Viewpoint

An optical lattice of flux

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> *A suitable optical lattice for cold atoms could produce a large effective magnetic field in which the atoms would realize analogs to quantum Hall states.*

Subject Areas: **Atomic and Molecular Physics**

Ultracold neutral atoms are among the most simple and flexible of quantum many-body systems. As such, they offer the capability to realize strongly interacting systems in their most fundamental form, absent the unwanted complexities that complicate (and sometimes enrich) their solid-state brethren. The question then arises: What classes of systems can be implemented with cold atoms, and of these, which can offer insight beyond that afforded in more conventional systems? In a paper published in *Physical Review Letters*, Nigel Cooper at the University of Cambridge, UK, proposes an elegant technique to take ultracold atoms to the extreme, where atoms moving about in a lattice potential experience an effective magnetic field with of order a unit flux quanta per lattice site [1]. This is a realm that is inaccessible in conventional materials and promises new types of quantized Hall effects where Landau-level quantization and band-structure effects are intertwined.

Magnetic fields are enigmatic. In our studies we learn the Lorentz force law: in a uniform magnetic field, the force on a moving, charged object is perpendicular to both the magnetic field $\mathbf{B}$ and the object's velocity $\mathbf{v}$. This doesn't fit in with our usual intuitive picture of forces derived from gradients of potentials, but instead requires a new type of potential: the electromagnetic vector potential $\mathbf{A}$. Like the usual scalar potential ϕ, the vector potential is related to the seemingly more physical fields via derivatives: the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$. These potentials are not just mathematical sleights-of-hand; in quantum systems, they take center stage.

In Schrödinger's wave mechanics, the evolution of a particle's wave function can be partially understood in terms of its quantum mechanical phase. Usually this phase can be divided into two parts: the dynamic phase acquired in proportion to the particle's kinetic energy, and the phase from scalar potentials. Each of these accumulates at a rate proportional to the associated energy.

If we consider a particle moving in a closed loop, with zero scalar potential, then the dynamic phase acquired upon traversing the loop will tend to zero as the velocity drops to zero.

The phase acquired in a magnetic field is different: it depends on the geometry of the particle's path. If our loop now encloses a magnetic flux $\Phi = \int \mathbf{B} \cdot d\mathbf{a}$, then the particle will acquire an additional phase proportional to Φ. This interpretation is uncomfortable: somehow the particle has a nonlocal knowledge of the magnetic field everywhere inside the loop. It is more natural to think of the vector potential, in which case the acquired phase is no more than the line-integral $\int \mathbf{A} \cdot d\mathbf{l}$ of the vector potential around the loop. This leads to the celebrated Aharonov-Bohm effect [2, 3] where a charged particle acquires a geometric phase as it moves in the completely field-free region outside an infinite solenoid.

Other physical situations produce geometric phases in which neutral particles can behave as if magnetic fields were present. This concept was introduced to quantum mechanics as Berry's phase [4] for particles with internal structure (like spin states) for which the energy is dependent on parameters in the Hamiltonian, such as position or momentum. If a particle starts in an eigenstate, it can acquire a geometric phase upon traversing a closed loop in parameter space, provided the "motion" is sufficiently slow that it adiabatically remains in the same eigenstate.

The simplest example of a Berry's phase is shown in Fig. 1 where a neutral spin-1/2 particle is moving in an inhomogenous magnetic field, giving rise to a position-dependent Zeeman shift—the difference in energy between the spin being oriented along or away from the magnetic field. The figure depicts the particle's trajectory, along with the orientation of its ground state on the Bloch sphere. (This sphere defines the allowed states of a spin-1/2 particle.) The particle accumulates a geometric phase of $\Omega/2$, one-half the solid area traced out on
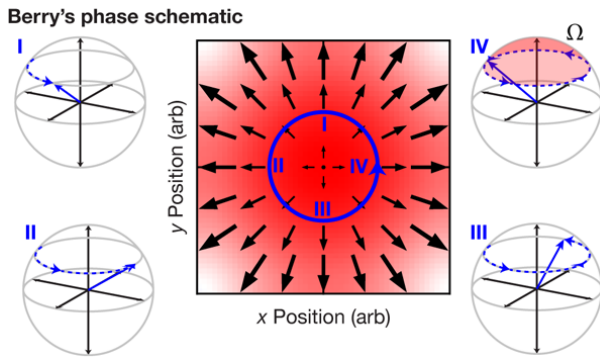
FIG. 1: The accumulation of phase as a spin-1/2 parti-
cle moves through an inhomogeneous (quadrupole) magnetic
field. The black vectors indicate the component of the nor-
malized effective Zeeman field in the $e_x - e_y$ plane and the
background color represents the e_z component. The Bloch
spheres show the enclosed solid angle, which sets the parti-
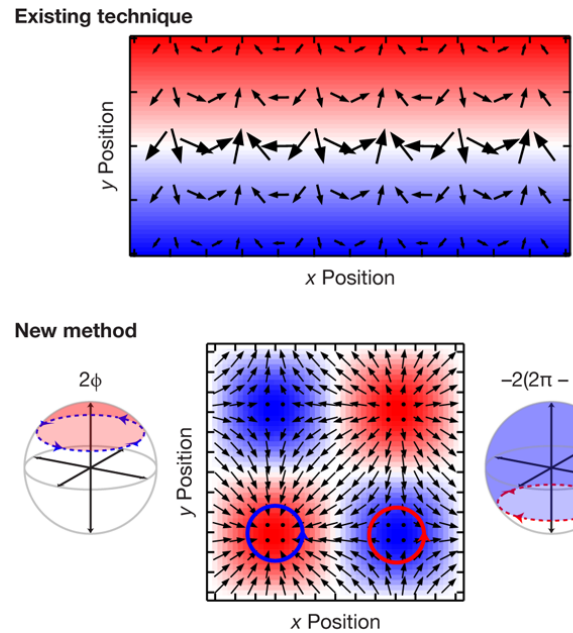cle's accumulated phase as it moves in a loop about the origin.

FIG. 2: Configurations that give rise to Berry's phases for cold
atoms in an optical lattice that generates an artificial gauge
field. (Top) Existing techniques can produce an infinitely
precessing Zeeman field along e_x, but not along e_y. (Bot-
tom) Cooper's proposal remedies this problem by allowing a
net positive Berry's phase in the lattice's unit cell. (Credit:
(Bottom) from [1])

the Bloch sphere as the particle moves in space. Such
a phase can be interpreted as arising from a geometric
gauge field, in analogy with the electromagnetic vector
potential.

Geometric phases are real. A system particularly well
suited for observing and studying their effects is a trap
of ultracold atoms. In these systems researchers can first
construct suitable geometries and then unambiguously
measure the result. Geometric phases in neutral atoms
were vividly demonstrated at MIT in 2002 when topolog-
ical phases were directly imprinted into the wave function
of a Bose-Einstein condensate (BEC) of sodium atoms in
a magnetic field by inverting the orientation of the field.
This produced 2π or 4π phase windings leading to vor-
tices in the final wave function [5].

Going beyond this elegant demonstration, the next
step is more powerful: constructing configurations where
the atomic system obeys a new Hamiltonian containing
steady-state gauge fields [6]. Such a system mimics that
of a particle in an effective magnetic field [7]. Instead of
using a magnetic field to generate a Berry's phase, these
ideas require a laser to couple different internal (spin)
states of atoms. Such coupling is formally equivalent to
a Zeeman magnetic field, but spatially structured on the
scale of the optical wavelength. In our group at NIST,
we followed these theory proposals with a series of ex-
periments demonstrating the effective mapping between
the Berry's phase and the electromagnetic vector poten-
tial, leading to artificial magnetic and electric fields (see
Refs. [8, 9] and references therein). The top of Fig. 2
depicts how these ideas lead to large Berry's phases, and
also highlights a key limitation. The illustrated Bloch
vector can wind an unlimited number of times around
its equator as an atom moves along $\mathbf{e}_x$, but it can tip to
the $\pm z$ axes of the Bloch sphere when the atom moves
along $\mathbf{e}_y$. This implies that we can create large "artificial
magnetic fields," but the maximum magnetic flux pass-

ing through the system scales as the length of the system,
not its area, making it difficult to scale the artificial field
to larger systems.

Cooper proposes a technique to overcome this limita-
tion [1] by creating a specific effective Zeeman field using
standing waves of light—a type of lattice. The essence of
his proposal is depicted in the bottom of Fig. 2: atoms
moving in the optical unit cell experience a Berry's phase,
both as a function of x and y (see also Ref. [10]). Usually
such ideas lead to staggered magnetic fields with equal
and opposite sign in neighboring lattice sites, with zero
average. The current work overcomes this limitation, al-
lowing for large effective magnetic fields with flux scaling
as the system's area, not its linear extent.

How does this work? The effective electromagnetic
vector potential created by this technique has a gauge-
dependent singularity called a Dirac string that effec-
tively concentrates magnetic field of one sign to isolated
points (with no physical effect). As a result, the effective
magnetic field is still staggered, but acquires a nonzero
average.

To more intuitively understand how this gives rise to
an effective field of the same sign, consider an atom mov-
ing along each of the two closed loops depicted in Fig. 2
(bottom). For the left loop, the Zeeman field traces out a
circle at the top of the Bloch sphere with a counterclock-
wise direction, while the right loop traces a circle at the
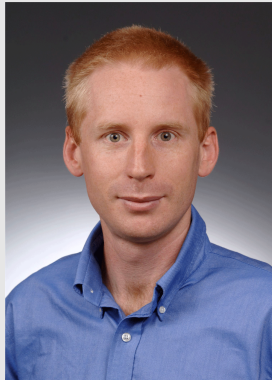bottom of the Bloch sphere with a clockwise direction. If

the top loop acquired a geometric phase ϕ, then the bottom loop acquired a phase of $-(2\pi - \phi)$ from the entire top portion of the Bloch sphere, but with the opposite sign. Since the acquired phase is defined only modulo 2π, these two curves enclose the same effective field.

This seemingly simple observation provides a straightforward path to realize larger effective magnetic fields than have hitherto been possible, yet with a comparatively simple set of lasers. (The pioneering proposals for large in-lattice gauge fields require more numerous lasers [11], and lack the simplicity and elegance of this approach.)

To complete the story, Cooper studied the properties of the lowest Bloch bands in this optical flux lattice and computed the Chern number. (Here, the Chern number enumerates the number of times the wave function's phase winds by 2π on a path running from one side of the Block band to the other: a loop on the torus). In some cases, the Chern number is ± 1, showing that these bands are topologically equivalent to the lowest Landau level (also with Chern number ± 1). Thus fermions completely filling the lowest band will be an integer quantum Hall state with a quantized Hall resistance $R_H = h/e^2$. The atoms used in cold-atom experiments are typically bosons and do not have a Fermi energy, but at fillings of around one per-lattice site—about one atom per magnetic flux quanta—they are expected to display interaction-driven bosonic fractional quantum Hall states.

Acknowledgments

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About the Author

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Ian Spielman in an experimentalist who received his Ph.D. in physics from the California Institute of Technology in 2004, studying quantum Hall bilayers. From there he moved to NIST in Gaithersburg, Maryland, for a two year NRC postdoc in the Laser Cooling and Trapping group, studying the physics of the superfluid-to-insulator transition in 2D atomic Bose gases. In 2006, he assumed his current position as a NIST physicist and a fellow of the newly founded Joint Quantum Institute (JQI). His research interests focus on using ultracold atomic systems to realize Hamiltonians familiar in condensed matter physics. This includes the pioneering work on creating artificial gauge fields for neutral atoms using laser atom interactions.