γ at every frequency point. Having determined the error matrix we can then proceed to the second part, using E to back out (or “de-embed”) the bare DUT S-parameters.²⁰

The matrix A in Eq. (5), can be calculated using Eqs. (1), (3), and (4). The result is given by

$$A = \begin{bmatrix} A^T & 0 & 0 \\ 0 & A^L & 0 \\ 0 & 0 & A^R \end{bmatrix}, \quad (\text{A1})$$

where

$$A^{T,R,L} = \begin{bmatrix} -S_a^{T,R,L} & I_2 & -S_m^{T,R,L}(S_a^{T,R,L})^t & -S_m^{T,R,L} \\ P_2 S_a^{T,R,L} & P_2 & -P_2 S_m^{T,R,L}(S_a^{T,R,L})^t & -P_2 S_m^{T,R,L} \end{bmatrix}, \quad (\text{A2})$$

where I_2 is the 2×2 identity matrix and P_2 is a 2×2 permutation matrix (the elements on the diagonal are equal to 0, elements off the diagonal are equal to 1). The calibration problem is solved by minimizing the cost function (Eq. (7)) with respect to $\mathbf{t}$, θ , and Γ . This nonlinear least squares problem can be simplified by first noticing that the system (Eq. (5)) is homogeneous. To avoid a trivial solution, we fix the first element of $\mathbf{t}$ to -1 :

$$\mathbf{t} = \begin{bmatrix} -1 \\ \mathbf{t}_r \end{bmatrix}, \quad (\text{A3})$$

so that the system (Eq. (5)) can be reduced to

$$A_r \mathbf{t}_r = \mathbf{b}, \quad (\text{A4})$$

where $A_r = A_{ij}$, $i > 1$ and $\mathbf{b} = A_{1j}$. We can explicitly calculate the covariance matrix C in Eq. (7) under the assumption that $\text{Cov}(S_m)$ is diagonal and $\langle S_m^2 \rangle = \langle S_m \rangle^2$. The result is:

$$C = \begin{bmatrix} C^T & 0 & 0 \\ 0 & C^L & 0 \\ 0 & 0 & C^R \end{bmatrix}, \quad (\text{A5})$$

where $C^{T,R,L} = X^{T,R,L}(X^{T,R,L})^H$ and:

$$X^{T,R,L} = \begin{bmatrix} (S_a^{11}t_5 + t_7)|S_m^{11}|^2 & S_a^{21}|S_m^{12}|^2t_6 & 0 & 0 \\ 0 & 0 & S_a^{12}|S_m^{21}|^2t_5 & (S_a^{22}t_6 + t_8)|S_m^{22}|^2 \\ 0 & 0 & (S_a^{11}t_5 + t_7)|S_m^{21}|^2 & S_a^{21}|S_m^{22}|^2t_6 \\ S_a^{12}|S_m^{11}|^2t_5 & (S_a^{22}t_6 + t_8)|S_m^{12}|^2 & 0 & 0 \end{bmatrix}. \quad (\text{A6})$$

The resulting nonlinear least-squares problem is quadratic in $\mathbf{t}$ and non-quadratic in the remaining parameters. As a result $\mathbf{t}$ can be found by a QR decomposition, while an iterative approach can be used for the remaining parameters. In particular, we compute the Cholesky decomposition of $C = LL^H$, where L is lower triangular and “ H ” indicates the conjugate transpose. Then we compute the QR decomposition of $LA_r = Q_AR_A$, where Q is orthogonal and R is upper triangular. By substituting the resulting decompositions in the cost function (Eq. (7)) and minimizing with respect to $\mathbf{t}$, we can obtain a reduced error function in the remaining parameters Γ and θ :

$$\epsilon_R(\theta, \Gamma) = \min_{\mathbf{t}} \epsilon = (LA_r \mathbf{t}_{R0} - L\mathbf{b})^H (LA_r \mathbf{t}_{R0} - L\mathbf{b}), \quad (\text{A7})$$

where $\mathbf{t}_{r0}$ is given by²⁵

$$\mathbf{t}_{r0} = R_A^{-1} Q_A^H L\mathbf{b}. \quad (\text{A8})$$

We employed a constrained interior-point optimization algorithm to minimize the cost function (Eq. (A7)).²¹ We constrained Γ as follows:

$$0 < \text{Re } \Gamma < 1, \quad (\text{A9})$$

$$-1 < \text{Im } \Gamma < 1. \quad (\text{A10})$$

The first condition corresponds to an open reflect standard. For a short standard the condition $-1 < \text{Re } \Gamma < 0$ should be used instead. The second condition guarantees passivity.

After the optimum values of $\mathbf{t}$ are known, the DUT S-parameters can be calculated by de-embedding the equivalent ABCD matrices T_1 and T_2 connecting the DUT to the measurement instrument. We can refer to Figure 2 and write the matrices as

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = T_1 \begin{bmatrix} a_3 \\ b_3 \end{bmatrix}, \quad (\text{A11})$$

$$\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = T_2 \begin{bmatrix} a_4 \\ b_4 \end{bmatrix}, \quad (\text{A12})$$

where T_1 and T_2 are suitable sub-matrices of E from Eq. (2). The actual DUT ABCD matrix T_a can be found from the measured DUT ABCD matrix T_m as $T_a = T_1^{-1} T_m T_2^{-1}$. Finally, the transformation between ABCD matrices and S-parameters can be found in other work.²⁶

- ¹A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, *Nature (London)* **431**, 162–167 (2004).
- ²M. Mück and R. McDermott, *Supercond. Sci. Technol.* **23**, 093001 (2010).
- ³D. Kinion and J. Clarke, *Appl. Phys. Lett.* **96**, 172501 (2010).
- ⁴D. H. Slichter, O. Naaman, and I. Siddiqi, *Appl. Phys. Lett.* **94**, 192508 (2009).
- ⁵A. D. O’Connell, M. Ansmann, R. C. Bialczak, M. Hofheinz, N. Katz, E. Lucero, C. McKenney, M. Neeley, H. Wang, E. M. Weig, A. N. Cleland, and J. M. Martinis, *Appl. Phys. Lett.* **92**, 112903 (2008).
- ⁶F. D. Parmentier, A. Mahe, A. Denis, J.-M. Berroir, D. C. Glatli, B. Placais, and G. Feve, *Rev. Sci. Instrum.* **82**, 013904 (2011).
- ⁷J. A. Schreier, A. A. Houck, J. Koch, D. I. Schuster, B. R. Johnson, J. M. Chow, J. M. Gambetta, J. Majer, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, *Phys. Rev. B* **77**, 180502 (2008).
- ⁸N. Bergeal, R. Vijay, V. E. Manucharyan, I. Siddiqi, R. J. Schoelkopf, S. M. Girvin, and M. H. Devoret, *Nat. Phys.* **6**, 296–302 (2010).
- ⁹N. D. Orloff, J. Mateu, A. Lewandowski, E. Roca, J. King, D. Gu, X. Lu, C. Collado, I. Takeuchi, and J. C. Booth, *IEEE Trans. Microwave Theory Tech.* **59**, 188–195 (2011).
- ¹⁰G. F. Engen and C. A. Hoer, *IEEE Trans. Microwave Theory Tech.* **27**, 987–993 (1979).
- ¹¹N. R. Franzen and R. A. Speciale, in *5th European Microwave Conference* (EuMA, Hamburg, Germany, 1975), pp. 69–73.
- ¹²D. F. Williams, C. M. Wang, and U. Arz, in *2003 IEEE MTT-S International Microwave Symposium Digest* (IEEE, Philadelphia, PA, USA, 2003), Vol. 3, pp. 1819–1822.
- ¹³Agilent Technologies (1998), note: Agilent application note AN 1287-3.
- ¹⁴W. Kruppa and K. F. Sodomsky, *IEEE Trans. Microwave Theory Tech.* **19**, 122–123 (1971).
- ¹⁵S. Rehnmark, *IEEE Trans. Microwave Theory Tech.* **22**, 457–458 (1974).
- ¹⁶Yeh Jen-Hao and S. M. Anlage, in *Precision Electromagnetic Measurements Conference (CPEM)* (IEEE, Washington, DC, USA, 2012), pp. 28–29.
- ¹⁷M. Scheffler and M. Dressel, *Rev. Sci. Instrum.* **76**, 074702 (2005).
- ¹⁸L. Spietz, K. Irwin, and J. Aumentado, *Appl. Phys. Lett.* **95**, 092505 (2009).
- ¹⁹This time can be shortened, increasing the amount of heat generated within a shorter amount of time. This may not present a significant problem since the cooling power increases with temperature in this range and it is therefore possible to optimize the calibration procedure time accordingly. We made no effort to minimize the total switch heating/cooling times in this work.
- ²⁰H. Van Hamme and M. Vanden Bossche, *IEEE Trans. Microwave Theory Tech.* **42**, 976–987 (1994).
- ²¹R. H. Byrd, J. C. Gilbert, and J. Nocedal, *Math. Program.* **89**, 149–185 (2000).
- ²²R. B. Marks, *IEEE Trans. Microwave Theory Tech.* **39**, 1205–1215 (1991).
- ²³C. Marki, private communication (2011).
- ²⁴L. Ranzani, L. Spietz, Z. Popovic, and J. Aumentado, *IEEE Trans. Appl. Supercond.* **22**, 1500606 (2012).
- ²⁵Gilbert W. Stewart, *Matrix Algorithms: Basic Decompositions* (SIAM, 1998), p. 251.
- ²⁶D. A. Frickey, *IEEE Trans. Microwave Theory Tech.* **42**, 205–211 (1994).