# Optimal Deployment of Pico Base Stations in LTE-Advanced Heterogeneous Networks

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## Abstract

As data traffic demand in cellular networks grows exponentially, operators need to add new cell sites to keep up; unfortunately, it is costly to build and operate macrocells. Moreover, it may not be possible to obtain the needed approvals for additional macrocell sites. Recently, the 3rd Generation Partnership Project (3GPP) introduced the concept of heterogeneous networks in Long Term Evolution (LTE) Release 10, where low-power base stations (BSs) are deployed within the coverage area of a macrocell to carry traffic. However, this new type of deployment can cause more severe interference conditions than a macro-only system, due to inter-cell interference; hence, enhanced inter-cell interference coordination (eICIC) has been actively discussed in LTE Release 10. Nevertheless, eICIC cannot completely eliminate interference and, to make matters worse, high-power transmissions of the reference signals from the BSs may increase the interference level. In this paper, we propose an algorithm for deploying low-power nodes within macrocell

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coverage areas in LTE heterogeneous networks, which aims to increase the system utility while minimizing the installation cost.

*Keywords:* LTE-Advanced heterogeneous network, Inter-cell interference, Utility, Cell Throughput, Installation cost

## 1. Introduction

To support the unrelenting growth in demand for data traffic, cellular networks have undergone huge changes, from second-generation (2G) technologies such as Global System for Mobile Communications (GSM) over twenty years ago to Third Generation Partnership Project (3GPP) Long Term Evolution (LTE) Release 8 [1] today. Release 8 offers higher spectral efficiency and throughput than its predecessor; however, its performance is not expected to keep up with the increase in data traffic demand. Thus, LTE Release 10, referred to as LTE-Advanced, has been standardized [2]. One of the new advanced technologies introduced by LTE-Advanced is the concept of heterogeneous networks, where low-power base stations (BSs) such as picocells, femtocells, and relay nodes are deployed within the coverage area of a macrocell. Thus, the traffic in the macrocell can be offloaded to the low-power nodes by reusing frequency resources across the different types of cells. In particular, the network operators can eliminate coverage holes in the macro-only system and improve capacity in hot-spots by deploying the low-power BSs. However, this new type of deployment can cause more severe interference conditions than the macro-only system due to inter-cell interference. Hence, in LTE Release 10, enhanced inter-cell interference coordination (eICIC) has been actively discussed.

ICIC was introduced in 3GPP Release 8 to solve intercell intra-frequency interference and its basic idea is to divide the overlapping area of neighboring cells into different frequency bands to reduce intercell interference. However, ICIC is not adequate for heterogeneous networks, resulting in the discussion of eICIC being started in LTE Release 10. A major component of eICIC is the use of Almost Blank Subframes (ABS), which enables BSs to transmit only during specific 1 ms subframes that are not used by neighboring BSs. That is, the UEs connected to pico cells can send their data during such ABS frames and avoid interference from the macro cell [3][4], Although the use of ABS is not mandatory in the LTE heterogeneous environment, several recent works considered an eICIC approach using ABS in the heterogeneous environment to improve user throughput (especially in urban area) [3][4]. This approach requires the user equipment (UE) to provide adequate resource-specific measurements and feedback and all the related BSs to transmit reference signals for channel quality measurement and cell acquisition signals in a predetermined schedule. Therefore, interference cannot be completely removed by using the ABS. Further, the interference level for the UE is affected by highpower transmissions of the Common Reference Signals (CRSs) from the BSs; future advanced UE designs may be able to do reference signal subtraction, but only if the various BSs' frames are aligned in the time domain [5].

Despite increasing interest in the interference problem in heterogeneous networks, there are few works available in the literature [3][7]. It is noteworthy that unplanned deployment of low-power BSs can lead to underutilization of air-interface resources due to the relatively small footprint of the low-power BSs compared to macro BSs and can lead to severe intercell interference. Thus, it is important to appropriately place low-power BSs within the macrocell. Nevertheless, previous studies on heterogeneous networks do not address how to place the low-power BSs in the macrocells in order to optimally reduce the interference. Although there has been a large amount of prior work on placement and configuration of BSs, the focus of this previous work was on the macro-only cellular network [8]-[12].

In this paper, we propose a deployment algorithm for low-power nodes within macrocells in LTE-Advanced heterogeneous networks, aiming to increase the system utility within a reasonable installation cost. Among the low-power nodes, picocells are considered in this study because they are typically used to extend coverage to indoor areas where outdoor signals do not reach well or to add network capacity in very dense traffic areas. The proposed deployment algorithm considers inter-cell interference and the configuration of ABS when maximizing the system utility. As far as we are aware, this is the first work that deals with the placement of pico BSs within the macrocell in LTE heterogeneous networks.

To avoid exponential growth in computation time with respect to the number of BSs, we propose a heuristic algorithm to efficiently solve the formulated problem and obtain the optimal picocell placement. We also conduct simulations to evaluate the performance of the picocell deployment obtained from the heuristic algorithm. The simulation results indicate that the proposed algorithm improves the utility in the network, especially in regions with high traffic density, while maintaining the installation cost at a reasonable level. The rest of the paper is organized as follows. Section 2 surveys previous related works. In Section 3, we formulate the new picocell deployment problem to optimize the throughput and the installation cost. In Section 4, we develop the system throughput model, considering the interference coordination technique in the LTE-Advanced heterogeneous network. Then, in Section 5, we formulate a heuristic algorithm to obtain the optimal picocell placement. We present the simulation results in Section 6. Finally, Section 7 concludes the paper.

### 2. Related Work

In current wireless cellular networks, mobile operators carefully choose the locations of new BSs so that they meet the increasing demand for wireless coverage and larger data rates. A large amount of prior work has focused on how to locate and configure new macro BSs. One of the first BS placement algorithms was presented in [8], which considered single and multiple transmitter problems. In [9], a genetic approach was used to find the nearoptimal location of the BSs. Amaldi et al. [10] proposed discrete optimization algorithms to support decisions in choosing the location of new BSs from a set of candidate sites, considering signal quality constraints in the uplink direction and fixed BS configuration. They also considered the downlink direction since 3G systems are especially intended to provide data services for users [11]. However, in practice, mobile operators usually have a very limited set of candidate sites due to authority constraints on new antenna installation and the sites acquisition cost is very expensive in urban areas. Thus, they investigated mathematical programming models for 3G radio planning, considering that modifying the configuration of the existing BSs can also provide improved wireless coverage for users [12].

3G cellular systems have evolved to the point of achieving near optimal performance, whereas the demand for capacity and wireless coverage is still increasing. To overcome the problem, LTE proposes heterogeneous networks, where low-power nodes are distributed throughout the coverage areas of macrocells. Since the **Release 8** [1] of LTE, new features have been introduced, and one of the key enhancements in LTE Release 10 is the adoption of eICIC [2]. The LTE-Advanced eICIC techniques enable interfering cells to configure ABSs [6]. Then, UEs under harsh interference conditions can be served in the ABS by their respective serving cells, while the interfering cells stay quiet during those subframes. In addition, LTE supports multiuser access because subcarriers can be allocated to different users within a transmission interval. The basic LTE radio resource that is **frequently** and dynamically addressable for data transmission in the two-dimensional time-frequency grid is called a resource block  $(0.5 \text{ms} \times 180 \text{kHz})$  [13]. The concept of radio resource block is not supported in traditional **cellular net**works based on Code Division Multiple Access (CDMA) and/or Time Division Multiple Access (TDMA). That is, in the LTE system, by allocating a variable number of resource blocks to a certain user and selecting a modulation and coding scheme to meet the current channel conditions, widely scalable transport block sizes are possible, resulting in a wide range of user-data rates [13][14]. Although there has been a large amount of prior work on placement and configuration of BSs, the focus of this work was on the macro-only cellular network [8]-[12] and hence, they did not consider

these LTE features. Therefore, the prior work on placement of BSs in the homogeneous network can no longer work for heterogeneous networks.

Concurrent with the increasing focus on the interference problem in heterogeneous networks by 3GPP, there is a small but growing body of works on this issue in the literature. In [6], joint cell-association and scheduling for picocells and femtocells have been discussed for downlink transmissions. In [7], location-based autonomous downlink power setting and uplink fractional power control schemes are proposed to mitigate cross-tier interference in femtocells. [3] presents system-level simulation results for eICIC techniques currently under discussion in 3GPP, identifying the major advantages and challenges of LTE-Advanced heterogeneous networks. [4] developed a theoretical framework of a heterogeneous cellular network based on random spatial models and demonstrated via simulations that essential intuition of these mathematical results hold in practice.

Several recent works such as [15] and [16] theoretically investigate heterogeneous cellular networks. [15] presents a general theoretical analysis of the distribution of the signal-to-interference-plus-noise-ratio (SINR) at an arbitrarily-located user, considering the downlink of a heterogeneous cellular network made up of multiple tiers of transmitters (e.g., macro-, micro-, pico-, and femtocells). The work also computes the probability that a UE can camp on either a macrocell or an Open Subscriber Group (OSG) femtocell when there are Closed Subscriber Group (CSG) femtocells in the network, and it provides insights into how to deploy femtocells so as to meet the camping requirements of the macrocells and the network as a whole, specifically deciding what fraction of the set of femtocells should be OSG femtocells. This



Figure 1: Placement of macro BSs in 2D space.

work, however, does not consider the fact that femtocells are often deployed by individual users and can appear and disappear randomly in the network as their owners switch them on and off, so operator control over them will be minimal [3]. Another analytical model for multiple-tier heterogeneous cellular networks, limited to downlink performance, develops mathematical expressions for performance metrics for a randomly located UE such as the outage probability and the average data rate, as well as the average load on each tier of BSs [16]. However, none of these studies addresses how to optimally place the different types of BSs to reduce the interference in the LTE-Advanced heterogeneous networks.

## 3. Pico BS Placement Problem Formulation for Optimizing Throughput and Cost

We aim to solve the pico BS placement problem within macrocell coverage areas in two-dimensional (2D) Euclidean space, (x, y)  $(0 \le x \le X, 0 \le y \le$  Y). That is, we seek the optimal placement and number of pico BSs within the macrocell coverage area so that the total throughput is maximized in the heterogeneous network and the installation cost is minimized. In the 2D space, we assume that the macro BSs are already deployed such that there are no coverage holes. Assuming a hexagonal shape for each cell, we can deploy the macro BSs with the cell range of  $R_m$  as shown in Fig. 1. We can place  $N_x$  and  $N_y$  macro BSs horizontally and vertically, respectively, to cover the 2D space as follows:

$$N_x = \lceil \frac{2}{3} \left( \frac{X}{R_m} + \frac{1}{2} \right) \rceil, \quad N_y = \lceil \frac{2Y}{\sqrt{3}R_m} + 1 \rceil.$$
(1)

For a macro BS,  $b_I$  with the index of  $I = \lfloor \frac{i}{2} \rfloor + \frac{N_x}{2}j$  (if  $N_x$  is even) or  $I = \lfloor \frac{i}{2} \rfloor + \frac{N_x+1}{2}j - \lfloor \frac{j}{2} \rfloor$  (if  $N_x$  is odd) for  $(0 \le i \le N_x - 1, 0 \le j \le N_y - 1, i + j$  is even number), its location is given by

$$x_I = \left(\frac{1}{2} + \frac{3}{2}i\right)R_m, \quad y_I = \frac{\sqrt{3}}{2}jR_m.$$
 (2)

Let  $B = \{b_I | 0 \le i \le N_x - 1, 0 \le y \le N_y - 1\}$  be the set the macro BSs deployed in the 2-D environment. We also consider the set of deployed pico BSs,  $\{p_{I,k}\}$  with  $1 \le k \le K_I$  and the cell range,  $R_p$ . That is,  $K_I$  is the total number of pico BSs deployed within the coverage area of  $b_I$ . The pico BSs  $\{p_{I,k}\}$  will be installed within the coverage area of the *I*th macrocell  $b_I \in B$ . Let  $(x_I, y_I)$  and  $(x_{I,k}, y_{I,k})$  denote the locations of the macro BS,  $b_I$  and the *k*th pico BSs,  $p_{I,k}$ , respectively. The cost function for the macro BS,  $b_I$ ,  $C(b_I)$ is

$$C(b_I) = C_I + \sum_{k=1}^{K_I} C_{I,k}$$
(3)

where  $C_I$  is the cost of installing macro BS  $b_I$ , and  $C_{I,k}$  is the cost for installing the *k*th pico BSs,  $p_{I,k}$ , within the coverage area of  $b_I$ . In Eq. 3, the

Service type		Interference ture	
Т	Ν	Interference type	
down	down	$N \rightarrow U$	
down	up	$U (in N) \rightarrow U (in T)$	
up	up	$\mathrm{U}  ightarrow \mathrm{T}$	
up	down	$N \rightarrow T$	

(T: Target macro/picocell, N: Neighboring macro/picocell, U: User equipment)

	1				, (	5		L J		
Configuration	Sub-frame number									
(c)	0	1	2	3	4	5	6	7	8	9
0	D	S	U	U	U	D	S	U	U	U
1	D	S	U	U	D	D	S	U	U	D
2	D	$\mathbf{S}$	U	D	D	D	S	U	D	D
3	D	$\mathbf{S}$	U	U	U	D	S	U	U	D

Table 2:	Uplink-downlink	configurations	[13]	
	1	0		

D: downlink, U: uplink, S: special subframe

pico BSs are installed at the locations with the highest traffic demand, the detailed algorithm for which will be explained later in section 5.

Let  $\rho(x, y)$  be the traffic demand from users at the point with coordinates (x, y), the unit of which is Mbit/s/km<sup>2</sup> in this study. Suppose that  $A_I$  and  $A_{I,k}$  are the coverage regions of  $b_I$  and  $p_{I,k}$ , respectively, and  $|A_I|$  and  $|A_{I,k}|$  are the areas of  $A_I$  and  $A_{I,k}$ , respectively. The total traffic demand arising in the coverage area of  $b_I$ ,  $\Gamma_I$ , is

$$\Gamma_I = \iint_{A_I} \rho(x, y) dx dy. \tag{4}$$

Denoting by  $S_I$  and  $S_{I,k}$  the maximum cell throughput for the BSs  $b_I$  and  $p_{I,k}$ , respectively we define the utility function at the location  $(x_I, y_I)$  as

$$U(b_I) = \frac{S_I + \sum_{k=1}^{K_I} S_{I,k}}{\Gamma_I}.$$
 (5)

We denote by  $A_{I,p}$  the candidate area where the pico BSs can be installed. The reason why we define the candidate area is that in practice, network operators may have a limited set of candidate sites due to regulatory constraints on new antenna installation locations, especially in urban areas. Thus,  $A_{I,p} \subset A_I$  is given as the finite set of all points at which pico BSs can be installed. Then, considering the installation cost in Eq. (3) and the system utility in Eq. (5), we formulate the pico BS deployment optimization problem as follows:

Opt<sub>B</sub>: Find 
$$K_I$$
 and  $(x_{I,k}, y_{I,k}) \in A_{I,p}$  for  $p_{I,k}$  in  $\forall b_I \in B$   
to maximize  $\sum_{\forall b_I \in B} (U(b_I) - \sigma C(b_I))$  (6)

such that

$$\frac{\sum_{\forall b_I \in B} (S_I + \sum_{k=1}^{K_I} S_{I,k})}{\sum_{\forall b_I \in B} (\Gamma_I)} \ge \bar{U}$$
(7)

$$d(p_{I_1,k_1}, p_{I_2,k_2}) \ge 2R_p, \ \forall p_{I_1,k_1} \in b_{I_1}, \ \forall p_{I_2,k_2}, \in b_{I_2}, \ \forall b_{I_1}, b_{I_2} \in B$$
(8)

where  $\sigma \geq 0$  is a trade-off parameter between the utility and the cost,  $\overline{U}$  is the minimum threshold on the total utility of the heterogeneous networking system, and  $d(p_{I_1,k_1}, p_{I_2,k_2}) = \{(x_{I_1,k_1} - x_{I_2,k_2})^2 + (y_{I_1,k_1} - y_{I_2,k_2})^2\}^{\frac{1}{2}}$  is the distance between two picocells,  $p_{I_1,k_1}$  and  $p_{I_2,k_2}$ . When deploying pico BSs in the LTE heterogeneous network, we ensure that the picocells' coverage areas do not overlap in order to reduce the inter-picocell interference, as expressed by the constraint to Eq. (8).



Figure 2: Interference cases in the heterogeneous network.

We also formulate the pico BS deployment optimization by putting a minimum threshold on the utility of each macro BS in the heterogeneous networking system, which will be referred to as  $Opt_I$ . Specifically,  $Opt_I$  is same as  $Opt_B$  except that the constraint of Eq. (7) is

$$\frac{(S_I + \sum_{k=1}^{K_I} S_{I,k})}{\Gamma_I} \ge \bar{U}, \ \forall b_I \in B.$$
(9)

## 4. Modeling System Throughput

#### 4.1. Interference Cases

We consider different interference cases in deriving expressions for the downlink and the uplink interference. Table 1 presents the possible intercell interference cases in the heterogeneous network. Assuming that 5 ms downlink-to-uplink switch-point-periodicity is supported, uplink-downlink configurations are shown in Table 2 [13]. Since different cells have different frame structures, the interference can occur in four different cases shown in Tables 1 and 2.

The downlink interference at the UE has different values depending on whether or not the UE is located in the picocell, as shown in Fig. 2. Let  $F_I(x, y)$  and  $F_{I,k}(x, y)$  denote the downlink interference at the UE located at (x, y) due to the macro BS,  $b_I$ , and due to the pico BS,  $p_{I,k}$ , respectively. The interferences due to the BSs are given by

$$F_I(x,y) = Pt_I(x,y)Pl_I(x,y)$$
(10)

$$F_{I,k}(x,y) = Pt_{I,k}(x,y)Pl_{I,k}(x,y)$$
(11)

where  $Pt_I(x, y)$   $(Pt_{I,k}(x, y))$  and  $Pl_I(x, y)$   $(Pl_{I,k}(x, y))$  are the transmit power and the path loss from the neighboring macro BS,  $b_I$ , (pico BS,  $p_{I,k}$ ) to the UE located at (x, y), respectively.

As can be seen in Table 1, the UE may also experience interference from another UE located in the same cell. Let  $F_I^{ue}(x, y)$  and  $F_{I,k}^{ue}(x, y)$  be the interference to the UE located at (x, y) from another UE located at  $(\bar{x}, \bar{y})$ , which is associated with the macro BS  $b_I$  or with the pico BS,  $p_{I,k}$ , respectively. Denoting by  $\rho_u$  the average traffic demand per UE, we define the estimated number of UEs at the location (x, y),  $N_{ue}(x, y)$ , as

$$N_{ue}(x,y) = \frac{\text{Traffic demand at } (x,y)}{\text{Traffic demand per UE}} = \frac{\rho(x,y)}{\rho_u}.$$
 (12)

Given the expected traffic demand  $\rho_u$  from the UE, we have

$$F_{I}^{ue}(x,y) = \iint_{A_{I}-\bigcup_{k=1}^{K_{I}}A_{I,k}} N_{ue}(\bar{x},\bar{y})Pt^{ue}(\bar{x},\bar{y})Pl((x,y);(\bar{x},\bar{y}))d\bar{x}d\bar{y}$$
(13)

and

$$F_{I,k}^{ue}(x,y) = \iint_{A_{I,k}} N_{ue}(\bar{x},\bar{y})Pt^{ue}(\bar{x},\bar{y})Pl((x,y);(\bar{x},\bar{y}))d\bar{x}d\bar{y}$$
(14)

where  $Pt^{ue}(\bar{x}, \bar{y})$  is the transmit power of the picocell's UE at  $(\bar{x}, \bar{y})$ , and  $Pl((x, y); (\bar{x}, \bar{y}))$  is the path loss from another UE's location,  $(\bar{x}, \bar{y})$  to the UE's location, (x, y). In Eqs. (10) - (14), the distance-dependent path loss

in the macrocell for urban sites is  $128.1 + 37.6 \log_{10}(R)$ , and the indoor path loss in the picocell is  $38 + 30 \log_{10}(R)$ , where R is the transmitter-receiver separation in km and m in the macrocell and the picocells, respectively [17].

## 4.2. Interference Probability

Let Pr(c, l) be the probability of uplink or downlink transmission for the configuration, c, where c = 0, 1, 2, or 3 in Table 2, and l indicates whether the transmission is uplink (l = U) or downlink (l = D). Let  $r_c$   $(\sum_{c=0}^{3} r_c = 1)$  be the probability that a randomly selected frame has the configuration, c. Given the value of  $r_c$  and the total number of BSs placed in the 2D space,  $N_{bs}$ ,  $r_c N_{bs}$  BSs use the configuration c. The values of the elements of  $\{r_c\}_{c=0}^{3}$  may be determined by the network operator as operator-specific data. In this study, we do not consider any specific constraints imposed on the inter-cell synchronization [19].

Considering the macrocell of BS  $b_I$ , the probability that the macrocells neighboring  $b_I$  have the transmission direction of l is

$$Pr(l) = \sum_{c=0}^{3} r_c Pr(c, l),$$
(15)

which yields  $Pr(U) = \frac{6}{10}r_0 + \frac{4}{10}r_1 + \frac{2}{10}r_2 + \frac{5}{10}r_3$  and  $Pr(D) = \frac{2}{10}r_0 + \frac{4}{10}r_1 + \frac{6}{10}r_2 + \frac{3}{10}r_3$  according to the values of Pr(c, l) in Table 2.

We denote by  $A_I^{(e)}$  and  $A_I^{(c)}$  the cell edge and cell center regions, respectively, in the area  $A_I$ , where  $A_I^{(c)}$  is assumed to be a circle whose center is located at macro BS  $b_I$  and whose radius is  $R_c$  ( $< R_m$ ). The available physical Resource Blocks (RBs) are assigned first to the UEs located in the cell edge,  $A_I^{(e)}$ , and then the remaining RBs are allocated to the UEs in  $A_I^{(c)}$ . Let |RB| denote the total number of RBs, which is set to 100 in the LTE system [17].

Let  $|RB|_l(x, y)$  be the number of RBs needed by the UEs at the location (x, y). Let  $N_{abs}$  be the average number of ABSs per subframe, which yields the ratio of ABSs to the available subframes,  $\tau = N_{abs}/8$ , considering all ten subframes in a frame, minus two special subframe. Then, we have

$$|\hat{RB}|_{l}(x,y) = \begin{cases} \left\lceil \frac{\rho(x,y)\rho_{l}(x,y)}{Pr(c,l)S_{l}(x,y)} \right\rceil, & \text{for } (x,y) \in A_{I,k}, \\ \left\lceil \frac{\rho(x,y)\rho_{l}(x,y)}{(1-\tau)Pr(l)S_{l}(x,y)} \right\rceil, & \text{otherwise} \end{cases}$$
(16)

where  $\rho_U(x, y)$  and  $\rho_D(x, y)$  are the ratios of uplink and downlink traffic, respectively, to the total traffic demand at the location (x, y) and  $S_l(x, y)$  is computed using Eq. (30) as the throughput per RB at the location (x, y); the formula for  $S_l(x, y)$  will be developed later in this section. In Eq. (16), the reason why we normalize by Pr(l) is to obtain the number of RBs (per frame) required for all the uplink/downlink sub-frames for the UEs at the location, (x, y).

Since more RBs should be offered as the traffic load increases in a cell, using Eq. (16), we can estimate the number of RBs used in  $A_I^{(e)}$  as

$$|RB|_{l}^{(e)} = \frac{\min(|RB|, |\hat{RB}|_{l}^{(I)})}{|\hat{RB}|_{l}^{(I)}} \iint_{A_{I}^{(e)} - \bigcup_{k=1}^{K_{I}} A_{I,k}} |\hat{RB}|_{l}(x, y) dx dy,$$
(17)

for  $l = \{U, D\}$ , where  $|\hat{RB}|_{l}^{(I)}$  is the number of RBs needed by the UEs serviced by  $b_{I}$  and can be obtained by integrating  $|\hat{RB}|_{l}(x, y)$  over  $A_{I} - \bigcup_{k=1}^{K_{I}} A_{I,k}$ . Likewise, we obtain the number of RBs used in  $A_{I}^{(c)}$ ,  $|RB|_{l}^{(c)}$ , by substituting  $A_{I}^{(c)}$  for  $A_{I}^{(e)}$  in Eq. (17). We also get the number of RBs used in  $p_{I,k}$ ,  $|RB|_{l}^{(k)}$ , by applying the integrals in Eq. (17) to  $A_{I,k}$ . Note that Fractional Frequency Reuse (FFR) is an attractive ICIC technique to improve the user experience at the cell edge through frequency reuse [18][4]. In [4], extensive simulations were conducted to investigate the performance of LTE heterogeneous system using FFR. In [18], use of FFR in the heterogeneous system is addressed. Thus, we assume that the LTE system utilizes FFR based on Power Bandwidth Profiles (PBPs) [18], where the UEs in the center of a cell can use the whole spectrum while the edge UEs can use only part of the spectrum. The bandwidth is divided into three non-overlapping bands, and each of three neighboring cells is assigned one of these bands to avoid inter-cell interference. In the cell edge, the macro BSs employ PBPs with the power split (PS) set to 3 dB for an N = 2/3 FFR scheme<sup>5</sup> presented in [18], where a frequency re-use of N = 1 implies that the BSs in macrocells transmit simultaneously on all available time-frequency RBs.

Let  $N_I$  be the set of six macrocells immediately adjacent to the coverage area of  $b_I$ . We name the six elements of  $N_I$  as  $\{b_1, \dots, b_6\}$ .  $|RB|_l^{(\hat{i},e)}$  and  $|RB|_l^{(\hat{i},c)}$ , the numbers of RBs used in the edge and the center region of the cell associated with macro BS  $b_{\hat{i}}$ , can be obtained as in Eq. (17). In the macrocell system, a macro BS,  $b_I$ , is allowed to use its maximum transmit power,  $P_{max}$ , for the UEs located in its edge of its cell for the specified subband, which we refer to as  $f_1$  hereafter. The UEs in the cell edge are assigned  $\frac{|RB|}{3}$  RBs from the specified sub-band,  $f_1$ , first. Likewise, the RBs from one

<sup>&</sup>lt;sup>5</sup>The analysis developed in this study can also be applied to another macrocell system model with different values of power split, Inter-Site Distance (ISD), and the number of sub bands.

of the other two sub-bands,  $f_2$  or  $f_3$  (we will refer to this subband hereafter as  $g_1$ ), are assigned to the UEs in the edge of the cell of  $b_i$  in order to avoid inter-macrocell interference. That is, in the cell edge of  $b_i$ , the RBs from  $g_1$ (which is subband  $f_2$  or  $f_3$ ), are assigned first; then, if all the RBs from  $g_1$ are used up, RBs from the remaining subbands  $g_2$  or  $g_3$  will be assigned to the UEs in the cell edge of  $b_i$ .

If the RBs from  $f_1$  and  $g_1$  are sufficient for the traffic demands in the cell edges of  $b_I$  and  $b_{\hat{i}}$ , respectively, (i.e.,  $|RB|_l^{(e)}, |RB|_l^{(\hat{i},e)} \leq \frac{|RB|}{3}$ ), no intermacrocell interference is expected to occur. However, if in addition to the RBs from  $f_1$ , more RBs are required by the UEs in  $A_{\hat{i}}^{(e)}$  (i.e., the edge of  $b_{\hat{i}}$ ) and the macro BS,  $b_I$  assigns some RBs (say *i* RBs) from  $f_2$  or  $f_3$  to the UEs in  $A_I^{(e)}$ , then there may be interference from  $b_{\hat{i}}$ .

For the location (x, y) in  $A_I$ , we derive the probability that the coverage area of  $b_I$  and its set of picocells,  $\{p_k^I\}$ , experience interference from both uplink and downlink transmissions (i.e.,  $l = \{U, D\}$ ) that occur in neighboring macrocells in the set  $\{b_i\}$  as follows:

$$Pr_{l}^{(\hat{i})}(f_{1}) = Pr(l) \sum_{j=1}^{3} Pr(f_{1}, g_{j}) \begin{cases} \frac{|RB|_{l}^{(\hat{i}, e)} - \frac{|RB|}{3}}{\frac{2|RB|}{3}}, & \text{if } |RB|_{l}^{(\hat{i}, e)} > \frac{|RB|}{3}\\ 0, & \text{otherwise.} \end{cases}$$
(18)

$$Pr_{l}^{(\hat{i})}(f_{i}) = Pr(l) \sum_{j=1}^{3} Pr(f_{i}, g_{j})$$

$$\times \begin{cases} 1, & \text{if } f_{i} = g_{1} \text{ and } |RB|_{l}^{(\hat{i},e)} > \frac{|RB|}{3} \\ \frac{|RB|_{l}^{(\hat{i},e)}}{3}, & \text{if } f_{i} = g_{1} \text{ and } |RB|_{l}^{(\hat{i},e)} \leq \frac{|RB|}{3} \\ \frac{|RB|_{l}^{(\hat{i},e)} - \frac{|RB|}{3}}{2|RB|}, & \text{if } f_{i} \neq g_{1} \text{ and } |RB|_{l}^{(\hat{i},e)} > \frac{|RB|}{3} \\ 0, & \text{if } f_{i} \neq g_{1} \text{ and } |RB|_{l}^{(\hat{i},e)} \leq \frac{|RB|}{3} \end{cases}$$
(19)

for i = 2, 3, where  $Pr_l(f_i, g_j)$  is the probability that an RB in the frequency band  $f_i$  of  $b_I$  overlaps with that from the frequency band  $g_j$  of  $b_i$ , and it is given by

$$Pr_l(f_i, g_j) = \begin{cases} 0, & \text{if } i = j \ (1 \le i, j \le 3) \\ \frac{1}{2}, & \text{otherwise.} \end{cases}$$
(20)

Eqs. (18) and (19) respectively give the probabilities of interference occurrence when the macro BS,  $b_I$ , uses only  $f_1$  and  $b_I$  allocates the RBs from  $f_i$ (i = 2 or 3) in addition to  $f_1$  while transmitting on uplink/downlink l.

We next compute the probability that the UE in the picocell of  $p_{I,k}$  experiences interference from the coverage area of  $b_I$ :

$$Pr_{l}^{(I)}(f_{1}) = Pr(l) \begin{cases} 1, & \text{if } |RB|_{l} > \frac{|RB|}{3} \\ \frac{|RB|_{l}^{(e)}}{\frac{|RB|}{3}}, & \text{otherwise.} \end{cases}$$
(21)

$$Pr_{l}^{(I)}(f_{i}) = Pr(l) \begin{cases} \frac{|RB|_{l} - \frac{|RB|}{3}}{\frac{2|RB|}{3}}, & \text{if } |RB|_{l} > \frac{|RB|}{3} \\ 0, & \text{otherwise,} \end{cases}$$
(22)

for i = 2, 3.

We assume a uniform allocation of RBs to each UE in the picocell. Then,

the probability of interference from the picocell of  $p_{I,k}$  is

$$Pr_{l}^{(k)}(f_{i}) = Pr(l) \begin{cases} 0, & \text{if } (x, y) \text{ is in } A_{I,k} \\ \frac{|RB|_{l}^{(k)}}{|RB|}, & \text{otherwise.} \end{cases}$$
(23)

for i = 1, 2, 3.

### 4.3. Cell Throughput

When placing the kth pico BS in the coverage area of  $b_I$  in the pico BS placement process, we note that interference may occur due to the new pico BS,  $p_{I,k}$ , the already placed pico BSs,  $p_{I,1}, p_{I,2}, \ldots, p_{I,k-1}$ , the macro BS,  $b_I$ , the six macrocells adjacent to  $b_I$ ,  $b_i(1 \le i \le 6)$ , and the pico cells deployed within each adjacent macro cell,  $b_i$ ,  $\{p_k^i\}$ .

We first consider the interference from the pico BS,  $p_{I,k}$  and the macro BS,  $b_I$ . Let  $Pr_l(f_1)$  be the probability that the UE at (x, y) in the coverage area of macro BS,  $b_I$  is allocated RBs from the frequency band  $f_1$ , for both transmission directions. This probability is expressed for different locations of (x, y) as follows. For  $(x, y) \in A_I^{(e)} - \bigcup_{k=1}^{K_I} A_{I,k}$ ,

$$Pr_{l}(f_{1}) = \begin{cases} 1, & \text{if } |RB|_{l}^{(e)} \leq \frac{|RB|}{3} \\ \frac{|RB|_{l}^{(e)}}{3}, & \text{otherwise.} \end{cases}$$
(24)

For  $(x, y) \in A_I^{(c)} - \bigcup_{k=1}^{K_I} A_{I,k}$ ,

$$Pr_{l}(f_{1}) = \begin{cases} 1, & \text{if } |RB|_{l} \leq \frac{|RB|}{3} \\ 0, & \text{if } |RB|_{l} > \frac{|RB|}{3} \text{ and } |RB|_{l}^{(e)} \geq \frac{|RB|}{3} \\ \frac{\frac{1}{3}|RB| - |RB|_{l}^{(e)}}{|RB|_{l}^{(e)}}, & \text{if } |RB|_{l} > \frac{|RB|}{3} \text{ and } |RB|_{l}^{(e)} < \frac{|RB|}{3}. \end{cases}$$
(25)

where  $|RB|_l = |RB|_l^{(e)} + |RB|_l^{(c)}$ .

Using  $Pr_l(f_1)$ , we can express the probability of allocating the RBs from the frequency band  $f_i$  (i = 2, 3) to the UE at (x, y) in  $b_I$  as

$$Pr_l(f_i) = \frac{1 - Pr_l(f_1)}{2}, \quad \text{for } j = 2, 3.$$
 (26)

For  $(x, y) \in A_{I,k}$ , under the assumption of the uniform allocation of RBs within the picocell, we have  $Pr_l(f_i) = 1/3$  for j = 1, 2, 3.

Supposing that the UE is at the location (x, y) within the coverage area of the macro BS,  $b_I$  (i.e.,  $b_I$  is the UE's serving cell), we denote with  $P_D(x, y)$  the downlink interference from the pico and/or macrocells to the UE and with  $P_U(x_I, y_I)$  the uplink interference from the UE and the picocells to the macro BS, and  $P_U(x_k, y_k)$  the uplink interference from the UE and the macrocell to the pico BS. Using Eqs. (10) - (14), (24), and (26), we can express the downlink interference, and the uplink interferences from the UE to the macro BS,  $b_I$  and to the pico BS,  $p_{I,k}$ , in Eqs. (27).

$$P_{l}(x,y) = \sum_{i=1}^{3} Pr_{l}(f_{i})$$

$$\times \begin{cases} Pr_{D}^{(I)}(f_{i})F_{I}(x,y) + Pr_{U}^{(I)}(f_{i})F_{I}^{ue}(x,y) \\ + \sum_{j=1,k\neq j}^{K_{I}} \left(Pr_{D}^{(j)}(f_{i})F_{I,j}(x,y) + Pr_{U}^{(j)}(f_{i})F_{I,j}^{ue}(x,y)\right), \text{ if UE is in } A_{I,k} \\ \sum_{k=1}^{K_{I}} \left(Pr_{D}^{(k)}(f_{i})F_{I,k}(x,y) + Pr_{U}^{(k)}(f_{i})F_{I,k}^{ue}(x,y)\right), \text{ otherwise.} \end{cases}$$

$$(27)$$

According to the constraint in Eq. (8), different picocells do not overlap with one another in the proposed problem formulation, but inter-pico cell interference may still occur. Thus, this interference is also reflected in Eq. (27). Considering the neighboring macro cells,  $b_{\hat{i}}$  and the pico cells,  $p_{\hat{k}}^{\hat{i}}$  within the macro cells,  $b_{\hat{i}}$ , where  $1 \leq \hat{i} \leq 6$ , we can express the expected interference power at the location, (x, y) in the coverage area of macro BS  $b_I$  as

$$P_{l}^{\hat{i}}(x,y) = \sum_{i=1}^{3} Pr_{l}(f_{i}) \left\{ (1-\tau) \left( Pr_{D}^{(\hat{i})}(f_{i})F_{\hat{i}}(x,y) + Pr_{U}^{(\hat{i})}(f_{i})F_{\hat{i}}^{ue}(x,y) \right) + \sum_{k=1}^{K_{\hat{i}}} \left( Pr_{D}^{(k)}(f_{i})F_{\hat{i},k}(x,y) + Pr_{U}^{(k)}(f_{i})F_{\hat{i},k}^{ue}(x,y) \right) \right\}.$$
(28)

Using Eqs. (27) and (28), at the location (x, y) within the coverage area of the macro BS,  $b_I$ , we have the downlink SINR to the UE as follows:

$$SINR_D(x,y) = \frac{P_{sig}}{\sum_{i=1}^6 P_D^i(x,y) + P_o + P_D(x,y)}$$
(29)

where  $P_{sig}$  is the received signal power and  $P_o$  is the noise power. Likewise, we can obtain  $SINR_U(x, y)$  by substituting  $P_D(x, y)$  with  $P_U(x_I, y_I)$  or  $P_U(x_k, y_k)$ .

Let  $SINR_{min}$  and  $SINR_{max}$  be the minimum and the maximum SINRs, respectively, for which the code set works. We also let  $S_{max}$  be the maximum throughput of the code set. Then, the throughput per RB at the location (x, y) can be derived by using Shannon's formula and Eq. (29):

$$S_{l}(x,y) = \begin{cases} 0, & \text{if } SINR_{l}(x,y) < SINR_{min} \\ \alpha \log_{2}(1 + SINR_{l}(x,y))C, & \text{if } SINR_{max} < SINR_{l}(x,y) \leq SINR_{min} \\ S_{max}, & \text{otherwise} \end{cases}$$

(30)

for  $l = \{D, U\}$ , where  $\alpha$  is the attenuation factor indicating implementation losses and C = 180 kHz is the bandwidth of a single RB [17][22]. Since there is no data transmitted during ABSs in the macrocell, we can express the maximum throughput that can be provided in the macrocell,  $b_I$ , as

$$S_I = (1 - \tau)(S_D^{(I)} + S_U^{(I)}), \qquad (31)$$

where

$$S_{l}^{(I)} = \frac{1}{|A_{I}| - \bigcup_{k=1}^{K_{I}} |A_{I,k}|} \times \iint_{A_{I} - \bigcup_{k=1}^{K_{I}} A_{I,k}} \left( \frac{|RB|_{l}^{(I)}}{|\bar{RB}|_{l}^{(I)}} |\bar{RB}|_{l} (x, y) Pr(l) S_{l}(x, y), \ \rho(x, y) \rho_{l}(x, y) \right) dxdy$$

$$(32)$$

Let  $\tau$  denote the portion of the throughput for ABS among the total throughput. Then, similar to Eq. (31), the maximum throughput in the picocell of  $p_{I,k}$  is given by

$$S_{I,k} = (1-\tau)(S_D^{(I,k)} + S_U^{(I,k)}) + \tau(\hat{S}_D^{(I,k)} + \hat{S}_U^{(I,k)}),$$
(33)

where

$$S_{l}^{(I,k)} = \iint_{A_{I,k}} \min\Big(\frac{|RB|_{l}^{(I,k)}}{|\bar{RB}|_{l}^{(I,k)}}|\bar{RB}|_{l}(x,y)Pr(c,l)S_{l}(x,y), \ \rho(x,y)\rho_{l}(x,y)\Big)\frac{dxdy}{|A_{I,k}|}$$
(34)

and, assuming that no interference occurs during the ABSs,  $\hat{S}_{U}^{(I)}$  and  $\hat{S}_{D}^{(k)}$  are obtained by applying  $P_{l}(x, y) = \sum_{i=1}^{3} Pr_{l}(f_{i}) \sum_{j=1, k \neq j}^{K_{I}} \left( Pr_{D}^{(j)}(f_{i})F_{I,j}(x, y) + Pr_{U}^{(j)}(f_{i})F_{I,j}^{ue}(x, y) \right)$  to Eq. (34) instead of Eq. (27).

## 4.4. Power Control

In the previous section, we assumed that power control is not utilized in the LTE system. In this section, the transmission power of UE (i.e.,  $Pt_{ue}(x, y)$  is revisited considering power control. Referring to [17], the macro and pico BSs use the transmit power at location (x, y) as follows:

$$Pt_{I}(x,y) = \begin{cases} P_{max} - 3 \text{ dB}, \text{ if } (x,y) \text{ is in } A_{I}^{(e)} \text{ and } f_{2} \text{ or } f_{3} \text{ is used} \\ P_{max}, \text{ otherwise,} \end{cases}$$
(35)

$$Pt_{I,k}(x,y) = P_{k,max} \tag{36}$$

where  $P_{k,max}$  is the maximum transmit power of the pico BS.

The UE's transmit power,  $Pt_{ue}(x, y)$ , is

$$Pt_{ue}(x,y) = \min(P_{max,ue}, 10\log_{10}M_{PUSCH} + P_{O\_PUSCH} + \gamma PL), \quad (37)$$

where  $P_{max,ue}$  is the UE's maximum power level,  $M_{PUSCH}$  is the bandwidth of the Physical Uplink Shared Channel (PUSCH) assignment expressed in the number of RBs valid for the subframe and the serving cell,  $P_{O_PUSCH}$  is a parameter composed of the sum of a cell specific nominal component,  $P_{O,nominal}$ and a UE specific component,  $P_{O,ue}$ ,  $\gamma \in \{0, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$  is a 3-bit cell specific parameter provided by higher layers for the serving cell, and PL is the path loss in the UE for the serving cell (i.e., either PL(x, y)or  $PL_k(x, y)$ ) [19].

## 5. Heuristic BS Placement Algorithm

In this section, we present a heuristic algorithm to solve the optimization problem. We denote by  $F_n^I$  the utility minus the cost when a new pico BS is installed, given that n pico BSs are already in the coverage area of  $b_I$ :

$$F_n^I = U(b_I) - \sigma C(b_I). \tag{38}$$

Algorithm 1 Heuristic Pico BS Placement Algorithm for  $Opt_B$  (Algo<sub>B</sub>)

- 1: while  $\sum_{\forall b_I \in B} (S_I + \sum_{k=1}^{K_I} S_{I,k}) / \sum_{\forall b_I \in B} (\Gamma_I) < \overline{U}$  or  $\exists (x, y)$  within  $A_{I,p}$ s.t.  $\Delta F_n^I > 0$  for  $\forall b_I \in B$  do
- 2: // Constraint in Eq. (7)
- 3: Select a BS  $b_I \in B$  randomly.
- 4:  $//U'(b_I)$  is the utility when a new pico BS is installed at (x, y).
- 5:  $// U''(b_I)$  is the utility when the selected pico BS is removed.
- 6: **if**  $\exists (x, y)$  within  $A_{I,p}$  s.t. ((Eq. (7) is not satisfied **and**  $U'(b_I) U(b_I) > 0$ ) **or**  $\Delta F_n^I > 0$ ) **and**  $d(p_{n+1}^I, p_k^{I'}) < 2R_p$  for  $1 \le k \le K_{I'}, \forall b_I, \forall b_{I'} \in B$ **then**
- 7: Install a new pico BS,  $p_{n+1}^I$  at the location, (x, y).

8: 
$$n = n + 1$$

- 9: end if
- 10: **if**  $\exists p_{I,k}$  within  $A_{I,p}$  s.t.  $U''(b_I) U(b_I) > 0$   $(1 \le k \le n)$  **then**
- 11: Remove the pico BS  $p_{I,k}$ .
- 12: n = n 1
- 13: end if
- 14: end while

To find the location that gives the biggest increase in the utility minus the cost of an additional pico BS, we define  $\Delta F_n^I$  as  $\Delta F_n^I = F_{n+1}^I - F_n^I$ . We find the location (x, y) for the first pico BS,  $p_{I,1}$ , such that  $\Delta F_0^I$  within  $A_{I,p}$  has the maximum value. Once  $p_{I,1}$  is installed at the location found by the algorithm, we can define  $\Delta F_1^I$  in the same fashion as  $\Delta F_0^I$  to find the location for the second pico BS,  $p_{I,2}$ , within  $A_{I,p}$  where  $\Delta F_1^I$  is maximized. However, a candidate location (x, y) that maximizes  $\Delta F_1^I$  within  $A_{I,p}$  cannot

be chosen for  $p_{I,2}$  if it would result in  $d(p_{I,2}, p_{I,1}) < 2R_p$ ; this satisfies the constraint in Eq. (8). We repeat this process with  $\Delta F_n^I$  and  $A_{I,p}$  until the constraints are satisfied. The detailed process of the algorithm for  $\text{Opt}_B$  is given in Algorithm 1. Similarly, the algorithm for  $\text{Opt}_I$  is given by replacing the condition of the while loop with  $(S_I + \sum_{k=1}^{K_I} S_{I,k})/\Gamma_I < \overline{U}$  or  $\exists (x, y)$ within  $A_{I,p}$  s.t.  $\Delta F_n^I > 0$  for  $\forall b_I \in B$  and replacing Eq. (7) in line 6 with Eq. (9). We will refer to the algorithm as Algo<sub>I</sub>.

Let  $|A_{I,p}|$  be the number of candidate points at which a pico BS can be installed. Denoting with  $N_k$ , the total number of pico BSs installed, the computational complexity of the proposed heuristic algorithm is  $|A_{I,p}| \times N_k$ , whereas the exhaustive search of the candidate area takes the computational complexity of  $\left(\sum_{\substack{\forall b_I \in B \\ N_k}} |A_{I,p}|\right)$ .

## 6. Performance Evaluation

This section evaluates the performance of the proposed heuristic picocell placement algorithm through simulations.

#### 6.1. Simulation Environment

We use the ns-3 simulator ver-3.13 [21] for the performance evaluation. We have extended the ns-3 simulator by implementing the following additional modules: a pico BS that uses lower transmit power than macro BS, LTE time division duplex (TDD), and eICIC including ABS and frequency reuse because the current implementation of ns-3 does not support these functions.

The simulation area considered is a rectangle with the bottom left corner at coordinates (X, Y) = (0, 0) and upper right coordinates (X, Y) =

Index	Region	Traffic density
1	$\frac{1}{9}X < x < \frac{2}{9}X; \ \frac{1}{9}Y < y < \frac{7}{18}Y$	25
2	$\frac{1}{9}X < x < \frac{3}{9}X; \ \frac{5}{9}Y < y < \frac{9}{9}Y$	15
3	$\frac{5}{9}X < x < \frac{8}{9}X; \ \frac{1}{9}Y < y < \frac{3}{9}Y$	17.5
4	$\frac{6}{9}X < x < \frac{8}{9}X; \ \frac{6}{9}Y < y < \frac{8}{9}X$	17
5	For the other region	2

Table 3: Traffic density (Mbit/s/km<sup>2</sup>) used in the simulation area.

(4 km, 4.33 km). The traffic density in the simulation area is set as shown in Table 3, where the ratio between the uplink and the downlink transmissions is set to 4:6. Each of the four regions with high traffic density in Table 3 forms a rectangle; for example, the four regions could be clusters of downtown buildings, where there are high traffic demands due to many indoor and outdoor pedestrians. They are identified as Regions 1 to 4 as shown in Table 3. Each user generates constant bit rate (CBR) traffic at 125 kbytes per second for both uplink and downlink transmissions.

The macro BS and pico BS installation costs  $C_I$  and  $C_{I,k}$  in Eq. (3), which are commonly identified as part of capital expenditures (CAPEX), may vary with location and the characteristics of the chosen site (e.g. open spaces versus buildings). Their values can be determined based on the network operator policy. For this study, based on some preliminary simulation runs, we set  $C_I = 1$  and  $C_{I,k} = 0.2$  to make their scale consistent with the utility factor in Eqs. (6) and (38). In the equations,  $\sigma$  is a trade-off parameter that quantifies the relative importance of maximizing the utility versus minimizing the total installation costs. The proposed algorithm will yield different

Table 4: Parameters used in the simulation [17],[22].					
Parameter	Value				
$SINR_{min}$ (dB)	-10				
$S_{max}$ (bit/s/Hz)	Downlink: 4.4; Uplink: 2				
$P_{max}$ (dBm)	46				
$P_{k,max}$ (dBm)	24				
System bandwidth (MHz)	20				

solutions depending on the importance that the network operator places on these two contrasting objectives. We ran the simulations for  $\sigma = 0.95$  and  $\sigma = 0.10$  to observe the effect of installation cost on picocell deployment. We set  $\bar{U}$  to 0.85 and 0.65. The main parameters and their values for the simulation, which are obtained from [17], are shown in Table 4.

#### 6.2. Results and Discussions

We place the pico BSs at the locations determined by the proposed algorithms, Algo<sub>I</sub> and Algo<sub>B</sub>, as shown in Fig. 3. We set the candidate points at which the pico BSs can be installed as  $A_{I,p} = \{(x, y) | (x, y) \in$ the candidate area with density  $\eta$ }, where  $\eta = 100$  (points per km<sup>2</sup>). The radii of macro and picocells are set to 1.0 km and 0.2 km, respectively, which gives an ISD of 1.7 km for the macrocells [20], as described in Section 3. The coordinates of the locations of the macro and the pico BSs and their configurations (i.e., c in Table 2) are:

Macro BSs: The coordinates are (0.5, 0.0), (3.5, 0.0), (2.0, 0.87), (0.5, 1.73), (3.5, 1.73), (2.0, 2.6), (0.5, 3.46), (3.5, 3.46), and (2.0, 4.33). The configurations in all the macrocells are set to c = 2.

• Pico BSs:

For Algo<sub>I</sub>/Algo<sub>B</sub> ( $\sigma = 0.10, 0.95, \overline{U} = 0.85$ ), the coordinates are (1) (0.6, 0.91); c = 1, (2) (1.1, 3.90); c = 0, (3) (3.47, 0.9); c = 0, (4) (0.83, 2.62); c = 0, (5) (0.17, 3.42); c = 0, (6) (3.17, 1.89); c = 0, (7) (2.83, 0.05); c = 0, (8) (2.67, 3.68); c = 0, and (9) (1.13, 0.02); c = 0.

For Algo<sub>B</sub> ( $\sigma = 0.10$ ,  $\overline{U} = 0.65$ ), the coordinates are (1) (0.83, 2.62); c = 0, (2) (0.6, 0.91); c = 1, (3) (3.47, 0.9); c = 0, (4) (0.17, 3.42); c = 0, (5) (3.17, 1.89); c = 0, (6) (2.83, 0.05); c = 0, (7) (1.1, 3.89); c = 0, and (8) (2.67, 3.68); c = 0.

For Algo<sub>B</sub> ( $\sigma = 0.95$ ,  $\overline{U} = 0.65$ ), the coordinates are (1) (0.6, 0.83); c = 3, (2) (1.2, 2.99); c = 0, (3) (2.77, 3.06); c = 1, (4) (3.47, 0.9); c = 0, and (5) (2.8, 4.15); c = 0.

For Algo<sub>I</sub> ( $\sigma = 0.10, 0.95, \overline{U} = 0.65$ ), the coordinates are (1) (0.83, 2.62); c = 0, (2) (3.37, 0.84); c = 3, (3) (0.17, 3.42); c = 0, (4) (0.6, 0.91); c = 1, (5) (2.6, 0.06); c = 0, (6) (1.1, 3.89); c = 0, (7) (3.17, 1.89); c = 0, (8) (2.67, 3.68); c = 0, and (9) (1.13, 0.02); c = 0.

As shown in Fig. 3(a), Algo<sub>I</sub> and Algo<sub>B</sub> stop installing pico BSs before  $\bar{U} = 0.85$  is met since they discover that the utility no longer increases due to intercell interference even if more pico BSs are installed. Thus, the locations of pico BSs are the same for the two algorithms irrespective of  $\sigma$ . We also note that Algo<sub>I</sub> produces the same pico cell placements for both  $\sigma = 0.10$  and  $\sigma = 0.95$  when  $\bar{U} = 0.65$ . This is because the utility of each BS should meet the minimum threshold,  $\bar{U}$  (i.e., the constraint of Eq. (9)) in Algo<sub>I</sub>, while maximizing  $\sum_{\forall b_I \in B} (U(b_I) - \sigma C(b_I))$ .

After installing the *n*th picocell according to  $\text{Algo}_B$  or  $\text{Algo}_I$ , we measure the average utility improvement per UE in the new *n*th picocell, which we denote by  $\Delta U$ . Considering all the UEs in the entire simulation area, we also measure the total utility after installing the *n*th picocell. Fig. 4 shows the total utility for  $\text{Algo}_B$  and  $\text{Algo}_I$ . When  $\overline{U} = 0.65$  and  $\sigma = 0.10$ ,  $\text{Algo}_B$ installs eight pico BSs as shown in Fig. 3(b). Three more pico BSs are installed for  $\sigma = 0.10$  than for  $\sigma = 0.95$ , as can be seen in Fig. 3 because more weight is put on the utility than the cost. When eight picocells are deployed,  $\text{Algo}_B$  with  $\sigma = 0.10$  improves the total utility by 42.9 %. We observe from Fig. 4



Figure 3: The simulation areas with the pico cells deployed by the proposed algorithms,  $Algo_I$  and  $Algo_B$ .



Figure 4:  $\Delta U$  and the total utility for the simulation areas illustrated in Fig. 3

Algorithm	$\Lambda_1 \ (\Lambda_1') \ (\%)$	$-\Delta U~(\%)$	$\Lambda_2 \ (\Lambda_2') \ (\%)$	$+\Delta U~(\%)$
$Algo_I/Algo_B \ (\bar{U} = 0.85)$	19.7(1.9)	10.2	22.7 (9.5)	40.0
Algo <sub>B</sub> ( $\sigma = 0.10, \bar{U} = 0.65$ )	23.9(1.1)	7.8	23.1 (10.2)	28.9
Algo <sub>B</sub> ( $\sigma = 0.95, \bar{U} = 0.65$ )	15.9(0.0)	9.4	19.3 (7.6)	39.9
Algo <sub>I</sub> ( $\bar{U} = 0.65$ )	21.2(1.9)	9.2	20.1 (8.7)	33.4

Table 5: The average utility degradation and improvement.

that  $\Delta U$  is improved by up to 534 % in the cell of pico BS (3) installed by the proposed algorithm. Algo<sub>I</sub> installs nine picocells in the order shown by the numerical labels in Fig. 3(d). Also, the total utility increases as each of the nine pico BSs is installed. After installing all the nine picocells, the total utility improvement of Algo<sub>I</sub> is 44.9 %, compared with the macrocell-only deployment where no picocells are installed.

Given  $C_{I,k} = 0.2$ , the average picocell installation cost per total utility improvement is 14.49 for the two algorithms when  $\bar{U} = 0.85$ , while 14.43 (15.08) and 13.80 for Algo<sub>B</sub> with  $\sigma = 0.10$  ( $\sigma = 0.95$ ) and for Algo<sub>I</sub>, respectively, when  $\bar{U} = 0.65$ . Although the total utility is improved more for  $\sigma = 0.10$  than for  $\sigma = 0.95$  due to the additional three picocells, we see from Fig. 4 that the utility does not continue to increase significantly by simply installing more pico BSs, due to the interference from other picocells.

We denote by  $\Lambda_1$  the average percentage of the UEs that experience utility degradation due to the installation of the pico BSs. Let  $\Lambda_2$  be the average percentage of the UEs that have utility improvements. Similarly, we denote by  $\Lambda'_1$  and  $\Lambda'_2$  the average percentage of the UEs that experience utility degradation and improvement, respectively, in the picocells. Thus, the UEs for computing  $\Lambda'_1$  and  $\Lambda'_2$  are a subset of those for  $\Lambda_1$  and  $\Lambda_2$ . Table 5 shows

Table of The average admity per eight addit region with inght traine admity.							
		$\mathrm{Algo}_I/\mathrm{Algo}_B$	$\operatorname{Algo}_B$	$Algo_B,$	$\mathrm{Algo}_I$		
Region	No picocell	$\bar{U} = 0.85$	$\bar{U} = 0.65$	$\bar{U} = 0.65$	$\bar{U} = 0.65$		
		$\sigma = 0.10,  0.95$	$\sigma=0.10$	$\sigma=0.95$	$\sigma = 0.10,  0.95$		
1	0.87	0.90	0.89	0.89	0.90		
2	0.70	0.73	0.74	0.72	0.73		
3	0.81	0.82	0.82	0.82	0.81		
4	0.84	0.86	0.86	0.88	0.86		

Table 6: The average utility per UE in each region with high traffic density.

the average utility degradation per UE (denoted by  $-\Delta U$ ) and the average utility improvement (denoted by  $+\Delta U$ ) associated with the UE percentages  $\Lambda_1$  and  $\Lambda_2$ , respectively. The table explains why  $\Delta U$  in the picocells (See Fig. 4) is higher than the total utility improvement shown in Table 5. The total utility does not improve as much as  $\Delta U$  in the picocells does due to the presence of utility deterioration (i.e.,  $-\Delta U$ ). We also see in Table 5 that the average utility reduction for the  $\sigma = 0.10$  is slightly higher than that for  $\sigma = 0.95$  due to the interference from the three more deployed pico BSs, whereas  $\Lambda_2$  and the corresponding  $+\Delta U$  for  $\sigma = 0.10$  are much higher than those for  $\sigma = 0.95$ .

In the simulation, we observe the effect of installing picocells on the UEs that have the lowest utility, which is 0.398. The UEs' utility is increased up to 0.998 for both  $Algo_I$  and  $Algo_B$ . Table 6 shows the average utility per UE in each region with high traffic density. We see from the table that, in all four regions with high traffic density, the proposed algorithms improve the utility compared to the only-macrocell deployment.

We now examine the fairness of utility per UE observed at the nine macro cells for the proposed algorithms. We employ Jain's fairness index since it has been used extensively as a fairness metric in the literature [23]. The Jain's index J for the utility per UE, x is defined as  $J = \frac{(\sum x)^2}{|B| \sum x^2}$ . We obtain J = 0.869, 0.820, and 0.906 for Algo<sub>B</sub> ( $\sigma = 0.10$ ), Algo<sub>B</sub> ( $\sigma = 0.95$ ), and Algo<sub>I</sub>, respectively. From the results for J, it is observed that Algo<sub>I</sub> distributes the pico cells over the macro cells more evenly than Algo<sub>B</sub> does because Algo<sub>I</sub> focuses on meeting  $\overline{U}$  based not on the total utility of the heterogeneous networking system but on the utility at each macro BS.

## 6.3. Quality of Optimal Solutions

To evaluate the quality of the optimal solution obtained from our proposed heuristic algorithm, an upper bound on the optimal solution of the BS placement optimization problem in Eq. (6) is necessary. A heuristic solution that is closer to the upper bound is considered to be of high quality. We use a Lagrangian Relaxation (LR) method [24] to obtain an upper bound on  $\sum_{\forall b_I \in B} (U(b_I) - \sigma C(b_I))$  for  $\text{Opt}_B$  and  $\text{Opt}_I$ , which are formulated in Section 3.

When  $\bar{U} = 0.85$ , the gap between the LR upper bound and the solution obtained from our heuristic algorithm for  $Opt_B$  and  $Opt_I$  is 8.64 % for the two algorithms. Note that the gap is computed by comparing the solution obtained from our heuristic algorithm not with the optimal solution but with the LR upper bound. When  $\bar{U} = 0.65$ , the gap is 3.52 % and 0.26 % for Algo<sub>B</sub> with  $\sigma = 0.10$  and 0.95, respectively, while 6.77 % for Algo<sub>I</sub> irrespective of  $\sigma$ . The reason why the gaps are the same for the two different values of  $\sigma$  in Algo<sub>I</sub> is that Algo<sub>I</sub> produces the same pico cell placements for both  $\sigma = 0.10$  and 0.95 due to the constraint of Eq. (9), as explained in Section 6.2. In Algo<sub>B</sub>, the gap for  $\sigma = 0.95$  is smaller than that for  $\sigma = 0.10$  because the higher  $\sigma$ , the more weight is put on the installation cost, leading to smaller number of BSs to be installed. That is, the dimensionality of the search space decreases with an increase in  $\sigma$ .

## 7. Conclusion

In this paper, we proposed an algorithm to deploy pico BSs throughout macrocell coverage areas in LTE-Advanced heterogeneous networks, aiming to increase the system utility with a reasonable installation cost. We formulated the problem of optimal picocell placement by considering the inter-cell interference and the configuration of ABS, the traffic demands in the network, and the picocell installation cost. The simulation study demonstrates that the proposed picocell placement algorithm increases the utility, particularly for the UEs located in areas of high traffic density, while minimizing the picocell installation cost.

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