12th CIRP Conference on Computer Aided Tolerancing

Reflections on the Role of Science in the Evolution of Dimensioning and Tolerancing Standards

V. Srinivasan*

National Institute of Standards and Technology, 100 Bureau Drive,Gaithersburg, MD 20899, U.S.A.

Abstract

Dimensioning and tolerancing standards originated about 75 years ago in the form of various national and company standards that governed engineering drafting and documentation practices. They served the purpose of communicating to manufacturers what geometric variations designers could tolerate in a product without compromising the product's intended function. These standards have evolved over time and are by now well entrenched in the engineering profession throughout the world. For several initial decades, this evolution was driven primarily by codification of best engineering practices without the benefit of any systematic scientific treatment. This trend encountered a major hurdle in early 1980s when the emergence of computer-aided design and manufacturing systems forced a drastic reexamination of these standards with a greater emphasis on mathematical formalism. Since then scientific principles to explain past practices and to guide future evolution have emerged, and the role of science has now become more prominent in the development of these standards. In this paper I outline some of the key scientific research results that have already made an impact, and future scientific trends that are likely to have an influence, on these evolving standards.

Keywords: dimensioning; tolerancing; standards; scientific developments; classification; theory; theorems

1. Introduction

In 1991 the Pennsylvania State University in the United States hosted a CIRP Computer-Aided Tolerancing Working Seminar. By some current count, it was the second of a series of what has come to be called the CIRP CAT Conferences (the first was a gathering on this topic at Jerusalem, Israel in 1989). The Penn State 'working seminar' was a timely event because several academic and industrial researchers had started working together to attack an important industrial problem using the computational power unleashed by the information age [1-4]. I was fortunate to be present at that workshop and spoke about how 'a geometer grapples with tolerancing standards' [5]. That workshop, and other similar events organized around that time, launched a series of initiatives that resulted in the creation of the ASME

*Corresponding author. Tel.: +1-301-975-3524; fax: +1-301-258-9749.
E-mail address: vsrini@nist.gov.

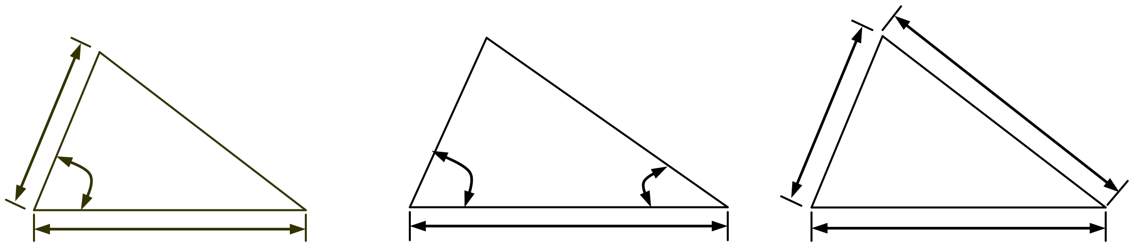
Y14.5.1 subcommittee and the ISO Technical Committee 213, both dealing with tolerancing standards. And they spurred one of the most creative research activities in mechanical and computational sciences, as chronicled in the proceedings of the twelve CIRP CAT Conferences – including this conference – and other similar conferences and research journals.

Looking back at the paper [5] written twenty-one years ago, two of its observations strike me as most consequential. The first was a call for mathematically sound definitions of the semantics of the standardized tolerancing language, because the lack of such a scientific basis was hampering the development of provably correct algorithms for computer-aided tolerancing. The second was a tentative mention of 'computational metrology' to refer to a set of computational techniques that were emerging to cope with processing large amounts of measured data coming out of coordinate measuring machines. It was not clear at that time if these were merely naïve observations that would be promptly forgotten or important topics that would be pursued with vigor. In fact, one prominent academic at the conference scoffed at the very idea of referring to tolerancing standards as defining a 'language with syntax and semantics.'

It is now heartening to reflect on the developments over the past two decades and see that both these observations have come to play an important role in the evolution of dimensioning and tolerancing standards. In this paper I will describe some of the scientific developments first (in Sections 2 through 5) and their impact on the evolution of dimensioning and tolerancing standards next (in Sections 6 through 8). Along the way, I will show how these scientific foundations have helped rationalize the past and current industrial practices, and how they are paving the way for important new avenues. Above all, I hope to communicate the excitement of one who has pursued geometric studies for both fun and profit.

2. Congruence Theorems

Congruence theorems dating back to Euclid provide an easy and powerful introduction to the notion of dimensioning (and parameterizing) geometric objects. To illustrate, let's start with the simple task of dimensioning and parameterizing triangles. Fig. 1 shows some successful attempts. It seems intuitively obvious that all these three schemes are valid ways to parameterize triangles, and we get valid dimensions when numerical values are assigned to the distances and angles indicated by arrows.



(a) (b) (c)

Fig. 1. Examples of dimensioning and parameterizing schemes for triangles.

We can provide a formal theoretical basis for our intuitive belief by associating each example in Fig. 1 with a famous triangle congruence theorem from Euclid's Elements [6]: Fig. 1(a) with the side-angle-side theorem (Book I, Proposition 4), Fig. 1(b) with the angle-side-angle theorem (Book I, Proposition 26), and Fig. 1(c) with the side-side-side theorem (Book I, Proposition 8).

The geometric notion of congruence is closely related to the practical engineering notion of interchangeability of parts, as they both belong to an equivalence class. More formally, they satisfy the following three axioms:

1. *Reflexivity*: *A* is congruent to (or, interchangeable with) *A*.
2. *Symmetry*: If *A* is congruent to (or, interchangeable with) *B*, then *B* is congruent to (or, interchangeable with) *A*.
3. *Transitivity*: If *A* is congruent to (or, interchangeable with) *B* and *B* is congruent to (or, interchangeable with) *C*, then *A* is congruent to (or, interchangeable with) *C*.

In fact, an entire formal theory of dimensioning can be built using congruence theorems [7, 8].

The taxonomy of such a modern theory of dimensioning is shown in Fig. 2. At the leaf nodes of the modern taxonomy of dimensioning we see intrinsic dimensions. At the intermediate nodes we see relational dimensions. The hierarchy can be built with as many levels as the product demands. Complex products may have more levels of hierarchy than simpler ones.

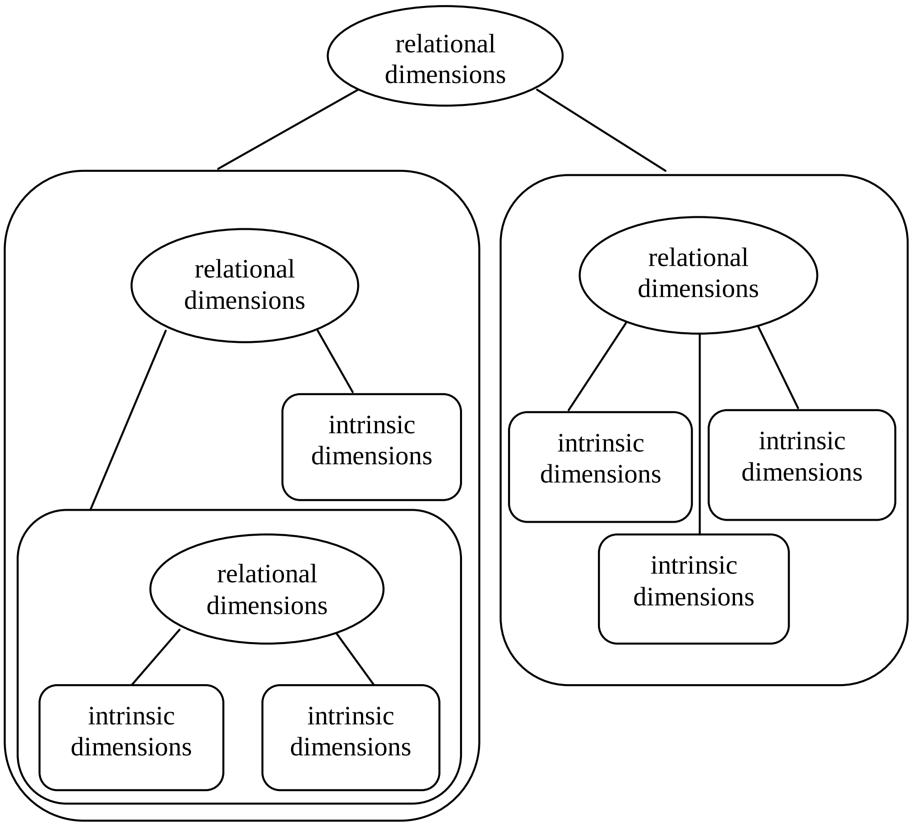


Fig. 2. A modern taxonomy of dimensioning.

Congruence theorems formalize the dimensioning scheme at each level of the dimensional taxonomy. In Section 3 we will see how this is accomplished for some of the commonly used intrinsic dimensions of surface features trimmed from quadric surfaces. Then in Section 4 we will repeat the exercise for relational dimensions. As we lay the scientific foundation for these dimensioning schemes, it is useful to look ahead to Section 8, where tolerancing standards have adopted a similar hierarchy for 'individual features' and 'related features' invoking the notion of datums. This point will be emphasized again in Section 8.

3. Classification of Quadrics

Most commonly used surfaces in engineering belong to second degree surfaces called quadrics. These are defined by a set of points with x, y, z coordinates as in

$$\{(x, y, z) : \mathrm{A}x^2 + \mathrm{B}y^2 + \mathrm{C}z^2 + \mathrm{D}xy + \mathrm{E}yz + \mathrm{F}zx + \mathrm{G}x + \mathrm{H}y + \mathrm{K}z + \mathrm{L} = 0\} \tag{1}$$

for real coefficients A, B, C, D, E, F, G, H, K and L, where at least one of A, B, C is nonzero. A well-known quadrics classification theorem [9] states that any surface of second-degree governed by an equation of the form (1) can be moved by purely rigid motion in space so that its transformed equation can assume one and only one of seventeen canonical forms. Of these seventeen, only twelve can have solutions for real values of x, y, z and they are shown in Table 1.

The classification theorem also provides a congruence theorem: two quadrics are congruent if and only if they have the same canonical equation. The last column in Table 1 lists the intrinsic parameters of these surfaces. The quadrics can be dimensioned by assigning numerical values to these parameters.

The complete classification of real quadrics plays an important role in the evolving standardized definition of ‘features of size’. Only six of the quadrics enumerated in Table 1 belong to 1-parameter family of surfaces; namely, sphere, (right circular) cylinder, (right circular) cone, parabolic cylinder, two parallel planes, and two intersecting planes forming a wedge. They also possess an important monotonic containment property; for example, a sphere with a larger size dimension contains the one with a smaller size dimension. These and other symmetry properties covered in the next section provide the strongest scientific rationale for a standardized definition of ‘features of size’ that will be discussed in Section 6.

4. Classification of Continuous Symmetry

The notion of symmetry greatly simplifies the task of relational dimensioning because we then need to prove only a limited number of congruence theorems. The simplification depends on some important classification theorems on the connected Lie subgroups of the rigid motion group, and their corollaries on the classification of continuous symmetry of surfaces [7, 8, 10, 11]. These and related theorems were rigorously proved only over the past fifteen years.

To gain an intuitive appreciation for the role of symmetry, consider the problem of positioning an arbitrary object, such as a chair, in three-dimensional space. It requires six dimensions – three for translation and three for rotation. Now consider positioning a sphere in space. It seems to require only three dimensions, which are needed to locate the center of the sphere. We don’t need any dimension to specify rotation because the symmetry of the sphere renders all rotations about its center irrelevant for positioning purpose. Finally, consider the task of positioning a sphere relative to a (unbounded) plane. A little reflection indicates that we need to specify only one dimension, namely the distance between the center of the sphere and the plane. This drastic reduction in the number of needed dimensions is due to the fact that the plane also possesses some symmetry because it remains invariant under all translations along the plane and all rotations about any axis perpendicular to the plane.

Table 1. Classification of real quadrics.

	Type	Canonical Equation	Intrinsic Parameters
Non-degenerate quadrics	Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	*a*, *b*, *c*
	Special case: Sphere	$x^2+y^2+z^2=a^2$	*a* = radius
	Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	*a*, *b*, *c*
	Hyperboloid of two sheets	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$	*a*, *b*, *c*
	Quadric cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	*a*/*c*, *b*/*c*
	Special case: Right circular cone	$\frac{x^2}{a^2}+\frac{y^2}{a^2}-\frac{z^2}{c^2}=0$	$\tan^{-1}(a/c)$ = semi apex angle of the cone
	Elliptic paraboloid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2z=0$	*a*, *b*
	Hyperbolic paraboloid	$\frac{x^2}{a^2}-\frac{y^2}{b^2}+2z=0$	*a*, *b*
	Elliptic cylinder	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	*a*, *b*
	Special case: Right circular cylinder	$x^2+y^2=a^2$	*a* = radius
	Hyperbolic cylinder	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	*a*, *b*
	Parabolic cylinder	$y^2-2lx=0$	*l*
Degenerate quadrics	Parallel planes	$x^2-a^2=0$	*a* = half of the distance between the parallel planes
	Intersecting planes	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$	$\tan^{-1}(b/a)$ = half of the angle between the intersecting planes
	Coincident planes	$x^2=0$	None

Table 2 shows the seven classes of continuous symmetry. This is a complete classification in the sense that any surface, in fact any set of points encountered in engineering, belongs to one and only one of these seven classes. These are also called invariant classes because symmetry is defined by invariance; for example, a cylinder remains invariant under all translational motions along its axis and all rotations about its axis. This is true about the axis as well, because it also remains invariant under these motions. There is a powerful theorem [7, 8] that reduces the problem of relative positioning any two sets to the relative positioning of their simple replacements shown in the last column of Table 2. These results provide the scientific basis for standardized datums and datum systems that will be described in Section 7.

Table 2. Seven classes of continuous symmetry

	Type	Simple Replacement
1	Spherical	Point (center)
2	Cylindrical	Straight line (axis)
3	Planar	Plane
4	Helical	Helix
5	Revolute	Straight line (axis) & point-on-line
6	Prismatic	Plane & straight line-on-plane
7	General	Plane, straight line & point.

5. Computational Coordinate Metrology

Computational coordinate metrology involves the development and implementation of reliable algorithms to fit, filter, and to perform other types of computations on discrete geometric data collected by coordinate measuring systems. Over the past twenty years it has grown into a separate research discipline. Several international conferences and journals now list computational metrology as a distinct topic of interest. Of the many research results that deal with computational coordinate metrology, those that address fitting and filtering are the most relevant to the verification standards for conformance to tolerance specifications. Mathematically and computationally, fitting is an optimization problem and most of filtering is a convolution problem [12-21]. The filtering techniques also form the mathematical basis for surface texture characterization.

The scientific and technical advances in hardware for coordinate measurements and software for processing the data collected from these measurements have forced a rethinking of tolerance specification standards. To illustrate this fact, let's consider the problem of fitting a plane to a set of points in space. If d_1, d_2, …, d_n are the perpendicular (Euclidean) distances of n input data points from a plane P, then we can define the distance between this set of points and the plane P using the generic l_p-norm

$$\left\{ \sum_{i=1}^{n} |d_i|^p \right\}^{1/p} . \tag{2}$$

The individual distances can be signed, in the sense that points lying on one side of the plane can be assigned positive distances and the points lying on the other side can be assigned negative distances. The l_1-norm is then just the sum of the absolute values of the individual distances. The l_2-norm is the square root of the sum of the squares of the distances. The l_∞-norm is the maximum of the absolute values of the distances; to see this we need to look at the definition of the l_p-norm as p tends to infinity. Table 3 presents a set of plane fitting problems posed as optimization problems for the objective function shown in (2). In the ISO parlance, the l_2-norm is known as the Gaussian norm and the l_∞-norm is known as the Chebyschev norm.

Table 3. Fitting a plane P to a set of points in space.

Objective	Constraints	Comments	Designation
Minimize l_1-norm	Points lie to one side of P	Minimum three-points touching P	Plane_1C
Minimize l_2-norm	None	Least-squares plane	Plane_2
	Points lie to one side of P	Constrained least-squares plane	Plane_2C
Minimize l_∞-norm	None	Minimax plane	Plane_∞
	Points lie to one side of P	Constrained minimax plane	$\text{Plane}_\infty C$

Algorithms for computing Plane_2, Plane_∞, and $\text{Plane}_\infty C$ in Table 3 have been well developed in literature [13]. Of these, the least-squares plane, designated as Plane_2, is the most widely implemented, tested, and used in industry [22]. Plane_1C conforms to the ASME Y14.5:2009 standard [23] for the establishment of primary planar datum because it guarantees a stable plane that touches at least three points. On the other hand, $\text{Plane}_\infty C$ is the default primary datum plane according to the ISO 5459:2011 standard [24]. Currently, ISO is mulling over future tolerance specification standards that will allow the designer to choose from several fitting objectives including the Gaussian norm and the Chebyschev norm. Such an expansion is also envisioned for a wide variety of geometric characteristics that will be described in Section 8.

6. Size Tolerancing

Historically, both the ASME and the ISO standards had recognized only sphere, cylinder, and two opposing parallel planes as the standardized features of size. These have been the only features to which size dimension and tolerance can be assigned; these have also been the only features that can be designated as datums with material conditions. These practices originated from engineering experience and were not based on any scientific rationale.

We can now provide a scientific basis for the definition of features of size and, in that process, expand their coverage [25-28]. ISO has recently adopted a total of five features of size, as illustrated in Fig. 3 from one of its recent standards. Five of the six 1-parameter family of quadrics discussed in Section 3 and five of the seven classes of symmetry (invariance classes) discussed in Section 4 are represented in this updated standardized definition of features of size; these features also possess the monotonic containment property discussed in Section 3. Parabolic cylinder, which belongs to the prismatic class, is the only 1-parameter family of quadrics that did not make the list – the engineering community did not consider its use to be sufficiently widespread to warrant the 'feature of size' designation.

Of the five features of size in Fig. 3, three – namely, cylinder, sphere, and two parallel opposite planes – have linear units as the intrinsic characteristics. These are the linear sizes, and the other two features of size have angular sizes. In a recently issued ISO 14405-1 standard [29], the linear size tolerances for cylinder and two parallel opposite planes have been expanded considerably. ISO 14405-1 allows tolerances to be specified on the following fourteen different types of sizes: two-point size, local size defined by a sphere, least-squares size, maximum inscribed size, minimum circumscribed size, circumferential diameter size, areal diameter size, volume diameter size, maximum size, minimum size, average size, median size, mid-range size, and range of sizes. Fig. 4 illustrates one of fourteen ways in which the size tolerance for a cylindrical feature can be specified.

The release of ISO 14405-1 standard in 2010 heralded a much needed revolution in the tolerance specification standards. Before the arrival of ISO 14405-1, there was an implicit expectation that size tolerances should be verifiable using the likes of micrometers, calipers, dial indicators, and functional gauges. (In fact, such expectations are prevalent even for all geometric tolerance specifications under the guise of 'open setups' for their verifications.) Using a computerized coordinate measuring system was deemed acceptable, as long as it could be run in the 'caliper-mode' and with some soft-gauging capabilities. In contrast, a size tolerance specification such as the one shown in Fig. 4 can only be verified by coordinate measuring systems employing least-squares fitting algorithms, which are now available thanks to scientific developments in computational coordinate metrology alluded to in Section 5, for cylinder fitting. Future ISO standards for geometric tolerance specifications will depend critically upon such fitting algorithms for their verification.

Feature of size	Invariance class	Intrinsic characteristic
Cylinder	cylindrical	diameter
Sphere	spherical	diameter
Two parallel opposite planes	planar	distance between the two planes
Cone	revolute	angle
Wedge	prismatic	angle

Fig. 3. Excerpt from ISO 5459:2011 [24] on features of size.

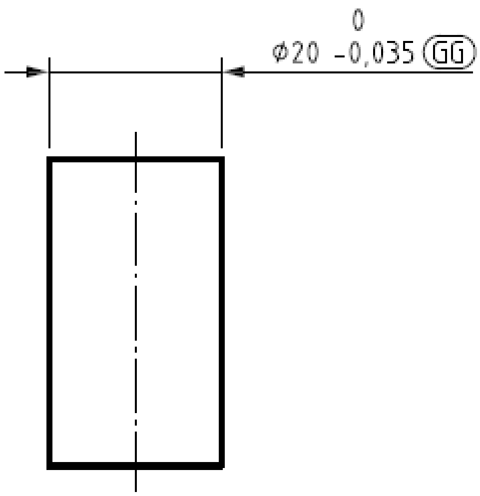


Fig. 4. Excerpt from ISO 14405-1:2010 [29] for least-squares size tolerancing of a cylinder. Here the GG modifier denotes that a Gaussian (least-squares) size is being toleranced for the indicated cylinder.

7. Datums and Datum Systems

Datums and datum systems are essential for specifying tolerable variations in the relative positioning of features. As we saw in Section 4, classification of continuous symmetry provides a scientific basis to define datums. This has been seized upon by the standards community in recent times, as illustrated in Fig. 5 excerpted from the ASME Y14.5 standard issued in 2009 and in Fig. 6 exceprted from the ISO 5459 standard issued in 2011. These figures clearly show the classification of standardized datums on the basis of the symmetry group classification of the nominal ideal geometric features that are designated by designers as datum features.

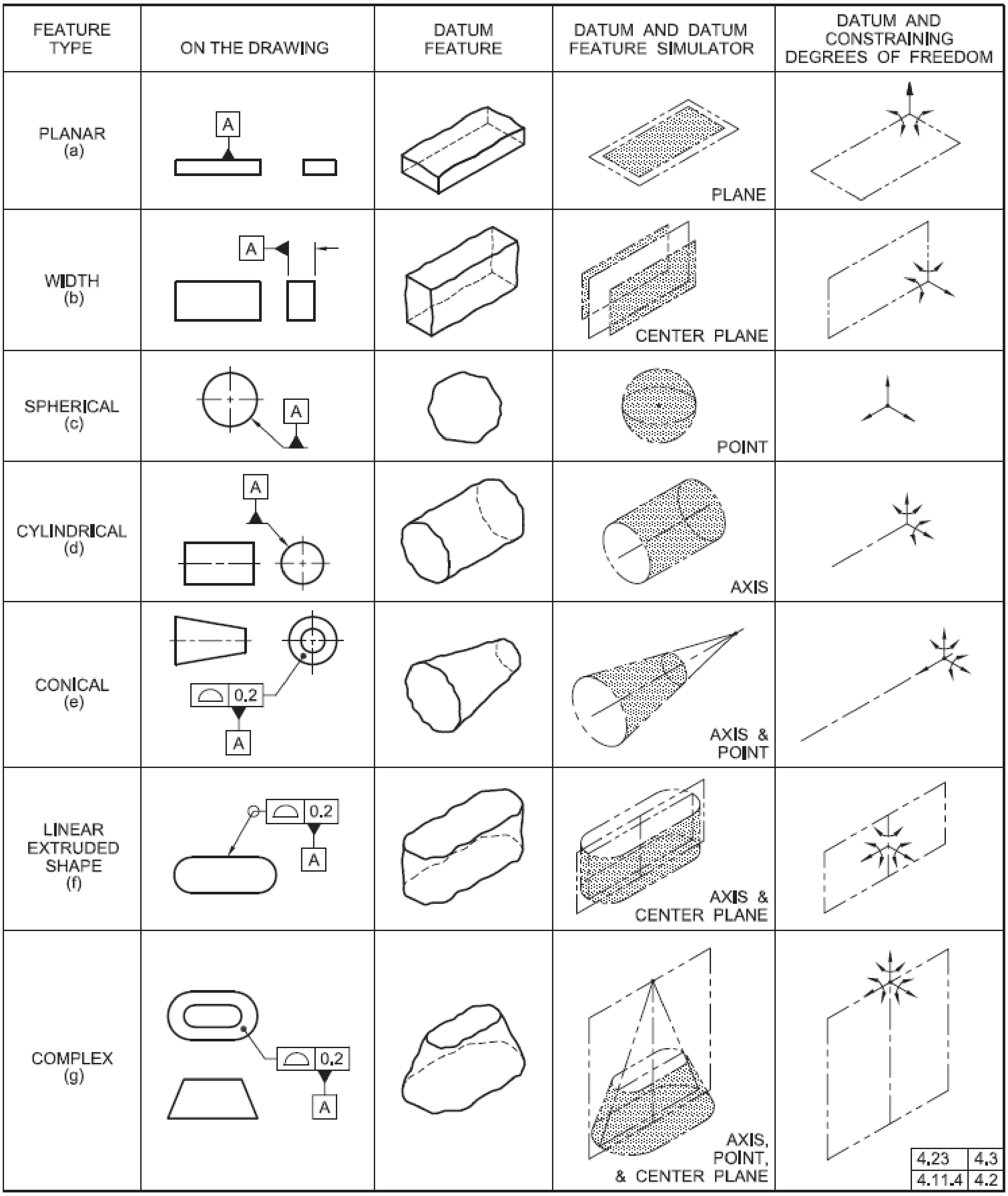
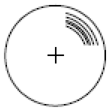
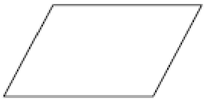
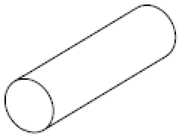
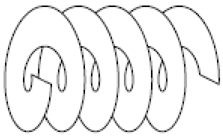
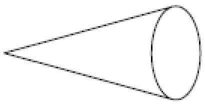
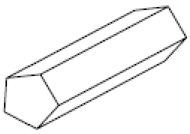
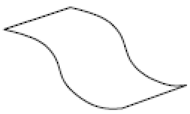


Fig. 5. Excerpt from ASME Y14.5-2009 standard [23] on primary datums and datum features.

A closer examination of Figs. 5 and 6 reveals the distinction between a datum feature and the associated datum. For example, a cylindrical feature can be designated as a datum feature, and its axis then becomes the datum. On an actual workpiece the features have non-ideal forms and so we need to fit ideal geometric features before the datums can be discerned. As described in Section 5, various norms can be used to define the objective function in the optimization problem involved in such fittings. The standards currently rely upon the l_1-norm and the l_∞-norm, with constraints; ISO is contemplating appropriate symbology to override the default so that other l_p-norms can be invoked for determining the datums.

Invariance class	Unconstrained degrees of freedom	Illustration	Situation features	Example of types of surfaces
Spherical	3 rotations around a point		Point	Sphere
Planar	1 rotation perpendicular to the plane and 2 translations along 2 lines of the plane		Plane	Plane
Cylindrical	1 translation and 1 rotation around a straight line		Straight line	Cylinder
Helical	Combination of 1 translation and 1 rotation around a single straight line		Straight line[a]	Helical surface with a basis of involute to a circle
Revolute	1 rotation around a straight line		Straight line Point	Cone Torus
Prismatic	1 translation along a line of a plane		Plane Straight line	Pentagonal prism
Complex	None		Plane Straight line Point	Bezier surface based on an unstructured cloud of points in space

[a] Helical surfaces as such are not considered in this International Standard. They are regarded as cylindrical surfaces because, in most functional cases where helical surfaces (threads, helical slopes, endless screws, etc.) are involved, the combined rotation and translation of the helix is not needed for datum purposes. In these cases, the pitch cylindrical surface is used for the datum; the major or minor cylindrical surface can also be considered and specified. Natively, the situation feature of a feature belonging to a helical invariance class is a helix, but in this International Standard we consider only its axis.

Fig. 6. Excerpt from ISO 5459:2011standard [24] on invariance classes to define datums.

8. Geometric Characteristics and Tolerancing

The dimensional taxonomy shown in Fig. 2 was useful in structuring the theoretical development of dimensioning and geometric parameterization. It also explains the classification of geometric characteristics and tolerancing as shown in Figs. 7 and 8 in the ASME and ISO standards, respectively, that have stood the test of time. The ASME classification, shown in Fig. 7, refers to individual features and related features in the same way intrinsic dimensions and relational dimensions are dealt with in Sections 2, 3 and 4. The ISO classification, shown in Fig. 8, is the same as that of ASME and it explicitly invokes the need for datums for tolerancing relative positioning.

APPLICATION	TYPE OF TOLERANCE	CHARACTERISTIC	SYMBOL
INDIVIDUAL FEATURES	FORM	STRAIGHTNESS	⏤
		FLATNESS	⏥
		CIRCULARITY	○
		CYLINDRICITY	⌭
INDIVIDUAL OR RELATED FEATURES	PROFILE	PROFILE OF A LINE	⌒
		PROFILE OF A SURFACE	⌓
RELATED FEATURES	ORIENTATION	ANGULARITY	∠
		PERPENDICULARITY	⊥
		PARALLELISM	//
	LOCATION	POSITION **	⌖
		CONCENTRICITY	◎
		SYMMETRY	⌯
	RUNOUT	CIRCULAR RUNOUT	↗ *
		TOTAL RUNOUT	⌰ *

* Arrowheads may be filled or not filled ** May be related or unrelated

Fig. 7. Excerpt from ASME Y14.5-2009 standard [23] on the classification of geometric characteristics.

Both the ASME and ISO tolerancing semantics of geometric tolerances shown in Figs. 7 and 8 are based on tolerance zones. ISO is now considering several ways to expand the syntax and semantics of geometric tolerances as it did for size tolerancing. For example, in the future, flatness tolerance can be specified to limit the root-mean-square deviation (i.e., standard deviation) of the points on an actual feature from a Gaussian (least-squares) plane fitted to the feature.

Tolerances	Characteristics	Symbol	Datum needed
Form	Straightness	—	no
	Flatness	⏥	No
	Roundness	○	No
	Cylindricity	⌭	No
	Profile any line	⌒	No
	Profile any surface	⌓	No
Orientation	Parallelism	//	Yes
	Perpendicularity	⊥	Yes
	Angularity	∠	Yes
	Profile any line	⌒	Yes
	Profile any surface	⌓	Yes
Location	Position	⌖	yes or no
	Concentricity (for centre points)	◎	Yes
	Coaxiality (for axes)	◎	Yes
	Symmetry	⌯	Yes
	Profile any line	⌒	Yes
	Profile any surface	⌓	Yes
Run-out	Circular run-out	↗	Yes
	Total run-out	⌰	Yes

Fig. 8. Excerpt from ISO 1101:2012 standard [30] on geometric characteristics.

9. Summary

In this paper, I have strived to describe the role of science in classifying and rationalizing some of the past and current dimensioning and tolerancing practices and in paving the way for future development of dimensioning and tolerancing standards. Motivated by the industrial need, the last two decades have witnessed considerable mathematical and algorithmic advances, and the dimensioning and tolerancing standards are well poised to exploit these advances in at least four distinct areas.

First, computer-aided dimensioning and tolerancing software systems can be based on data models that are provably complete and algorithms that are provably correct. Second, these data models can form the basis for standardized exchanges [31] that enable interoperability among engineering information systems. Third, computer-aided manufacturing systems can consume the tolerancing information automatically for smarter numerical control of machine tools. Fourth, computer-aided inspection systems can use the tolerancing information to generate and execute inspection plans automatically.

In the next few years we are likely to witness major expansions in the ISO tolerancing standards [32], assisted by scientific developments similar to those outlined in this paper and subsequent codification of concepts and terminology [33-35]. Industry will struggle with the magnitude of such changes, and there will be a great demand for education of the industrial workforce and college students in these new standards and practices. We might well look forward to another two decades of fun and profit.

Acknowledgements

I would like to acknowledge the contributions of various subject matter experts in the ASME and ISO standards committees to the developments reported in this paper. Any mention of commercial products within this paper is for information only; it does not imply recommendation or endorsement by NIST.

References

[1] Requicha AAG. Toward a theory of geometric tolerancing. *Int J Robotics Research* 1983; **2**: 45-60.

[2] Requicha AAG. Representation of tolerances in solid modeling: Issues and alternative approaches. In Pickett MS, Boyce JW, editors. *Solid modeling by computers*, New York: Plenum; 1984, p. 3-22.

[3] Jayaraman R, and Srinivasan V. Geometric tolerancing: I. Virtual boundary requirements. *IBM J Research and Development* 1989; **33**: 90-104.

[4] Tipnis VA, editor. *Research opportunities in mechanical tolerancing*. New York: The American Society of Mechanical Engineers; 1989.

[5] Srinivasan V. A geometer grapples with tolerancing standards, *CIRP Workshop on Computer-aided Tolerancing*, Pennsylvania State University, State College, PA, U.S.A.; 1991.

[6] Joyce D. *Euclid's elements*, http://aleph0.clarku.edu/~djoyce/java/elements/elements.html

[7] Srinivasan V. *Theory of dimensioning: An introduction to parameterizing geometric models*. New York: Marcel-Dekker; 2004.

[8] Srinivasan V. A mathematical theory of dimensioning and parameterizing product geometry, *Int J Product Lifecycle Management* 2005; **1**:70-85.

[9] Olmsted JMH. *Solid analytic geometry*. New York: Appleton-Century-Crofts; 1947.

[10] Clement A, Riviere A, and Temmerman, M. *Cotation tridimensionnnelle des systemes mechaniques*, Ivry-sur-Seine: PYC Edition, France; 1994.

[11] O'Connor MA, Srinivasan V, and Jones A. Connected Lie and Symmetry Subgroups of the Rigid Motions: Foundations and Classifications, *IBM Research Report RC 20512,* Yorktown Heights, New York; 1996.

[12] Srinivasan V. Elements of computational metrology. In: Janardan R, Smid M, Dutta D, editors. *Computer aided design and manufacturing*, DIMACS book series, vol. 67. RI: American Mathematical Society; 2005, p. 79–116.

[13] Srinivasan V. Computational metrology for the design and manufacture of product geometry: A classification and synthesis. *ASME Journal of Computing and Information Science in Engineering* 2007;**7**: 3–9.

[14] Muralikrishnan B, and Raja J. *Computational surface and roundness metrology*. Berlin: Springer; 2008.

[15] Krystek M. *The implementation of the Gaussian filter for dimensional metrology*. Berlin: Beuth; 2012.

[16] Scott PJ. The mathematics of motif combination and their use for functional simulation. *Int. J. Mach. Tools Manufact.* 1992; **32**: 69-73.

[17] Scott PJ. Pattern analysis and metrology: The extraction of stable features from observable measurements. *Proceedings of the Royal Society, A*. 2004; **460**: 2845-2864.

[18] Zhang X, Jiang X and Scott PJ. A new free-form surface fitting method for precision coordinate metrology. *Wear* 2009; **266**: 543-547.

[19] Jiang X, Cooper P and Scott PJ. Freeform surface filtering using the diffusion equation. *Proceedings of the Royal Society, A*. 2011; **467**: 841-859.

[20] Jiang X, Scott PJ and Whitehouse D. Freeform surface characterisation - a fresh strategy. *CIRP* Annals. 2007; **56**: 553-556.

[21] Jiang X, Scott PJ, Whitehouse D. and Blunt L. Paradigm shifts in surface metrology. Part II. The current shift. *Proceedings of the Royal Society, A*. 2007; **463**: 2071-2099.

[22] Srinivasan V, Shakarji CM, and Morse EP. On the enduring appeal of least-squares fitting in computational coordinate metrology. *ASME Journal of Computing and Information Science in Engineering* 2012; **12**:011008-1-15.

[23] ASME Y14.5 *Dimensioning and Tolerancing*. New York: The American Society of Mechanical Engineers; 2009.

[24] ISO 5459:2011 *Geometrical product specifications (GPS) – Geometrical tolerancing – Datms and datum systems*. Geneva: International Organization for Standardization; 2011.

[25] Voelcker HB. Let's talk about 'size'. *mfg magazine* 1995; **2**: 40-41.

[26] Voelcker HB. 'Size' revisited. *mfg magazine* 2001; **8**: 30-33.

[27] Voelcker HB. Whither size in geometric tolerancing? *Proc ASPE Summer topical meeting in Tolerance Modeling and Analysis* 2002, Charlotte, NC, U.S.A.

[28] Morse EP, Srinivasan V, and Voelcker HB. Size tolerancing revisited: A basic notion and its evolution in standards. *12th CIRP Conference on Computer Aided Tolerancing* 2012; Huddesfield, U.K.

[29] ISO 14405-1:2010 *Geometrical product specifications (GPS) – Dimensional tolerancing – Part 1: Linear sizes*. Geneva: International Organization for Standardization; 2010.

[30] ISO 1101:2012 *Geometrical product specifications (GPS) – Geometrical tolerancing – Tolerances of form, orientation, location and run-out*. Geneva: International Organization for Standardization; 2011.

[31] Srinivasan V. Standardizing the specification, verification and exchange of product geometry: Research, status and trends. *Computer-Aided Design* 2008: **40**: 738-749.

[32] Nielsen HS. Recent developments in ISO-GPS standards and strategic plans for future work. *12th CIRP Conference on Computer Aided Tolerancing* 2012; Huddesfield, U.K.

[33] ISO 17450-1:2011 *Geometrical product specifications (GPS) – General concepts – Part 1: Model for geometrical specification and verification.* Geneva: International Organization for Standardization; 2011.

[34] Ballu A and Mathieu L. Analysis of dimensional and geometrical specifications: standards and models. *CIRP Computer Aided Tolerancing*, 3rd Seminar, Cachan, France, 1993, pp. 157-170.

[35] Ballu A and Mathieu L. Univocal expression of functional and geometrical tolerances for design, manufacturing and inspection. *CIRP Computer Aided Tolerancing*, 4th Seminar, Tokyo, Japan, 1995, pp. 31-46.