

A Novel Method for Determining the Lower Bound of Antenna Efficiency

Jason B. Coder ^{#1}, John M. Ladbury ^{*2}, Mark Golkowski ^{#3}

[#] *Department of Electrical Engineering, University of Colorado Denver
1201 5th St. Denver, CO 80204 USA*

¹ jason.coder@nist.gov

³ mark.golkowski@ucdenver.edu

^{*} *National Institute of Standards and Technology*

Mail Stop 687.02, 325 Broadway St. Boulder, CO 80305 USA

² john.ladbury@nist.gov

Abstract—Determining absolute transmitting efficiency has been a difficult task since the inception of the antenna itself. While methods that can measure transmitting efficiency do exist, most are complicated and prone to high uncertainties. A new method is presented for determining the lower bound of absolute transmitting efficiency by use of a reverberation chamber. This method is able to characterize both the transmitting and receiving efficiency of an antenna. After the method is derived, numerical simulations are presented. These simulations can provide insight into the behavior of the equations and necessary assumptions. Then, by use of measurement data, the method for transmitting efficiency is compared to efficiency data obtained from another reverberation chamber method. Data for this comparison will come from two different types of antennas: a wide band dual-rigged horn, and a narrow-band meta-material inspired antenna. Following the measurement data, possible areas for improvement of the method and its optimization are discussed.

I. INTRODUCTION

In [1], a new model for antennas was defined and discussed from a theoretical perspective. The proposed model accounts for the loss and mismatch characteristics of an antenna by combining them into an unknown two-port network. In theory, this conceptual network is placed in front of an ideal (lossless and perfectly efficient) antenna, as shown in Figure 1. When the model was presented in [1], a series of equations, derived from waveguide junction theory, were developed to explain the basic behavior of the unknown two-port network. From these original equations, a new series of equations has been derived that yield the absolute transmitting and receiving efficiency of an antenna modeled by an unknown two-port network. This derivation was facilitated by two key assumptions: (1) the model proposed in [1] is mathematically correct, and (2) the environment in which the antenna is placed has relatively little loss (environment absorbs less than 70% of the transmitted power). The second assumption is met by use of data collected in a reverberation chamber at low frequencies, where the losses due to chamber walls and contents are sufficiently low.

Antenna efficiency, as a concept, is well defined in textbooks [2] and standards [3], but few practical measurements exist that can accurately yield absolute efficiency [4], [5]. Of the

methods that yield absolute efficiency, a reverberation chamber is seldom the measurement environment of choice [6]. Most methods utilize a free-space environment (i.e., radiation pattern integration) or a metal sphere/hemisphere (i.e., Wheeler cap). Of the methods that do make use of the reverberation chamber, most yield relative efficiency [7]. Unless the absolute efficiency of the reference antenna is known, the user can calculate only the relative efficiency of the antenna [8].

Measurements of relative efficiency in reverberation chambers usually have another undesirable characteristic: high uncertainties. These large uncertainties are due to the low number of stirring positions (i.e., paddle, frequency, and/or platform stirring), the low number of independent samples, and electric fields that are not statistically uniform. The method presented here attempts to combat these high uncertainties with a simplified measurement method that is less dependent on the statistical uniformity of the reverberation chamber. Another advantage of this method is that it requires only one antenna. Relative methods typically require three antennas (in two separate measurements) to determine the relative efficiency of one antenna compared to that of another.

We begin by reviewing the IEEE standard definition of transmitting efficiency and how the proposed method fits in, then briefly discuss the two-port antenna model originally proposed in [1]. After this method is presented, numerical simulations are used to demonstrate the performance and accuracy of this method. Measured data are analyzed and compared to data processed by means of relative efficiency methods.

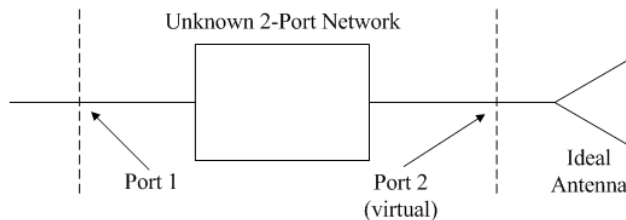


Fig. 1. The configuration of the unknown two-port network and ideal antenna.

II. DEFINITION OF EFFICIENCY

There are two common definitions of transmitting efficiency. The main difference between the two definitions lies in the treatment of an impedance mismatch. The IEEE 145 standard defines radiation efficiency (referred to here as "transmitting efficiency") as the ratio of power radiated by the antenna to the power accepted by the antenna [3]. This definition allows for the case where there is very high mismatch, but a very efficient antenna. For example, assume 1 W of incident power into an antenna and 0.95 W is reflected. Of the 0.05 W accepted by the antenna, 0.025 W is radiated, and 0.025 W is lost. Under the IEEE definition of transmitting efficiency, this antenna is 50% efficient, even though 95% of the incident power is reflected.

The alternative definition of transmitting efficiency (used in [4]) considers reflection of power by the antenna to be a loss mechanism. In this definition, efficiency is defined as the ratio of power radiated by the antenna to the power incident to the antenna. Returning to the above example, the antenna would be 2.5% efficient because 1 W was incident on the antenna and 0.025 W was radiated.

Although there is a large difference between these two definitions, neither one is necessarily wrong (or right). The method presented here utilizes the IEEE standard definition of transmitting efficiency. This difference is pointed out as an aid to future comparisons between this method and any other method of finding transmitting efficiency.

III. TWO-PORT ANTENNA MODEL

The two-port model used here was first published by Ladbury and Hill [1]. This antenna model is novel, in that it models a real antenna as an ideal antenna (lossless and perfectly matched) connected to an unknown two-port network.

The parameters of the two-port network model the mismatch and loss characteristics of the antenna. Thus, knowledge of the two-port network parameters allows for full characterization of the antenna. Note that the radiation pattern and scattering characteristics of the antenna are not modeled as part of this two-port network, and are assumed to be identical to those of the real antenna. The unknown two-port model is defined such that the intrinsic impedance (Z_0) of port 2 is the same for port 1. Additionally, the network is defined to be reciprocal, that is: $S_{21}=S_{12}$ [1].

By examining this model in a free-space environment, an equation for transmitting and receiving efficiency can be derived. If we transmit into port 1 of the setup shown in Figure 1, there will be no reflections from our ideal antenna in free-space. Therefore, the network will appear to be terminated using a non-reflecting load [1]. This results in the reflection coefficient of the antenna being the same as the S_{11} of the two-port network. Given an incident power P_{inc} , the power reflected from the two-port network is $P_{inc}|S_{11}|^2$, and the net power into the ideal antenna is $P_{inc}(1 - |S_{11}|^2)$. Because an ideal antenna is placed after the two-port network, net power into it (all of which will be transmitted) is $P_{inc}|S_{21}|^2$ [1]. The transmitting efficiency η_T can be written as

$$\eta_T = \frac{|S_{21}|^2}{1 - |S_{11}|^2}. \quad (1)$$

The receiving characteristics of this model can be expressed in a similar fashion. The model assumes that a non-reflecting load can represent the receiver connected to port 1. This results in a definition of receiving efficiency that is very similar to the transmitting efficiency equation:

$$\eta_R = \frac{|S_{12}|^2}{1 - |S_{22}|^2}. \quad (2)$$

Though the receiving efficiency can be defined algebraically, measuring S_{22} of the model is difficult because it is a theoretically defined quantity. Though it cannot be directly measured, a procedure to calculate it is shown below. It would be inappropriate to assume that $|S_{11}| = |S_{22}|$, because the two-port network used to model the antenna can be *any* reciprocal, realizable network. Not assuming that the reflection coefficients are equal challenges a common assumption: receiving and transmitting efficiencies may not be the same.

In [1], Ladbury and Hill noted that ϕ_{21} and ϕ_{12} , the phase angles of S_{21} and S_{12} , respectively, are generally unknown, but the actual phases are unimportant in the subsequent analysis. Because the model was defined to be reciprocal, we will assume that $\phi_{21} = \phi_{12}$.

IV. LOWER BOUND OF EFFICIENCY

If we consider the case where the antenna is within a reverberation chamber, the two-port model described in the previous section can be expanded. This expansion will yield an algebraic solution for transmitting and receiving efficiency. In order to develop these algebraic solutions, an assumption must be carefully made. We assume that, for at least a few paddle positions, the reverberation chamber absorbs very little power. That is, nearly all energy transmitted into it is reflected back to the antenna. This assumption is achieved as long as (a) the losses due to chamber wall and contents are sufficiently low, and (b) a sufficient number of samples are taken. These assumptions force our calculations to be only a lower bound for transmitting or receiving efficiency, rather than an estimate.

Figure 2 shows an expanded view of the two-port network being used to model the parameters of the antenna. When S -parameters are referred to in this section, they are in reference to the network shown here, rather than to the ideal antenna.

The Γ_L parameter shown in Figure 2 represents the connection to the reverberation chamber for a perfectly matched and

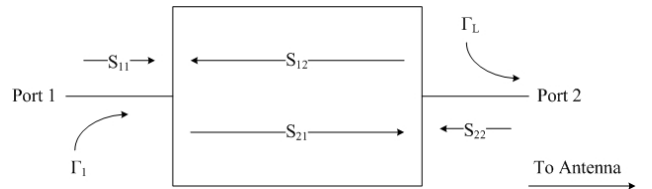


Fig. 2. An expanded view of the two-port network used to model an antenna.

lossless antenna. With Γ_L connected to a lossy reverberation chamber, Γ_L is assumed to have a uniformly distributed phase and a Rayleigh distributed magnitude. A truncated Rayleigh distribution will be used to account for the fact that the magnitude of the reflection coefficient cannot be greater than 1. For a known Γ_L , Γ_1 can be found by use of the following equation from [9]:

$$\Gamma_1 = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}. \quad (3)$$

In a reverberation chamber, the magnitude and phase of the electric field are assumed to be independent of each other. This allows for the magnitude and phase of Γ_L to change separately, thus simplifying the analysis. If the magnitude of Γ_L is held constant while the phase is varied, we would effectively be attaching a sliding termination to the model two-port network [1]. This concept has been well characterized, and we show that (3) will transform a Γ_L circle (centered at the origin with radius r) to a new circle with the following radius R_1 and center C_1 [9] given by:

$$R_1 = \frac{|S_{12}S_{21}\Gamma_L|}{1 - |S_{22}\Gamma_L|^2}, \quad (4)$$

$$C_1 = S_{11} + R_1 \overline{S_{22}} |\Gamma_L| e^{j(2\phi_{21})}. \quad (5)$$

The notation $\overline{S_{nm}}$ is used to denote the complex conjugate of S_{nm} .

Since Port 2 in Figure 2 is a virtual port, only Γ_1 represents a physical measurement. All other parameters are determined from (3), (4), and (5). These equations can be simplified by applying the assumption that the reverberation chamber is almost perfectly reflective at a few paddle positions. This allows $|\Gamma_L| \approx 1$. This assumption does not come without cost. By assuming that $|\Gamma_L| \approx 1$, an error is introduced into the calculation. This error is equivalent to assigning all of the chamber loss to the antenna. This will result in an overestimate of the antenna losses, which corresponds to an underestimate of efficiency.

Further simplification is possible when we apply the assumption that the model network is reciprocal ($S_{12} = S_{21}$). Here, the phases of S_{21} and S_{12} (ϕ_{21} and ϕ_{12} , respectively) were set to zero. In [1], the authors showed that because the phase of Γ_L is uniformly distributed and independent of the magnitude, S_{11} can be estimated by taking the ensemble average of Γ_1 :

$$E(\Gamma_1) = S_{11}. \quad (6)$$

With these definitions and assumptions, a lower bound for the receiving and transmitting efficiency of the model two-port network can be found.

A. Receiving Efficiency

Utilizing the assumptions discussed in the previous section, the receiving efficiency can be found by manipulating (4):

$$R_1 = \frac{|S_{12}S_{21}\Gamma_L|}{1 - |S_{22}\Gamma_L|^2} = \frac{|S_{12}S_{21}|}{1 - |S_{22}|^2} = \frac{|S_{21}|^2}{1 - |S_{22}|^2} = \eta_R. \quad (7)$$

To calculate receiving efficiency, we can take a set of Γ_1 data and find the circle of minimum radius that bounds all data points. According to (7), the radius of the minimum bounding circle is equal to the receiving efficiency of the antenna/two-port model network. This result is consistent with [9].

B. Transmitting Efficiency

Finding the transmitting efficiency is not as easy as finding the receiving efficiency. In this process, the same assumptions that were used in finding the receiving efficiency will be used again. Referring to (1), both the transmitting efficiency and $|S_{22}|$ are unknown. To find both parameters, a simple system of equations can be used. First, S_{22} can be derived from (5):

$$S_{22} = \frac{1}{R_1}(\overline{C_1} - \overline{S_{11}}), \quad (8)$$

which yields

$$|S_{22}|^2 = \frac{1}{R_1^2} |C_1 - S_{11}|^2. \quad (9)$$

With $|S_{22}|^2$, (1) and (2) can be manipulated to yield the transmitting efficiency:

$$\eta_T = \frac{\eta_R(1 - |S_{22}|^2)}{1 - |S_{11}|^2}. \quad (10)$$

By use of (7), (9) and (10), transmitting efficiency can be determined with the same minimum radius bounding circle that was used to find the receiving efficiency.

V. NUMERICAL SIMULATIONS

To simulate these equations numerically, we generated several random sets of Γ_L data consistent with our hypothetical distribution, as shown in Figure 3. Next, specific values were assigned to the S -parameters, subject to the realizability constraints [7] of the two-port network (see Figure 1). Then, by use of (3), the data were transformed to Γ_1 . Finally, the Γ_1 data were processed by means of the procedures given above to establish the lower bound of transmitting and receiving efficiencies. This allows the script to calculate the efficiencies from data where the correct answers are already known.

When calculating the transmitting and receiving bounds, the first step is to calculate the circle of minimum radius that bounds all Γ_1 points. Figure 4 shows the random Γ_L data transformed to Γ_1 . The red circle around the Γ_1 data is the minimum radius bounding circle. For this example, 1,000 random points were generated from 1,000 normal real and 1,000 normal imaginary samples with a standard deviation of 0.5. Points that lie outside the unit circle are discarded and new points are generated until all points lie within the unit circle. This high standard deviation is necessary to maintain the truncated Rayleigh distribution that models the low-loss reverberation chamber. This standard deviation also ensures

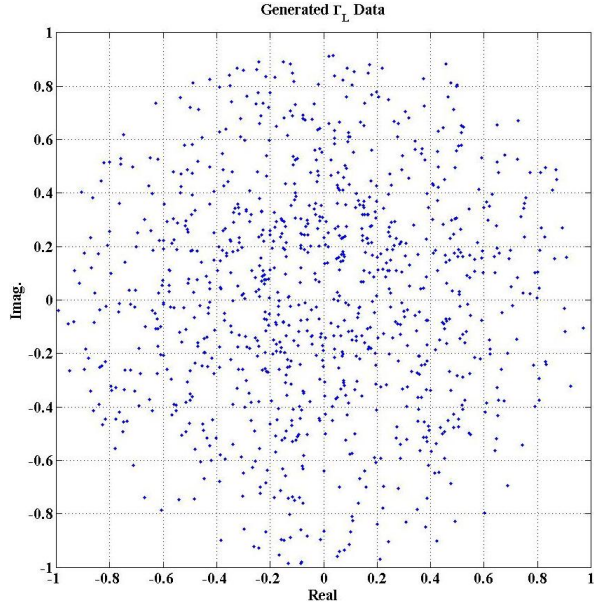


Fig. 3. Randomly generated Γ_L data that has been generated to have a uniform phase distribution and truncated Rayleigh distributed magnitude.

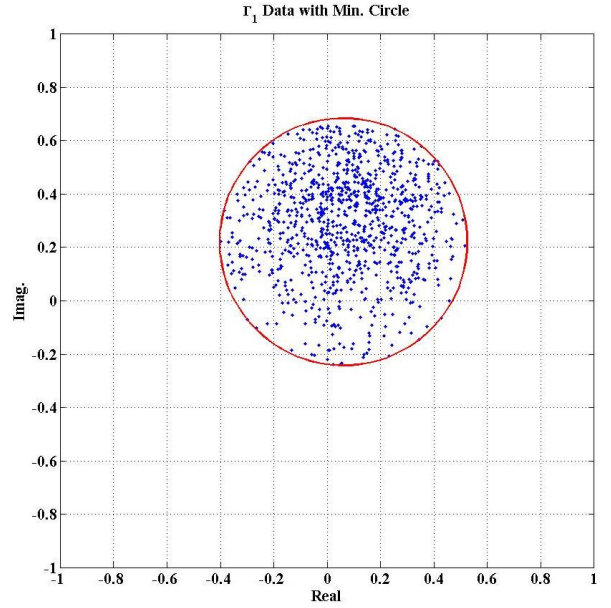


Fig. 4. Γ_L data from Figure 3 transformed to Γ_1 with the minimum radius bounding circle in red.

that a few points (at least) will be obtained where almost all of the energy transmitted into the reverberation chamber is reflected back to the antenna.

The receiving efficiency bound is estimated directly from the radius of the bounding circle. The complex average of the Γ_1 data becomes an estimate of S_{11} of the model network. The center of the bounding circle is used to find S_{22} and the transmitting efficiency (with (9) and (10)). The transmitting and receiving efficiencies are calculated and compared to the values of the original S -parameter matrix.

Adjusting the standard deviation of the Γ_L data being generated can effectively change how much loss occurs in the reverberation chamber. By decreasing the standard deviation, the amount of loss in the chamber increases. Eventually, the standard deviation will be low enough that too few Γ_L values are close to 1, and the efficiency calculations described here will become increasingly biased.

These numerical simulations allow two important conclusions to be drawn. First, the amount of loss the reverberation chamber has strongly influences the accuracy of the efficiency calculations. The lower the loss, the more accurate the calculations will be. Second, the number of points (or paddle positions) is key to properly modeling the truncated Rayleigh distribution of the magnitude. Too few paddle positions, even with very low loss, can generate large errors in the efficiency calculations.

VI. MEASURED DATA

As a practical application of the proposed method, data collected from a relative efficiency measurement are analyzed. This allows for a direct comparison between efficiencies determined with the relative method, and absolute efficiency bounds

determined with the proposed algebraic method. The measured relative efficiency data being used were collected with two antennas inside a reverberation chamber. The reverberation chamber has dimensions of 4.2 m in length, 3.6 m in width, and 2.9 m in height. This chamber is equipped with two paddles, one extending horizontally from wall to wall, and the other vertically from floor to ceiling. The paddles are similar in design, and are approximately 0.7 m wide. The reference antennas used for this measurement were a pair of dual-ridged horns with a specified operating frequency of 200 MHz to 2 GHz. The antenna under test (AUT) was a meta-material inspired antenna with a nominal resonant frequency of 305 MHz. Data were obtained with a vector network analyzer and averaged over 100 paddle positions. Because these data were acquired with the intent of calculating relative efficiency, two antennas were present in the chamber.

Figures 5 and 6 show the lower bound of transmitting and receiving efficiency (respectively) of the dual-ridged horn antenna. The receiving efficiency is shown for purposes of comparison to the transmitting efficiency. We expect them to be similar, but not necessarily identical. Because there is no way to measure the receiving efficiency of an antenna, the algebraic method proposed here cannot be wholly validated. Future measurements may allow for the validation of the method for calculating receiving efficiency. Until then, only transmitting efficiency is compared to the relative measurements.

The second antenna measured has very different characteristics. As a meta-material inspired antenna it is designed to have a very narrow-band resonance. The benefit of this antenna style is that it is significantly smaller (physically) than a wavelength at its resonant frequency. Figures 7 and 8 show the respective

calculated transmitting and receiving efficiency bounds for the meta-material inspired antenna. When compared to the horn antenna, the different characteristics of the meta-material inspired antenna are clear. But why aren't the transmitting and receiving efficiencies as close as they were in the previous case? The difference is the result of a statistical sampling and distribution problem. This problem will be discussed in more detail in the next section. One key result is that at the resonant frequency, the calculated transmitting and receiving efficiencies are close to the same value.

Uncertainties for the lower bound calculations in Figures 5 to 8 are based on the uncertainties of the measurement. In this case, the reflection coefficient S_{11} , is the only measured parameter. The uncertainty in the measurement of S_{11} is 0.1 dB (with a coverage factor of 2). This translates into a 2.3% uncertainty in the efficiency calculations. This does not mean that the true transmitting or receiving efficiency of the antenna is within the uncertainty bound, and should not be confused with a definite estimate.

A. Comparison to Relative Efficiency Method

To compare the proposed algebraic method with the relative method, a single frequency can be examined. For this comparison, the resonant frequency of the meta-material inspired antenna will be selected: 307.8 MHz. At this frequency, data processed with the relative method show that the horn antenna should have a transmitting efficiency 2.3% higher than the meta-material inspired antenna. However, looking at the difference in absolute efficiencies as they have been calculated here, the transmitting efficiency of the horn antenna is 7.7% higher than the meta-material inspired antenna. We do not expect the numbers to be the same because the algebraic method only produces a lower bound. The difference between

these two methods can most likely be attributed to a difference in measurement uncertainties. The through parameters used to calculate the relative efficiency number are subject to an uncertainty of 1.2 dB (with a coverage factor of 2) based on equipment uncertainties, field uniformity, and number of paddle positions. This gives the final relative efficiency an uncertainty of 31.8%.

The relative efficiency method requires that two antennas be in the chamber for both measurements because it uses the through parameter (S_{12} or S_{21} , respective to global measurement variables, not specific to the two-port model) to calculate relative transmitting efficiency. The proposed algebraic method requires only one antenna in the chamber, because it uses the reflection parameter (S_{11}) when efficiency is calculated. In the reverberation chamber (particularly at low frequencies) measurements of through parameters have a higher uncertainty than measurements of reflection parameters. As the frequency increases and field uniformity inside the chamber improves, the uncertainty when through parameters are measured decreases. In addition to the inherent uncertainty encountered when through parameters are measured, the second antenna being in the chamber creates a source of loss that is not accounted for.

Although the second antenna in the chamber increases the losses attributed to the antenna under test, we need to use a data set with two antennas in order to make a direct comparison to the relative method. The second antenna in the chamber will result in slightly lower estimates of transmitting and receiving efficiency.

VII. DISCUSSION

The difference between the transmitting and receiving efficiencies of the meta-material inspired antenna point out an

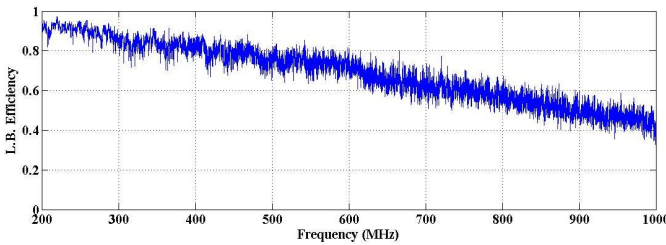


Fig. 5. Calculated lower bound of transmitting efficiency for the dual-ridged horn from 200 MHz to 1 GHz.

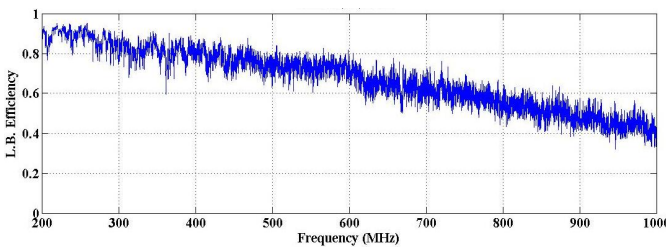


Fig. 6. Calculated lower bound of receiving efficiency for the dual-ridged horn from 200 MHz to 1 GHz.

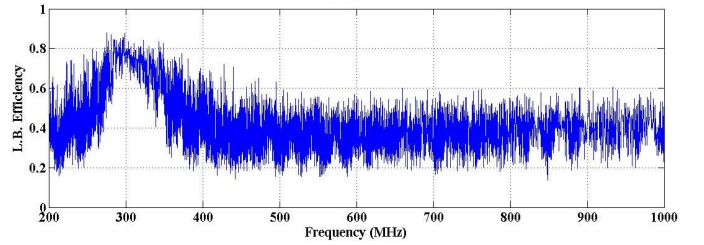


Fig. 7. Calculated lower bound of transmitting efficiency for the meta-material inspired antenna from 200 MHz to 1 GHz.

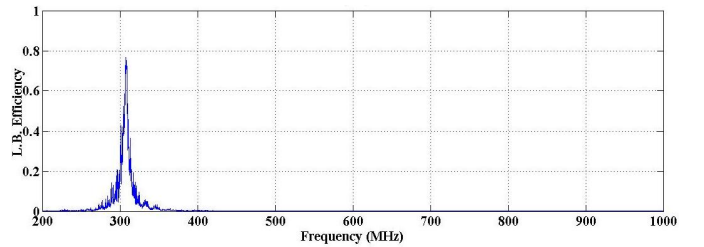


Fig. 8. Calculated lower bound of receiving efficiency for the meta-material inspired antenna from 200 MHz to 1 GHz.

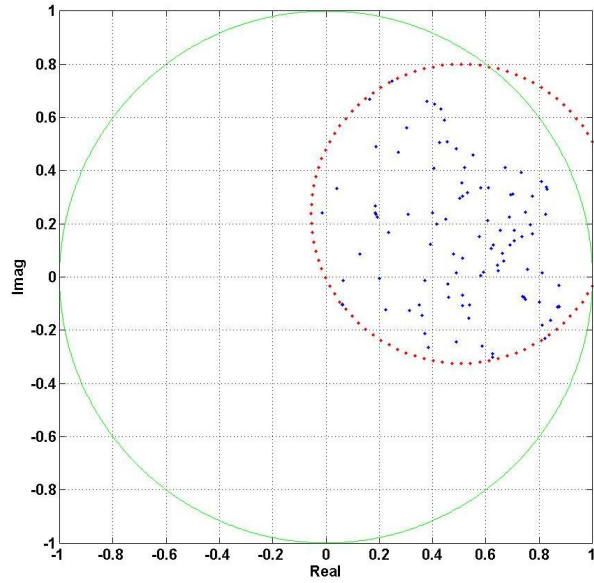


Fig. 9. An example of a measured Γ_1 distribution that forces the minimum bounding circle to exceed the unit circle.

important issue with the proposed method. With only 100 paddle positions, the truncated Rayleigh magnitude and uniform phase distributions of Γ_1 are not properly represented. This has a direct impact on the radius of the minimum bounding circle, which causes an underestimation in the lower bound of efficiency. As the number of paddle positions increases, the statistical distributions of Γ_1 will be better represented, and the estimate of the lower bound will improve.

Although additional paddle positions would improve the accuracy of the calculations across all frequencies, this problem is especially prevalent when the following three conditions are met: low number of Γ_1 points, high mismatch, and high efficiency. Consequently, the efficiency calculations for the horn antenna are more accurate than those for the meta-material inspired antenna, because these conditions do not apply. Since the meta-material antenna has very high mismatch outside of the resonant frequencies, the minimum bounding circle will be poorly estimated, and the accuracy of the transmitting and receiving efficiency calculations will be degraded.

Another issue encountered in the comparison to the relative efficiency data are cases where the distribution of Γ_1 forces part of the minimum-radius bounding circle outside the unit circle. This case typically occurs when the measured $|\Gamma_L|$ is very close to 1 (greater than 0.99). Figure 9 shows an example of this problem, where the center of the bounding circle is $0.2254-0.0844i$ and the radius is 0.8185. In this case, the calculated transmitting efficiency is erroneous because it is outside of the 0-100% bound.

Future work may show that this problem can be overcome, perhaps with an optimization method that forces the circle to be within the unit circle by weighting particular Γ_1 points. In the results shown, these cases were removed from the final data set. In the horn antenna calculations, these occurrences

were rare. However, in the meta-material data, these cases were more prevalent, as the reflection coefficient outside of resonance is almost always very close to 1. This problem would be alleviated by an increase in sampling (i.e., more than 1,000 paddle positions).

VIII. CONCLUSION

A new method for determining the lower bound of absolute transmitting and receiving efficiency by means of a reverberation chamber is presented. This method was derived algebraically based on equations from Ladbury and Hill [1], and waveguide theory [9]. Numerical simulations were used to model the equations used in the efficiency calculations and to explore their behavior when the assumptions are not met. The most important of these assumptions is that the reverberation chamber must not be the dominant source of loss, and the antennas must be reasonably well matched. A comparison with transmitting efficiency values calculated from a relative measurement method produced similar results. The differences between these two methods were indistinguishable, due to measurement uncertainty. The disadvantage of the new method is that it provides only a lower bound on efficiency. This is due to the fact that chamber does have some loss at all paddle positions. Part of the proposed algebraic method remains unvalidated as there is no way to measure the receiving efficiency of an antenna. This problem may eventually be solved, but our results indicate that the difference between the transmitting and receiving efficiencies is usually small.

ACKNOWLEDGMENT

The authors thank Dr. Chris Holloway at the National Institute of Standards and Technology for his input and the use of his meta-material inspired antennas for our measurements.

REFERENCES

- [1] J. M. Ladbury and D. Hill, "An improved model for antennas in reverberation chambers," in *Proc. IEEE International Symposium on EMC*, Ft. Lauderdale, FL, July 2010.
- [2] J. D. Kraus and K. R. Carver, *Electromagnetics*, 2nd ed. New York, NY: McGraw-Hill, 1973.
- [3] A. S. Committee, *IEEE Standard Definitions of Terms for Antennas, Standard 145*, The Institute of Electrical and Electronics Engineers Std. 145, Rev. 145-1993, 1993.
- [4] H. A. Wheeler, "The radiansphere around a small antenna," *Proceedings of the IRE*, pp. 1325–1331, August 1959.
- [5] Y. Huang, S. Boyes, H. Chattha, and N. Khiabani, "Wideband antenna efficiency measurements," in *International Workshop on Antenna Technology*, March 2010.
- [6] S. Lee and C. Lee, "Antenna efficiency measurements in a reverberation chamber without the need for a reference antenna," *IEEE Antennas and Wireless Propagation Letters*, vol. 7, pp. 448–450, 2008.
- [7] M. Piette, "Antenna radiation efficiency measurements in a reverberation chamber," in *Proc. of the Asia-Pacific Radio Science Conference*, 2004, pp. 19–22.
- [8] A. Erentok and R. W. Ziolkowski, "Metamaterial-inspired efficient electrically small antennas," *IEEE Transactions on Antennas and Propagation*, vol. 56, no. 3, pp. 691–707, March 2008.
- [9] D. M. Kerns and R. W. Beatty, *Basic Theory of Waveguide Junctions and Introductory Microwave Network Analysis*, 1st ed. New York, NY: Pergamon Press, 1967.