Johnson Noise Thermometry Measurement of the Boltzmann Constant with a 200 Ω Sense Resistor

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***Abstract* — In 2010, NIST measured the Boltzmann constant *k* with an electronic technique that measured the Johnson noise of a 100 Ω resistor at the triple point of water (TPW) and used a voltage waveform synthesized with a quantized voltage noise source (QVNS) as a reference. In this paper, we present measurements of *k* using a 200 Ω sense resistor and appropriately modified QVNS circuit and waveform. Preliminary results show agreement with the previous value within the statistical uncertainty.** [1]

***Index Terms* — Boltzmann equation, Josephson array, measurement units, noise measurement, standards, temperature.**

I. Introduction

The Johnson-Nyquist equation (1) defines the thermal noise power (Johnson noise) $\langle V^2\rangle$ of a resistor in a bandwidth Δf through its resistance R and its thermodynamic temperature T:

$$\langle V^2\rangle = 4kTR\Delta f. \quad (1)$$

Therefore, it is possible to obtain the value of the Boltzmann constant k by measuring the Johnson noise and the resistance value of a sense resistor at a defined temperature such as the triple point of water (TPW). NIST has developed a Johnson-noise thermometer (JNT)-based cross-correlation electronics to optimize the measurement of the noise power of a sense resistance at the TPW. The measurement electronics are calibrated by use of a pseudo-noise voltage waveform synthesized with the quantized voltage noise source (QVNS) that acts as a reference signal [1-2].

After dramatically improving the system by reducing nonlinearities, electronic distortion, and EMI coupling [3], it was possible to obtain the first practical electronic measurement of the Boltzmann constant [4]. The value of k is obtained from fitting the ratio between the noise power spectra measured across the sense resistor and the one measured across the QVNS with a two-parameter fit equation $a_0+a_2f^2$, where k corresponds to a_0 at zero frequency. This previous measurement was done by use of a 100 Ω sense resistor and obtained a difference of 0.6×10^{-6} between the measured k and the 2006 CODATA value, with a relative combined total uncertainty $u(k) = 12.1 \times 10^{-6}$.

In this experiment, we test the JNT measurement technique under different measurement conditions by doubling the sense resistor value to 200 Ω to produce higher noise power, in the expectation this will yield a lower statistical uncertainty for the same integration period. The primary disadvantage expected for this higher resistance is that the transfer function of the input transmission lines will roll off faster. The 650 kHz cutoff frequency of the low-pass filters in the correlator amplifier chain still defines the measurement bandwidth.

II. Transmission Line Matching

In order to perform the measurements with the higher 200 Ω resistance, the voltage amplitude of the QVNS synthesized reference signal was first increased by $2^{1/2}$ to match the Johnson noise of the sense resistor. Different QVNS circuits were also required that had 200 Ω resistances on each output lead. Fortunately, the superconducting integrated circuits had been designed with this experiment in mind, because each lead resistor was constructed from four pairs of parallel 50 Ω resistors. To double the lead resistances, one resistor from each pair was mechanically opened.

In order to have well-matched transmission lines, the QVNS lead resistances must be within 1 % of the sense resistance. This required soldering chip resistors of a few ohms to each of the four output leads of the QVNS flex cryopackage. Metal film resistors were used because their values don't change significantly when immersed in liquid helium.

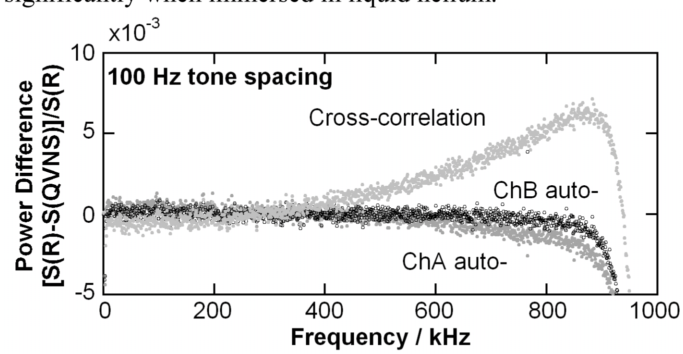

Fig. 1. Differences in power spectra [S(R)–S(QVNS)]/S(R) between the sense resistor and QVNS vs. frequency for the auto-correlation of channels A and B and the cross-correlation.

The last fine-tuning necessary was to match the transmission line impedances [5-6]. This modification was necessary for several reasons. First of all, the new QVNS chip was mounted in a new probe that is longer than the one previously used. The longer QVNS cable lengths and the different sensors changed the total impedance connected to the amplifiers. Matching was obtained by changing the length of the cable that connected the probes to the amplifiers and by modifying the dielectric between the conductors. Lead resistances were adjusted a second time to match the auto-correlation power of the sense-resistor and QVNS signals; 2.2 Ω resistors were added to all four sense resistor leads.

III. MEASURED DATA

The ratio of the acquired noise power spectra of the resistor and the QVNS was calculated over a measurement bandwidth of 640 kHz, from 10 kHz to 650 kHz. As in previous measurements, QVNS waveforms composed only of odd harmonics tones and having two different code lengths produced frequency combs with correspondingly different tone densities and amplitudes. The first code, of length 3×2^{23} bits, produced tones spaced at 800 Hz, whereas the eight-times longer code had tones spaced at 100 Hz.

The difference between the auto-correlated noise signals of the QVNS and the sense resistor for each channel is showed in Fig. 1. One can see that the two auto-correlation differences are matched to better than 2 parts in 10^3, whereas the cross-correlation has a quadratic behavior with increasing frequency. The correlated noise of the sense resistor is larger at higher frequencies for larger sense resistor because of the frequency dependence of the current noise of the input circuit. No correlated noise exists in the QVNS.

The ratios of the cross-correlated resistor and QVNS signals are also quadratic. Fig. 2 shows the residuals of the fitted ratios for both sets of data. Each data set in this plot is integrated over a 15-hour period. There is negligible (or "flat") frequency response of the ratio-fitting residual across the 640 kHz measurement bandwidth, and is especially visible on 800 Hz tones spacing curve which has smaller scatter. 800 Hz tones spacing curve

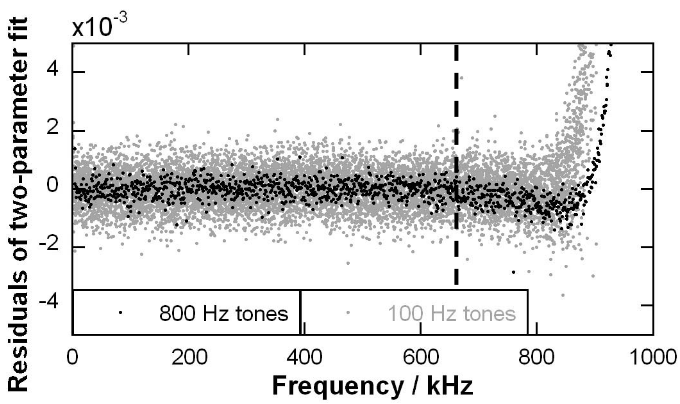

Fig. 2. Residuals of the two-parameter fit of the ratio between the Johnson noise of the resistance and QVNS noise for the 800 Hz and 100 Hz tone spacing of the two QVNS codes.

In order to analyze the JNT system's frequency response, we fit the data over different bandwidths starting at 10 kHz and stepping the ending frequency up to 650 kHz. Fig. 3 shows the difference between the calculated value a_0^{2006} and the coefficient a_0 obtained from fitting the acquired data [4].

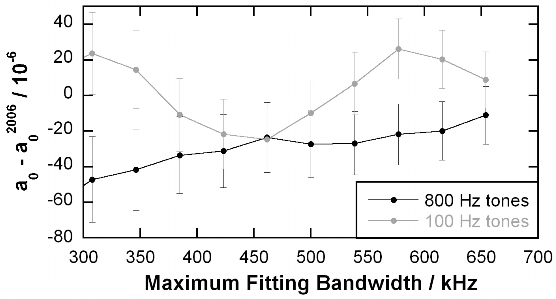

Fig. 3. Difference of a_0 fitted over different bandwidths from the 2006 CODATA value for QVNS signals of both tone spacings.

IV. CONCLUSION

Our measurements of the 200 Ω sense resistor show differences of -10×10^{-6} and $+27\times10^{-6}$ between the measured and 2006 CODATA values of k for the 800 Hz and 100 Hz tones spacing, respectively. For both sets of data, the statistical uncertainty is 16×10^{-6}. The larger current noise of this higher resistance JNT measurement appears not to affect the measurement of k, probably because all associated effects are removed by fitting, as are the other quadratic effects. As in the previous measurement [4], the combined relative uncertainty must account for variations in a_0 with fitting bandwidth. This effect is being investigated [7].

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