Flat frequency response in the electronic measurement of the Boltzmann constant

Jifeng Qu[1], Samuel P. Benz[2], Yunfeng Fu[1], Jianqiang Zhang[1], Horst Rogalla[2] and Alessio Pollarolo[2]

[1]National Institute of Metrology, Beijing, 100013 China
E-mail: qujf@nim.ac.cn

[2]National Institute of Standards and Technology, Boulder, CO 80305 USA

***Abstract* — A new quantum voltage calibrated Johnson noise thermometer (JNT) was developed at NIM to demonstrate the electrical approach that determines the Boltzmann constant *k* by comparing electrical and thermal noise power. A measurement with an integration period of 10 hours and bandwidth of 640 kHz results in relative offset of 0.5×10^{-6} from the current CODATA value of *k*, and type A relative standard uncertainty of 23×10^{-6}. The quadratic fitting parameters of the ratio spectrum show a flat response with respect to the measurement bandwidth. This flat response is a dramatic improvement compared to the response produced by the NIST JNT system that dominated the relative combined uncertainty of previous measurements of *k*.**

***Index Terms* — Boltzmann constant, Correlation, Josephson junction arrays, Noise, Quantization, Thermometry.**

I. Introduction

There has been much interest in measuring the Boltzmann constant *k* with comparable uncertainty by use of methods based on physical principles other than acoustic gas thermometry [1]-[2]. Recently, NIST reported the first practical electronic measurement of the Boltzmann constant with Johnson noise thermometer (JNT) that compared the thermal noise of a resistor at the triple-point of water (TPW) temperature to the pseudo-voltage-noise synthesized by a quantum voltage noise source (QVNS) [3]. The result of $k = 1.380\ 651(17)\times10^{-23}$ J/K was consistent with the current CODATA value. The 12×10^{-6} relative combined uncertainty of this previous measurement was dominated by (1) "systematic effects that produce aberrations in the ratio spectra" (10.4×10^{-6}) and (2) random statistical uncertainty (5×10^{-6}) by combining two data sets, each having 116.6 hours of integration. Neither the systematic error producing the spectral aberration nor the statistical uncertainty is currently believed to represent a fundamental limit of this measurement.

In order to demonstrate the reproducibility of this electrical approach to measuring *k* and to pursue even lower measurement uncertainties, it is important to perform an independent measurement with different QVNS-JNT systems. In this paper, we describe the newly developed QVNS-JNT system at NIM and report a recent measurement result that shows flat frequency responses for both the noise-power ratio and the quadratic fitting parameters.

II. NIM QVNS-JNT System

Figure 1 shows the schematic of the NIM QVNS-JNT system. The QVNS is constructed with a NIST-fabricated chip consisting of two arrays that each contains 10 Josephson junctions (JJs). The sense resistor (R) is mounted in a vacuum-sealed probe that is inserted into a cell that maintains the TPW temperature. Both the R and JJ are equally divided and grounded by a center tap to provide differential source noise signals. The noise sources are connected to the two nominally identical low-noise (~1 nV/Hz$^{1/2}$) high-gain (70 dB) and high-CMRR (100 dB at 100 kHz) amplifiers (Amp) through latching relays. The nonlinearity of the amplifier is minimized by integrating a compensation circuit that nulls the DC offset of the signal before the differential stage [4]. Between the two buffer stages of the Amp, 800 kHz low-pass filters (LPF) define the measurement bandwidth in each channel. Two commercial analog-to-digital converters (ADC) sample the signal synchronously at 2 MHz with a resolution of 20 bits and a 1 MHz anti-alias digital filter. The data are optically transmitted to computer. The software cross-correlates the data that are acquired for every 1 s, accumulates these data for 100 s, and then switches the relays to measure the other noise source.

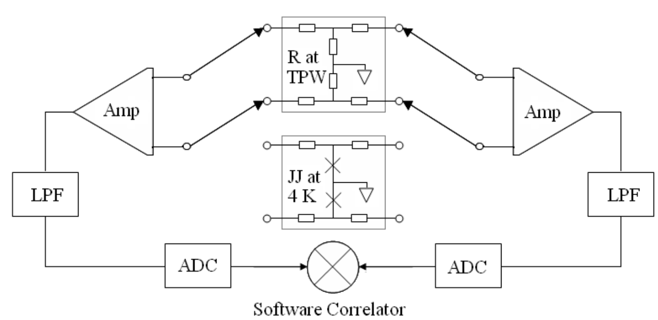

Fig. 1. Schematic of the QVNS-JNT system.

The comb-like quantum voltage noise waveform is synthesized by the bipolar pulse-driven technique, which consists of odd tones from 1 kHz to 999 kHz with tone spacing of 2 kHz [3]. The voltage spectral density is set to V_Q= 1.2282 nV/Hz$^{1/2}$ to closely match the thermal noise V_R= 1.228272 nV/Hz$^{1/2}$ of the 100.0065 Ω resistor at the TPW.

The measured cross-correlation power spectra are summed and compared over discrete 2 kHz intervals centered at the frequency of the tones of the QVNS-synthesized waveform.

III. Flat Frequency Response

Because different transmission lines are used to transmit the noise signals from R and JJ to the amplifiers, different transfer functions may occur and result in deviation of the noise power ratio from unity response. The resulting expected quadratic frequency of the noise-power ratio is corrected by least-squares fitting with a two-parameter formula $a_0+a_2f^2$. The fitting coefficient a_0 and associated standard deviation are used to determine k and its relative statistical uncertainty. The coefficient a_2 characterizes the remaining quadratic response.

Fig. 2(a) shows our measurement result with flat frequency response of the noise power ratio over the entire measurement bandwidth. The deviation of the ratio from unity is less than 1×10^{-3} up to 800 kHz. This is achieved by inserting additional resistors in both the R and JJ transmission lines, as shown in Fig. 1, and carefully trimming the values of the inserted resistors, as well as the length of the transmission lines. Matching both the noise power and the impedance of the transmission lines is important to minimize the effects of nonlinearities and frequency-dependent corrections. In Fig. 2(b), the residuals of the two-parameter fit over a bandwidth from 5 kHz to 600 kHz appear to be reasonably flat. Absence of significant curvature in the frequency dependence of the residuals indicates very good linearity of the electronics.

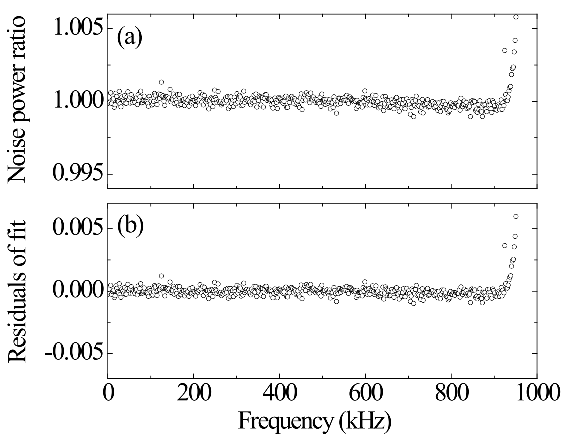

Fig.2 Flat frequency responses of (a) the power ratio and (b) the residuals of the two-parameter fit.

In Fig. 3, we plot the results of fitting the data with different bandwidths over different ranges starting from 5 kHz and ending at successively higher frequencies. The difference a_0-a_0^{2006} represents the relative offset of k from the 2006 CODATA value, and $a_0^{2006} = (V_R/V_Q)^2$. For the highest fitting bandwidth 640 kHz, a_0-$a_0^{2006} = 0.5\times10^{-6}$, $a_2 = 1.4\times10^{-9}$. It is important to note that both a_0 and a_2 show flat behavior as a function of fit bandwidth. The values of a_0-a_0^{2006} are consistent with each other within the error bars for the standard deviations of the different fitting bandwidths. The flat responses demonstrate the self-consistency of the data, and indicate that the systematic error that leads to the spectra aberrations in [2] is greatly reduced in the NIM system.

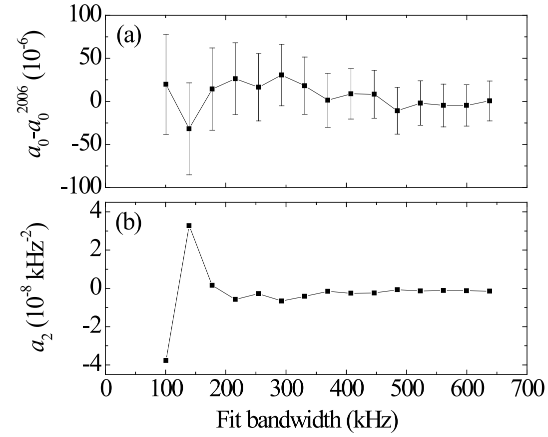

Fig.3 Flat responses of the fit parameters, a_0 and a_2, on the fitting bandwidth.

IV. Conclusion

A new QVNS-JNT system was developed at NIM to pursue lower uncertainty for the electronic measurement of the Botzmann constant. A measurement with an integration period of 10 hours resulted in a relative offset of 0.5×10^{-6} from the current CODATA value of k, and a type A relative standard uncertainty of 23×10^{-6}. Most importantly, the quadratic fitting parameters of the power-ratio spectra showed extremely flat frequency response for different fitting bandwidths, indicating that the systematic errors that produced the spectral aberrations in the NIST system are greatly reduced. With further improvements and longer integration periods we anticipate reaching the goal of an electronic measurement of k at a combined relative uncertainty less than 10×10^{-6}.

Acknowledgement

We thank P. Dresselhaus and C. Burroughs for chip fabrication and packaging. The work is supported by NSFC and Public Welfare Scientific Research Project.

References

[1] P.J. Mohr, *et al*, “CODATA recommended values of the fundamental physical constants: 2006,” *Rev. Mod. Phys.*, vol. 80, pp. 633–730, 2008.

[2] J. Fischer, *et al*, “Preparative steps towards the new definition of the kelvin in terms of the Boltzmann constant,” *Int. J. Thermophys.*, vol. 28, pp. 1753–1765, 2008.

[3] S.P. Benz, *et al*, “An electronic measurement of the Boltzmann constant,” *Metrologia*, vol. 48, pp. 142-153, 2011.

[4] J.F. Qu, *et al*, “Reduced nonlinearity effect on the electronic measurement of the Blotzmann constant,” *IEEE Trans. Instrum. Meas.,* vol. 60, pp. 2427-2433, 2011.