Theoretical Failure Envelopes of Open Hole Composite Laminates With A- and B-basis Allowables Estimated from Smooth Specimen Properties (*)

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Abstract

Composite material property databases are important information to guide designers. Aeronautical companies spend a great deal of effort in terms of money and labor to build such databases, which are considered confidential. The development of a single fiber-resin system design allowables database can take years and cost in the order of tens of million dollars. Based on this information, theoretical failure theories can generate failure envelopes which will define the reliability of composites components. However, most handbook-based composite material property databases contain incomplete information. Usually only the test average values of material properties are given. Failure envelopes generated from these databases are thus deterministic. It would then be desirable to include uncertainties in order to increase the reliability of the design. A-basis (99% coverage; 95% confidence) and B-basis (90% coverage; 95% confidence) design allowables are a standard way to introduce uncertainties into the experimental data. In this paper, a case study is presented in which an average design allowable failure envelope of open hole specimens was obtained numerically for a quasi-isotropic carbon fiber-epoxy laminate from smooth specimens material property database. Using the method of statistical design of experiments, it is then shown how the average design allowable can be supplemented with uncertainty estimates. In addition, A-basis and B-basis design allowables failure envelopes are presented using DATAPLOT, a public-domain statistical analysis software package available online. Application of this methodology to predicting reliability of composite structures is discussed.

Keywords: A-bass; B-basis; carbon fiber-epoxy laminate; composite materials; design allowables; design of experiments; factorial orthogonal design; failure envelopes; failure theories; open-hole strength; reliability; tolerance limits theory; uncertainty quantification.

INTRODUCTION

This paper is a continuation of an investigation by Shah, Melo, Cimini, and Fong [1][1], who presented a case study where a failure envelope of open hole specimens was first obtained numerically for a quasi-isotropic carbon fiber-epoxy laminate from the average material properties of smooth specimens. Using the method of statistical design of experiments and finite element method-based simulation of failure at estimated variation of the material properties, they showed how the average-property-based failure envelope can be supplemented with uncertainty estimates by using DATAPLOT [2], a public-domain statistical analysis software package available online. In this paper, we will go one step further by scaling up the specimen-based failure envelope to the full-scale structures with the estimation of the A-basis (99% coverage; 95% confidence) and B-basis (90% coverage; 95% confidence) design allowables from the theory of tolerance limits.

For completeness, we begin this paper with the same observation made by Shah, Melo, Cimini Jr, and Fong in their 2010 paper [1] that lightweight engineering structures such as aircraft fuselage contain a number of openings and notches due to the presence of windows, doors, access points, bolt holes, rivet joints, etc. Stress concentrations developed around these notches and openings make them a potential source for failure initiation. Thus numerous studies have been conducted to investigate the strength of notched laminates including open holes and filled holes [3-13].

In the aerospace industry, open hole tests have been standardized as a method to generate a design allowable [8-11]. However, standards for such open hole tests are limited to uniaxial tension or compression loadings. This is primarily due to the fact that testing for biaxially loaded specimens is not only very difficult but also expensive. Thus often, data generated from the open hole tension and compression tests, designated as Category 3, are coupled with criteria such as maximum stress criterion to estimate the strength in biaxial loads. Such an approach may produce either an over-conservative or unsafe design [11]. Further, such a design allowable generated using notched specimens are very much specific to the layup definition and geometry and therefore cannot be generalized. It is then recommended the notched design allowable be computed using data-set from smooth specimens, producing Category 2 data using Category 1 data. This not only eliminates the erroneous approach of using uniaxial data to empirically estimate biaxial data but also reduces the experimental costs involved in design and optimization phase by one order of magnitude [12].

In order to obtain Category 2 data from Category 1 data, computational methods such as finite element analysis can be used [13]. However, results obtained from such numerical computation rely heavily on the accuracy of data supplied for Category 1, which are among many, mechanical and hygro-thermal properties of the composite. Although the standard practice in measuring these Category 1 data consists of the testing of a number of coupons, the results are often reported as average values without any statistical variation information. The material properties reported in industrial datasheet as well as standard handbooks only consists of such average values.

During numerical simulation to generate Category 2 data, the reported average values for Category 1 data, without any statistical variation, are taken into consideration, which leads to numerical results lacking statistical uncertainties. Also, the effect of the statistical variation of any property on the final result was ignored. Even with a state of the art numerical prediction tool, the

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[1]Figure in square brackets denotes a reference listed at the end of this paper.

results might be different from those expected as they do not include the statistical variations unlike experimental observations. It is apparent that the completeness of Category 2 data generated from Category 1 data is significantly dependent upon the completeness of the supplied raw data for the smooth specimens. Thus there is a need for including such statistical variation in Category 1 data.

In this work, the numerical computation for a design allowable of a smooth specimen from Category 1 ply strength data is coupled with a statistical tool in order to estimate the open hole strengths with their respective statistical parameters for a quasi-isotropic laminate for biaxial loading condition. In addition, failure envelopes for A-basis and B-basis design allowables are also presented.

II. TOOL FOR STATISTICAL ANALYSIS (DATAPLOT)

The statistical analysis performed in this work was based on the software DATAPLOT from NIST (National Institute of Standards and Technology) [2, 14-16]. DATAPLOT [2] is a multiplatform software for scientific visualization, statistical analysis and non-linear modeling. The option for Design of Experiments (DEX) was used [18]. It allows associating an experimental design to a virtual finite element method simulation including variation on the input parameters in order to perform an estimation within 95 % confidence interval.

III. SMOOTH SPECIMEN PROPERTIES – A NUMERICAL EXAMPLE

An open hole notched plate was considered as reported in an experimental study [17]. A square plate of 32mm x 32mm size has a central circular opening with a diameter of 6.35 mm as shown in Figure 1. The laminate was made up of quasi-isotropic $[45/90/-45/0]_{4s}$ IM7/8552 composite, with overall thickness of 4mm. The mechanical properties of the material are as shown in Table 1 [17], which forms Category 1 data.

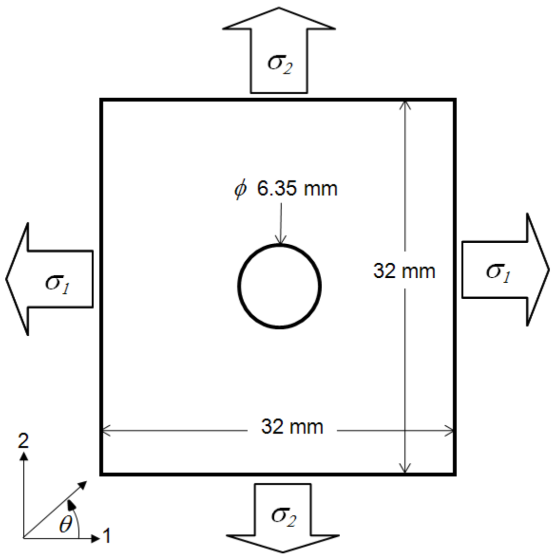

Figure 1 – Open hole IM7/8552 plate.

In Table 1, E_{11}, E_{22} and G_{12} are the longitudinal, transverse and shear modulus of elasticity, respectively; ν_{12} is the main Poisson's ratio; X, X', Y, Y', and S, are longitudinal tensile, longitudinal compressive, transverse tensile, transverse compressive and interlaminar shear strengths, respectively.

Table 1 - Ply properties of IM7/8552 (Category 1) [17]

E_{11} GPa	E_{22} GPa	G_{12} GPa	ν_{12}	X/X' MPa	Y/Y' MPa	S MPa	Thickness mm
150	11	4.6	0.3	2,400/ 1,690	111/ 250	120	0.125

It should be noted that Category 1 data reported in [17] are average values and do not have any statistical information (e.g., standard deviation, coefficient of variation, sample size, etc.). In order to consider the data for this work along with statistical variations based on the reference values in Table 1, four ply strengths were varied within feasible range as shown in Table 2. For a statistical design of experiments [18], we choose a 2-level, full factorial design for four factors plus a center point. This means we need a total of $2^4 + 1$, or, 17 runs. A total of 17 combinations, designated as Runs 1 though 17, were considered which would serve as input data for computational runs performed in this study. The first combination in Table 2 represents the average values.

It can be observed, in Table 2, that the variations were included only for tensile and compressive longitudinal strengths (X and X') and for tensile and compressive transverse strengths (Y and Y'). The coefficient of variations included were $\pm$14% for X, $\pm$21% for X', $\pm$11% for Y, $\pm$20% for Y', obtained using arbitrary two standard variations (2σ) for the average data from reference [17]. Such arbitrary variation was chosen primarily to observe the effect of such variations on the final result from the simulation, the result being the ultimate strengths of the specimen. Although in this work statistical

Table 2 - Ply properties variations for IM7/8552

	X MPa	X' MPa	Y MPa	Y' MPa
Run-1	2400	1690	110	250
Run-2	2064	1335	98	200
Run-3	2736	1335	98	200
Run-4	2064	2045	98	200
Run-5	2736	2045	98	200
Run-6	2064	1335	122	200
Run-7	2736	1335	122	200
Run-8	2064	2045	122	200
Run-9	2736	2045	122	200
Run-10	2064	1335	98	300
Run-11	2736	1335	98	300
Run-12	2064	2045	98	300
Run-13	2736	2045	98	300
Run-14	2064	1335	122	300
Run-15	2736	1335	122	300
Run-16	2064	2045	122	300
Run-17	2736	2045	122	300

variations considered are limited to these four ply strengths, the same should be considered for other properties in Table 1 for data completeness.

With inputs from Table 2, numerical analysis was performed to extract failure envelopes for the open hole specimen under inplane (σ_1,σ_2) biaxial loading conditions. For this, finite element method (FEM) based tool MicMac/FEA [13] was chosen. The code interfaces with ABAQUS Student Edition to perform failure analysis of the given specimen at various biaxial loading combinations, plotting as result a failure envelope for the given specimen. The FEM model was created with conventional shell elements of quadrilateral S4R type. Since delamination was not observed in the experimental study [17] for the considered plate thickness, no delamination was simulated. A structured meshing scheme was implemented with 512 elements and 544 nodes as shown in Figure 2. A simultaneous degradation scheme was chosen to compute the tensile or compressive ultimate strength of the specimen. In this scheme, once a failure initiation was noticed as per Tsai-Wu criterion, elastic properties of the whole laminate are degraded to obtain the ultimate ply failure strength. This method was chosen for demonstration purposes of statistical analysis due to its computational efficiency, although results obtained generally yields to relatively conservative predictions as compared to more accurate methods such as progressive damage models.

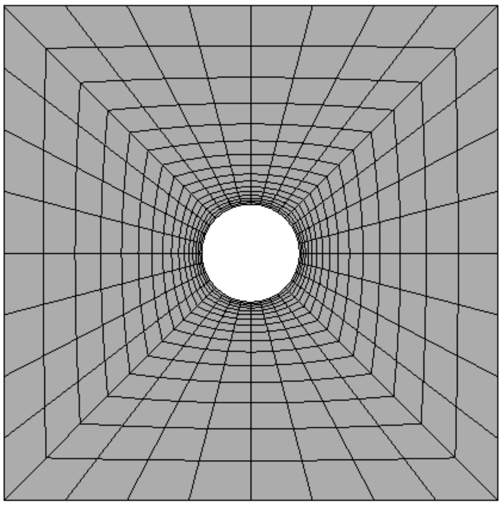

Figure 2 – A finite element mesh design for the open hole specimen.

IV. SMOOTH SPECIMEN PROPERTIES WITH UNCERTAINTY ESTIMATION

A total of 17 failure envelopes were obtained for the open hole specimen. Each set corresponds to the input data (Runs-1 to 17) as reported in each row of Table 2. In Figure 3, a failure envelope obtained using average ply strengths (Run-1) is plotted along with the data obtained from entire combinations considered (Runs-2 to 17) in Table 2. It is evident that the design allowable generated using average values do not address the possibility of the data scatter as would be covered by inclusion of statistical variation in the input strengths.

Similarly, Figure 4 represents inner and outer limits for possible failure plot data points with all the statistical variation considered for the input ply level strength parameters (Runs 1-17).

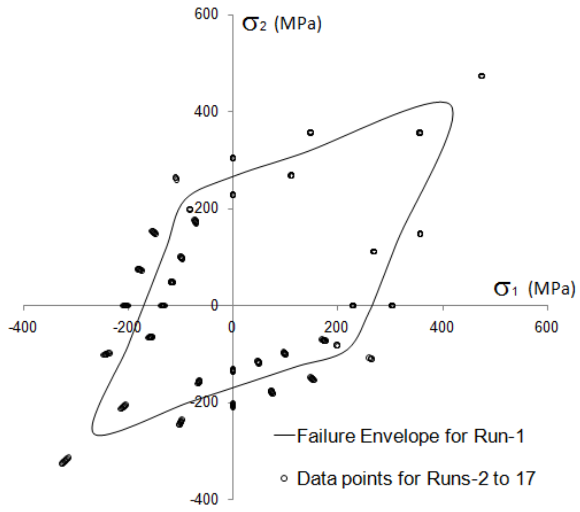

Figure 3 – Failure envelope for the open hole specimen.

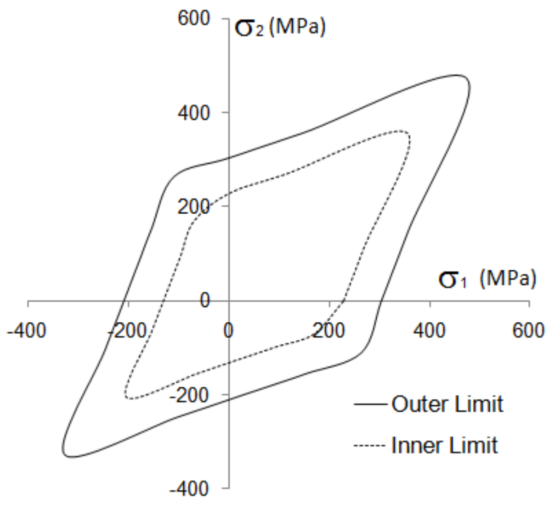

Figure 4 – Limits for possible failure plot data points with all the statistical variation considered (Runs 1-17).

In Table 3, biaxial strengths (σ_{1ULT}, σ_{2ULT}) of the specimen with properties considered in Run-1 (average values), Run-5 and Run-16 are given for varying load ratio. These given values are taken from the corresponding locus of the failure envelopes. It is observed that the computed biaxial strength with statistical variation in consideration can vary up to 22% of that of the strength obtained from average value only (Run-1).

Table 3 - Typical Computed biaxial strengths for 3 of the 17 Runs of FEM Simulations

Load ratio σ_1:σ_2	(σ_{1ULT}, σ_{2ULT}) MPa		
	Run-1	Run-5	Run-16
1:0	(266,0)	(304,0)	(228,0)
1:1	(415,415)	(474,474)	(356,356)
0:1	(0,266)	(0,304)	(0,228)
-1:1	(-125,125)	(-150,150)	(-152,152)
-1:0	(-170,0)	(-204,0)	(-207,0)
-1:-1	(-265,-265)	(-318,-318)	(-323,-323)
0:-1	(0,-170)	(0,-204)	(0,-207)
1:-1	(125,-125)	(150,-150)	(152,-152)

The statistical procedure adopted was to associate each experimental design to its respective finite element simulation in order to predict the selected strengths within a 95% confidence interval. Using the finite element analysis results, it was possible to estimate the statistical strength parameters for all the eight load cases studied. Therefore, Figures 5 to 9 graphically depict these

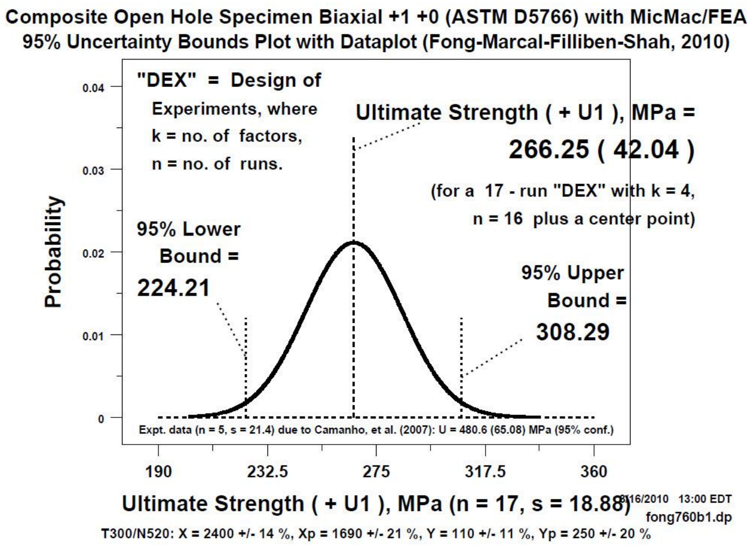

Figure 5 – Estimated open hole laminate statistical tensile strength for Case (a): Load ratio σ_1:σ_2 = 1:0

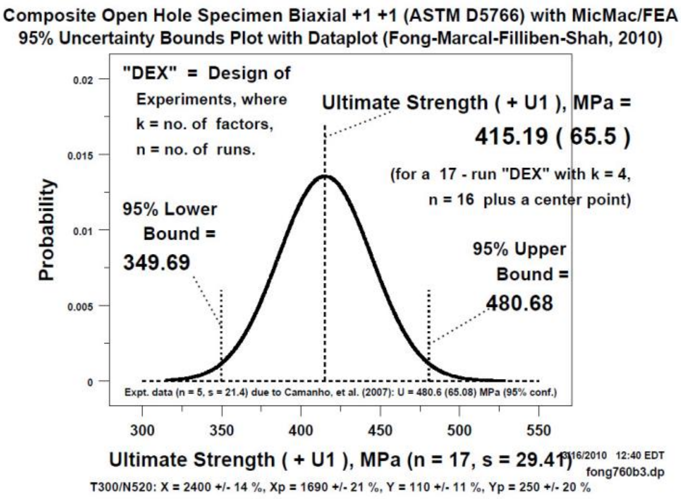

Figure 6 – Estimated open hole laminate statistical tensile strength for Case (b) Load ratio σ_1:σ_2 = 1:1

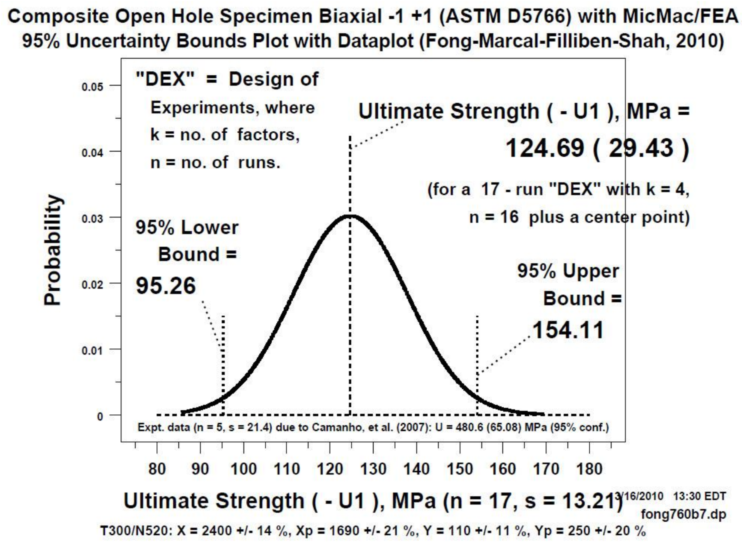

Figure 7 – Estimated open hole laminate statistical compressive strength for Case (c): Load ratio σ_1:σ_2 = -1:1

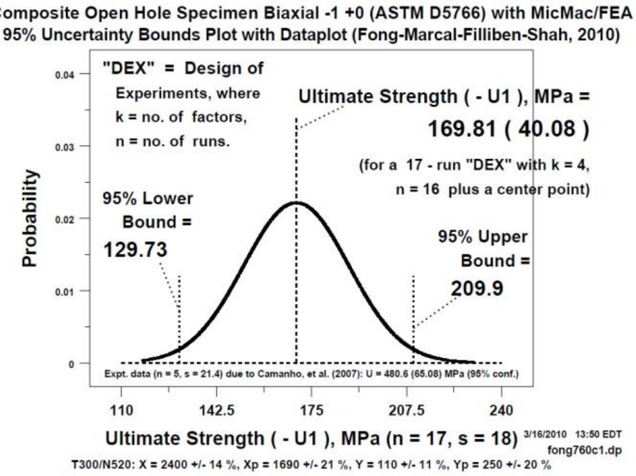

Figure 8 – Estimated open hole laminate statistical compressive strength for Case (d): Load ratio σ_1:σ_2 = -1:0

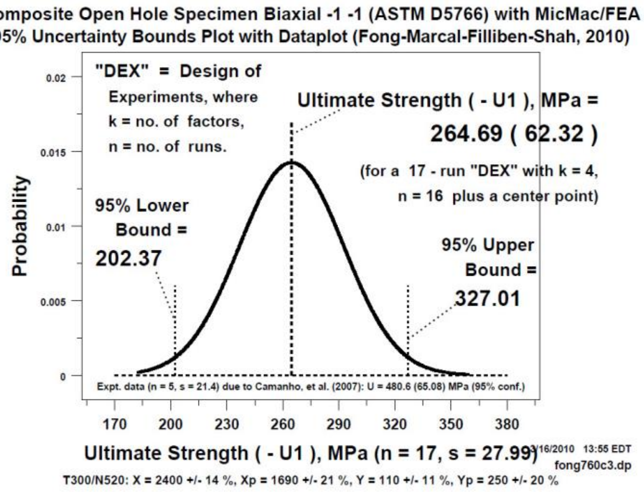

Figure 9 – Estimated open hole laminate statistical compressive strength for Case (e): Load ratio σ_1:σ_2 = -1:-1

parameters in a normal distribution, which we choose to model as a first approximation. Here, Figure 5 presents the strength estimated with respective statistical parameters for load ratio σ_1:σ_2 of 1:0 (unidirectional tensile). Figure 6 presents the strength estimated with respective statistical parameters for load ratio σ_1:σ_2 of 1:1 (first quadrant in σ_1-σ_2 domain). Figure 7 presents the strength estimated with respective statistical parameters for load ratio σ_1:σ_2 of -1:1 (second quadrant in σ_1-σ_2 domain). Figure 8 presents the strength estimated with respective statistical parameters for load ratio σ_1:σ_2 of -1:0 (unidirectional compressive). Figure 9 presents the strength estimated with respective statistical parameters for load ratio σ_1:σ_2 of -1:-1 (third quadrant in σ_1-σ_2 domain).

Table 4 shows a summary of the main estimated statistical strength parameters for all load ratio cases, which are mean, standard deviation, coefficient of variations and 95% bounds. The coefficient of variations is given by the standard deviation divided by the mean. It can be observed that the coefficient of variations for the three first load ratio cases σ_1:σ_2 of 1:0, 1:1, and 0:1 presented the same value of 0.071. On the other hand, the coefficient of variations for the remaining five load ratio cases (-1:1, -1:0, -1:-1, 0:-1, and 1:-1) also presented the same value, 0.106, however different from the previous three load ratio cases. This occurs because the open hole biaxial strengths for the first three load ratio cases were controlled by the ply tensile strength, which had lower variation, while the strengths for the remaining five load ratio cases were controlled by the ply compressive strength, which had higher variation (Table 2).

Table 4 - Estimated open hole laminate statistical strengths based on a 4-factor, 2-level, orthogonal factorial design of a 17-run finite element simulation experiment simulations

Load ratio $\sigma_1 : \sigma_2$	Estimated open hole laminate strength MPa			
	Mean	Standard Deviation	Coefficient of Variations	95% Bounds
1:0	266.25	18.88	0.071	42.04
1:1	415.19	29.41	0.071	65.50
0:1	266.25	18.88	0.071	42.04
-1:1	-124.69	13.21	0.106	29.43
-1:0	-169.71	18.00	0.106	40.08
-1:-1	-264.69	27.99	0.106	62.32
0:-1	-169.71	18.00	0.106	40.08
1:-1	124.69	13.21	0.106	29.43

V. THEORY OF TOLERANCE LIMITS AND MIL-HANDBOOK-17-1F, CHAPTER 8

To show how we extrapolate the smooth specimen properties of a certain material with a prediction 95% confidence interval such as those given in the last column of Table 4, to the integrity assessment of a full-scale structure of the "same" material, we apply the classical theory of tolerance limits (see, Prochan [19], Eisenhart, Hastay and Wallis [20], Natrella [21], and Beyer {22]) and its implementation in the MIL-Handbook-17-1F [23]. As stated in the introductory section of Chapter 8 of this handbook [23],

> *"Variability in composite material property data may result from a number of sources including run-to-run variability in fabrication, batch-to-batch variability of raw materials, testing variability, and variability intrinsic to the material.*
>
> *"It is important to acknowledge this variability when designing with composites and to incorporate it in design values of material properties. Procedures for calculating <u>statistically-based</u> material properties are provided in this chapter."*

For some background information on the notion of a confidence interval, we refer our readers to engineering statistics textbooks such as Nelson, Coffin, and Copeland [24], and Vining and Kowalski [25], where the former includes an excellent presentation of not just one but three types of statistical interval estimates [24, pp. 165-181], namely, (1) confidence interval, (2) predictive interval, and (3) tolerance interval. Some interesting properties of the tolerance interval, as noted in [24, p. 178], are quoted below:

> *"The width of a tolerance interval depends mostly on the amount of spread (e.g., standard deviation) in the population.*

"If the members of the population vary widely, tolerance intervals will necessarily be wide, no matter how large a sample is taken.

"The simplest situation for constructing tolerance intervals is when the underlying population is (at least approximately) normally distributed."

In the handbook [23], details of calculating the tolerance intervals and design bases were given for three commonly-used distributions, namely, the normal, the lognormal, and the two-parameter Weibull. In this paper, we will illustrate our methodology by assuming the simplest underlying distribution, namely, the normal. Furthermore, we assume that our data sets are "unstructured" as defined in [23], namely, "data for which all relevant information is contained in the response measurements themselves," and "these measurements are all that is known."

VI. A- AND B-BASIS DESIGN ALLOWABLES

We adopt the definitions of the A-basis and B-basis values or design allowables as follows [23]:

"*A-basis value -- A statistically-based material property; a 95 % lower confidence bound on the first percentile of a specified population of measurements*," or, "a *95 % lower tolerance bound for the upper 99 % (coverage) of a specified population*." The two definitions are identical to each other.

"*B-basis value -- A statistically-based material property; a 95 % lower confidence bound on the tenth percentile of a specified population of measurements*," or, "a

95 % lower tolerance bound for the upper 90 % (coverage) of a specified population." The two definitions are identical to each other.

Assuming our data set is unstructured and normally distributed, we wrote a macro in DATAPLOT to calculate the A-basis and B-basis design allowables from the set of means and standard deviations of smooth specimen properties given in Table 4. Our results are given in Table 5. A listing of the DATAPLOT macro is given in Appendix A. A typical output file from the macro is given in Appendix B.

Table 5 - Design allowables of open hole composite laminates estimated from smooth specimen properties assuming normal distribution

Distinct Case	Load ratio $\sigma_1 : \sigma_2$	Estimated open hole laminate strength MPa				Design Allowables	
		Mean	Standard Deviation	Coefficient of Variations	95% Bounds	A-basis design allowable	B-basis design allowable
(a)	1:0	266.25	18.88	0.071	42.04	201.8	228.5
(b)	1:1	415.19	29.41	0.071	65.50	314.8	356.3
a	0:1	266.25	18.88	0.071	42.04	201.8	228.5
(c)	-1:1	-124.69	13.21	0.106	29.43	- 79.6	- 98.3
(d)	-1:0	-169.71	18.00	0.106	40.08	- 108.3	- 133.7
(e)	-1:-1	-264.69	27.99	0.106	62.32	- 169.1	- 208.7
d	0:-1	-169.71	18.00	0.106	40.08	- 108.3	- 133.7
c	1:-1	124.69	13.21	0.106	29.43	79.6	98.3

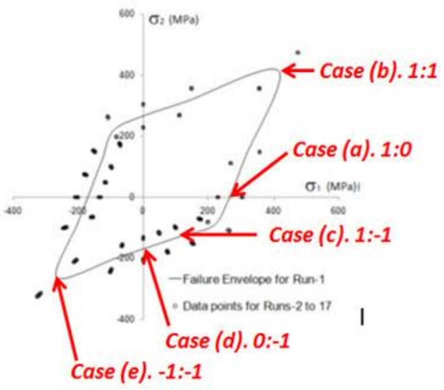

Figure 10 – The five distinct cases (a) to (e) of the failure envelope.

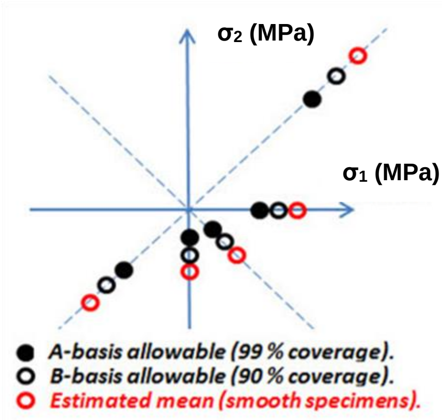

Figure 11 – Mean value and two design allowables for each case.

A graphical representation of the five distinct cases of a failure envelope is given in Fig. 10. The relationship between the mean value and two design allowables for each case is shown in Fig. 11.

VII. SIGNIFICANCE AND LIMITATIONS OF RESULTS

Two sets of results are presented in this paper. We use the design of experiments methodology to estimate uncertainty in smooth specimen properties (first set of results), and the theory of tolerance limits to extrapolate the smooth specimen properties to full-scale structures (second set of results). Both sets are significant in accomplishing three fundamental objectives in engineering and materials science, namely, (1) to resolve the data scatter problem of a material characterization test, (2) to report the results of such a task with an error bar, and (3) to apply a laboratory measurement to a full-scale structure with an expression of uncertainty. As shown by Fong, et al, in two recent articles on failure analysis [26, 27], the availability of an error bar in material property measurements such as yield strength, ultimate tensile strength, buckling strength, and fracture toughness provides a basis for using a non-deterministic and physically more realistic model to simulate progressive weakening and partial or complete failure of a structure or component in an abnormal or aging-related collapse scenario.

Clearly, the two uncertainty-estimation methodologies as applied to the design problem are not without limitations. For the first methodology, i.e., the use of the powerful method of design of experiments (DEX) to reduce any number of factors to two or less dominant ones (as shown in a typical output plot in Fig. 12) , the primary limitation stems from the difficulty of reducing a large list of test-related factors to a small and manageable number. A second and perhaps more serious limitation is the judgment-based requirement of high and low settings of each factor, making the experimental runs more difficult and the analysis outcome less rigorous when the setting is not achieved to a desired value.

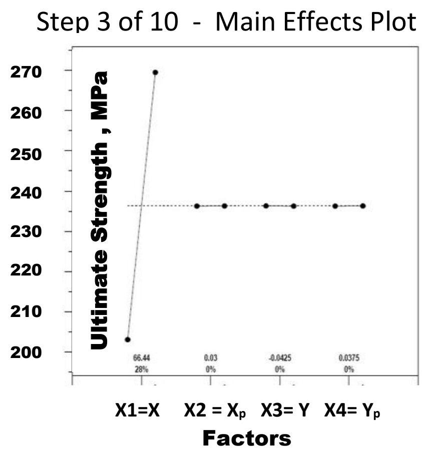

Figure 12 – A typical DEX-analysis output plot file from one of 16 runs.

For the second methodology (tolerance limits), the user needs to be aware that the validity of the tolerance factors furnished by Natrella [21], or Beyer [22] and estimated in MIL-Handbook-17-1F [23] hinges on the assumption that the measured data is normally distributed. This means that all such experimental data need to be tested for normality, and if they fail, we need to apply methods for non-normal populations. Nevertheless, as long as the user is aware of those limitations and interprets the results of the analysis accordingly, we believe the two methodologies are useful additions to the engineer's tool box.

VIII. CONCLUDING REMARKS

The results presented in this paper establish not only the importance of inclusion of statistical variation in Category 1, but also the usefulness of the methodology in evaluating A-basis and B-basis design allowables. Ignoring the statistical variation for smooth specimen data readily affects the completeness of design allowable generated as Category 2 data. Deterministic results obtained from the traditional average-value-type of calculations are inadequate as a basis for a reliability analysis of composite structures.

We wish to make two more points on the limitations of the uncertainty quantification (UQ) methodology described in this paper. First of all, we chose a very simple path to estimating the uncertainties of all stress variables and material parameters by endowing each quantity with two numbers, namely, mean and standard deviation. In reality, this is only a first step in a long journey toward a proper way to address the uncertainties of composites failure strength prediction problem, as shown in an ISO guide to the expression of uncertainty in measurement [28] and a recent review paper by Kacker, Sommer, and Kessel [29]. Secondly, the stochastic nature of the ultimate failure of a specimen or a full-scale composite structure requires an understanding of the stochastic nature of the progressive weakening of their microstructure due to various failure modes such as debonding and fibre breaking, as shown in a companion paper by Fong, Marcal, Heckert, and Filliben [30]. Nevertheless, our simple-minded approach does offer a statistically-based and computationally-friendly method to assess composites failure theories versus experimental data and to generate an evidence-based ultimate strength design allowables for aircraft design.

IX. ACKNOWLEDGMENT AND DISCLAIMER

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APPENDIX A

```
. ---------------------------------------------------------------------------------------------------
,     THIS IS A DATAPLOT FILE.   DATE OF EXECUTABLE FILE:  JUNE 14, 2012
, ---------------------------------------------------------------------------------------------------
. Date:   Aug. 20, 2012          Filename:      fong820a.dp
.
. By:        Jeffrey T. Fong,     Div. 771, NIST, fong@nist.gov
.
.       and N. Alan Heckert,     Div. 776, NIST, alan.heckert@nist.gov
. ---------------------------------------------------------------------------------------------------
.   Purpose:  Given      mean (xmean), standard deviation (xsd), and
.                        sample size (n) of a univariate distribution, and
.               assume   the distribution is normal,
.               find     the one-sided lower tolerance limits for
.
.                        90%, 95%, 99% confidence levels, and
.                        50%, 75%, 90%, 95%, 99%, 99.9% coverages
.                        in a tabular format.
. ---------------------------------------------------------------------------------------------------
.    Step 1:  Input sample size (n), mean (xmean), and stand. dev. (xsd)
. ---------------------------------------------------------------------------------------------------
echo on
capture fong820a.out
set write decimals 6
.
let n = 17
let xmean = 266.25
let xsd = 18.88
. ---------------------------------------------------------------------------------------------------
.    Step 2:  Calculate one-sided lower tolerance limits (display table format)
. ---------------------------------------------------------------------------------------------------
normal summary lower tolerance limits xmean xsd n
. ---------------------------------------------------------------------------------------------------
.    Step 3:  Calculate and display A-basis (A2) and B-basis (B2) allowables
. ---------------------------------------------------------------------------------------------------
. -----  Note: Input specific confidence level (alpha) and coverage (gamma)
let alpha = 0.95
. ------- one-sided tolerance limits for A-basis allowable (gamma = 0.99)
let gamma = 0.99
let xmeanvec = data xmean
let xsdvec   = data xsd
let nvec     = data n
let A2 = summary normal toleranc one sided lower limit xmeanvec xsdvec nvec
.
. ------- one-sided tolerance limits for B-basis allowable (gamma = 0.90)
let gamma = 0.90
let B2 = summary normal toleranc one sided lower limit xmeanvec xsdvec nvec
.
end of capture
echo off
Exit
. ---------------------------------------------------------------------------------------------------
.   END OF A DATAPLOT CODE.     NAME OF CODE:  fong820a.dp   Aug 20, 2012
. ---------------------------------------------------------------------------------------------------
```

APPENDIX B

A typical output file of the computer code using DATAPLOT [2] to calculate the A-and B-basis design allowables for a biaxial load ratio and the associated sample size, mean and standard deviation of the estimated open-hole laminate strength is given below:

Name of output file: fong820a.out

** let n = 17 **
THE COMPUTED VALUE OF THE CONSTANT N = 0.1700000E+02

** let xmean = 266.25 **
THE COMPUTED VALUE OF THE CONSTANT XMEAN = 0.2662500E+03

** let xsd = 18.88 **
THE COMPUTED VALUE OF THE CONSTANT XSD = 0.1888000E+02

One-Sided Lower Normal Tolerance Limits:

Summary Statistics:
Number of Observations: 17
Sample Mean: 266.250000
Sample Standard Deviation: 18.879999

Confidence = 95%

Coverage Value (%)	k Factor	Lower Limit	
50.0	0.423438	258.255472	
75.0	1.220493	243.207081	
90.0	2.001713	228.457646	--- B-basis
95.0	2.486268	219.309256	
99.0	3.414407	201.785994	--- A-basis
99.9	4.471363	181.830662	

THE COMPUTED VALUE OF THE CONSTANT A2 = 201.7860 (A-basis)

THE COMPUTED VALUE OF THE CONSTANT B2 = 228.4576 (B-basis)