INFLUENCE OF NONSINUSOIDAL WAVEFORMS ON VOLTMETERS, AMMETERS, AND PHASE METERS

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ABSTRACT

The operating principles of various voltmeters, ammeters, and phase meters are described. The results of tests on these instruments at different levels of distortion indicate that phase meters are subject to large, often unpredictable errors while most voltmeters and ammeters respond to the rms value, independent of waveshape.

INTRODUCTION

The influence of waveform distortion on induction watthour meters has been well documented. The performance of power system voltmeters, ammeters, and phase meters under distorted conditions has received less attention. While these instruments are generally not used for revenue purposes, they are often used in control circuits which, in some cases, detect faults and shut down power systems to prevent damage. It is generally felt that modern rms-responding voltmeters and ammeters are relatively immune to the influences of waveform distortion and the data presented in this paper generally supports that position. On the other hand, modern phase meters operate by timing threshold crossings and these instruments may be very susceptable to distortion.

DEFINITIONS [1]

Peak Value: Y_p = maximum instantaneous value of waveform Y with fundamental period T.

Average Value:

$$Y_a = 1/T \int_a^{a+T} Y dt$$

Y_a of a sine wave = 0.

Average Absolute Value:

$$Y_{aav} = \frac{1}{T} \int_a^{a+T} |Y| dt$$

Y_{aav} of a sine wave $\simeq 0.637\, Y_p$.

Root Mean Square Value:

$$Y_{rms} = \left[\frac{1}{T} \int_a^{a+T} Y^2 dt\right]^{1/2}$$

Y_{rms} of a sine wave $= Y_p/\sqrt{2} \simeq 0.707\, Y_p$.

Form Factor: $\frac{Y_{rms}}{Y_{aav}}$

Crest Factor: $\frac{Y_p}{Y_{rms}}$

OPERATING PRINCIPLES [2]

The following are commonly used techniques for measuring the various parameters of voltage and current waveforms:

A. In the peak voltage detector shown in Fig. 1, amplifier A_1 compares the output v_o to the input v_{in}. If the input is greater, capacitor C is charged until the output exceeds the input and the capacitor charging circuit is turned off. A_2 senses the voltage on C without draining charge. For a peak current detector, the circuit is modified by adding shunt resistor R. The voltage drop across R is then applied to the peak voltage detector. A peak-responding instrument could be configured to indicate the rms value of a sine wave by dividing the peak value by the crest factor (1.414); however instruments based on this technique are vulnerable to waveform distortion. The peak detector is generally used in power-line monitors where the peak value is of interest.

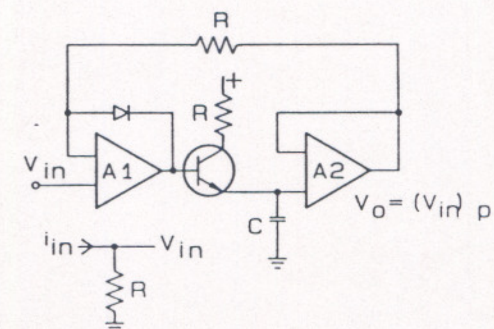

Fig. 1. Simplified diagram of a circuit to detect the peak value of voltage v_{in}. The output, v_o, is a dc voltage proportional to the peak value of v_{in}. Peak current is measured by converting the current to a voltage in shunt resistor R.

B. In the average voltage detector, a low-pass filter continuously integrates the signal providing an indication of it's dc component. If it is sufficiently large, the dc component of a signal can saturate transformers, damage certain equipment, and cause metering errors; thus, the average value is also of interest in power-line monitors.

C. In the average-absolute-value (aav) voltage detector shown in Fig. 2, the input v_{in} is split into positive and negative components in a full-wave operational rectifier. The negative component is applied to an inverting output stage and combined with the positive component to give a dc output that is proportional to the aav of the input voltage. The aav of a current waveform is derived from the simple analogous circuit also shown in Fig. 2. In the past, aav detectors were used as rms-indicators by multiplying their output by the form factor of the waveform (1.111 for sine waves). However, form factor varies with waveform and thus an aav circuit set up to indicate rms for sine waves may be in error by ±20% when measuring a waveform with 40% third harmonic. The aav of a waveform has little physical meaning and with the advent of inexpensive "true" rms detectors, aav instruments are becoming rare.

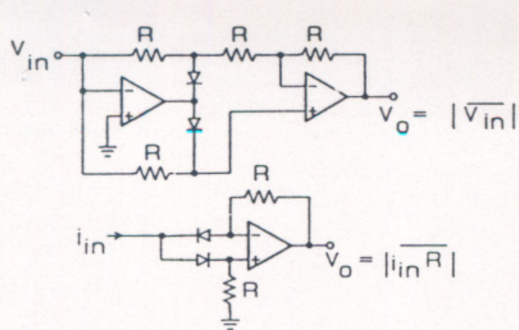

Fig. 2. Simplified diagram of circuits that convert the average-absolute-value of voltage v_{in} or current i_{in} to a dc voltage v_o.

D. In the rms-responding circuits described below, the input voltage is processed using an electronic multiplier. Commonly used multiplier techniques are "variable transconductance", "log/antilog", "time-division", "thermal", and most recently "digital sampling". In the past, electromechanical multipliers such as "electrodynamic" circuits were used extensively. All of these techniques can be configured to respond to the rms value of voltage or current waveforms, independent of harmonic amplitude or phase, as long as the harmonics are within the operating bandwidth of the instrument.

1. A block diagram of an rms-responding circuit using two analog multiplier modules is shown in Fig. 3. The input signal is squared in the first stage, averaged in a low-pass filter, and the square root is extracted by using the second multiplier as a feedback element in the third stage.

   A log/antilog rms-responding circuit is shown schematically in Fig. 4. The output is a dc voltage proportional to the rms value of v_{in}. The input signal is first rectified by the absolute value circuit. The next circuit takes twice the log of the rectified signal. The log of the output is subtracted in the next circuit block and this is followed by an antilog block to give v_{in}^2/v_{out}. The final circuit block takes the average, completing the rms-to-dc conversion. Log/antilog circuit modules are commonly used as rms-sensors in commercial digital multimeters.

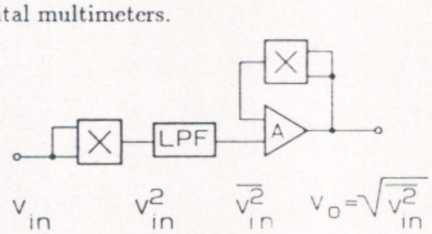

Fig. 3. Simplified diagram of a circuit that uses an analog multiplier to converts the rms value of v_{in} to a dc voltage v_o.

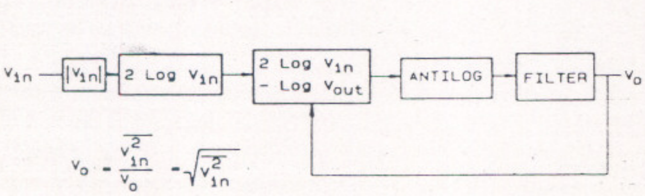

Fig. 4. Simplified diagram of a circuit that uses logrithmic modules to convert the rms value of v_{in} to a dc voltage v_o.

2. The time-division multiplier, in its simplest form, generates a series of pulses whose widths are proportional to the X input and heights are proportional to the Y input. The pulse area is then proportional to the product XY. Versions of this multiplication technique are used extensively in commercial electronic watt/watthour meters and in a few multifunction (V,I,P,J) meters.

3. The "thermal/electronic" rms-to-dc converter shown in Fig. 5 compares the heating power of the unknown voltage to the heating power of the dc voltage, v_o. A differential amplifier drives the difference between a matched pair of thermal-voltage-converters to zero by producing the output voltage v_o which is proportional to the rms value of the input voltage v_{in}.

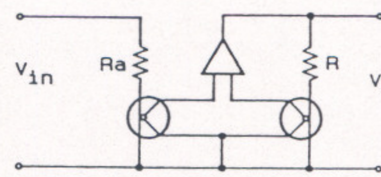

Fig. 5. Simplified diagram of a circuit that uses a differential pair of thermal voltage converters to convert the rms value of v_{in} to a dc voltage v_o.

4. In the sampling instrument (Fig. 6), instantaneous values along the input waveform v_{in} are held (using a sample/hold circuit (S/H)) so that an analog-to digital converter (ADC) can transform the sampled voltage to digital code. Data are then processed in a computer to give the rms voltage by calculating the square root of the mean-squared value of all of the sample points v_i. An advantage of this technique is that it is also capable of computing other waveform parameters such as the peak value, the aav, and total harmonic distortion.

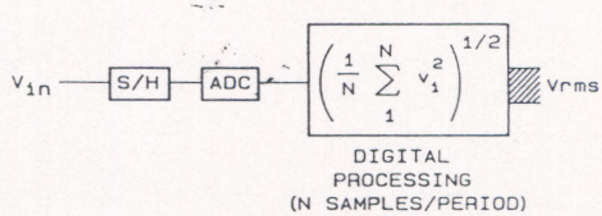

Fig. 6. Simplified diagram of a circuit that uses digital sampling to compute the rms value, v_{rms}, of v_{in}.

E. The phase angle between two signals may be computed by measuring the ratio of the time, t_1, between the zero-crossings of two signals and period, t_2, of one of the signals. In Fig. 7, the phase relationship between the two signals is defined as: $\Phi = (t_1/t_2) \times 360°$. Many commercial phase meters also measure the time between negative going zero-crossings (t_3) which is averaged with t_1 to compensate for dc components or nonsymmetrical waveforms caused by even harmonics. However, odd harmonics can lead to errors that are a function of the phase relationship of the harmonic to the fundamental. Figures 8–10 are oscillographs of a 100-Hz sine wave, and a sine wave with 40% third harmonic at 0, 90, and 180 degrees respectively. In Fig. 8 the zero-crossings are coincident and the phase meter should read correctly. In Figs. 9 and 10 the difference in zero-crossings is indicated by the timing markers, with "delta t" shown below the trace. From Fig. 9, assuming a period of 10 ms (t_2) and a delta t of 400 μs (t_1), the approximate phase meter error, $\Phi = (0.4/10) \times 360° = 14.4°$. Similarly, from Fig. 10, the phase meter error, $\Phi = (0.56/10) \times 360° = 20.2°$. Manufacturers often specify the maximum error (in % radians) as equal to the percentage of odd harmonic distortion (e.g. a 40% 3rd harmonic produces a maximum error of $0.4 \times 57° \simeq 23°$).

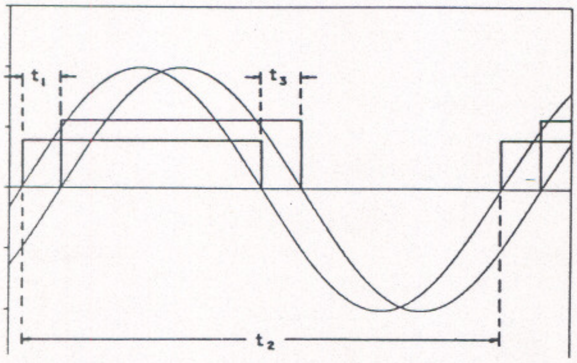

Fig. 7. Timing diagram showing the times t_1, t_2, and t_3 used by digital phase meters to compute the phase relationship between two signals.

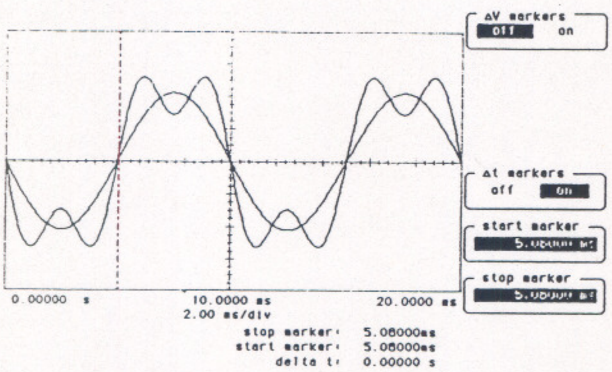

Fig. 8. Oscillograph of two waveforms used to test phase meters: a nearly pure sine wave and a waveform consisting of a sine wave plus 40% 3rd harmonic at 0° relative to the fundamental (zero-crossings are coincident).

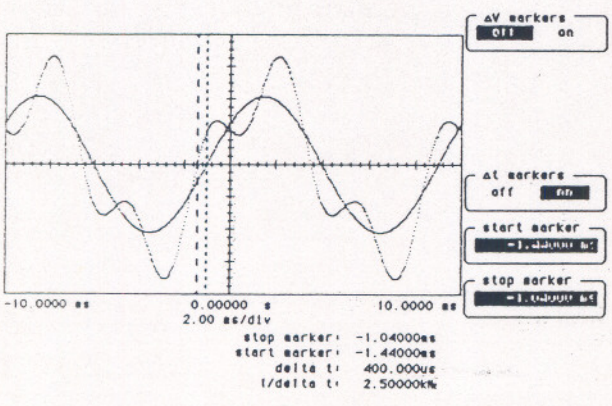

Fig. 9. Same waveforms as Fig. 8, with the harmonic phase-shifted 90° (dashed timing markers show the difference between zero-crossings).

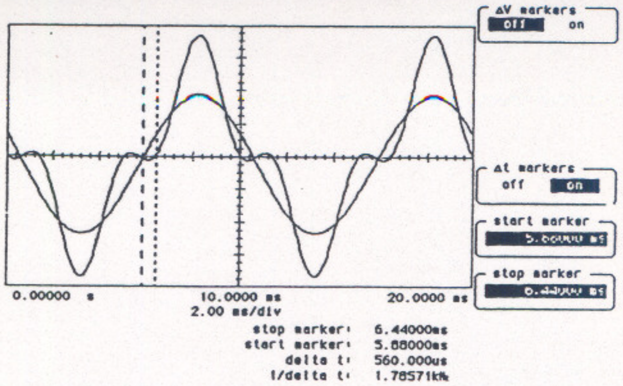

Fig. 10. Same waveforms as Fig. 8, with the harmonic phase-shifted 180° (dashed timing markers show the difference between zero-crossings).

TEST RESULTS

To measure the effect of nonsinusoidal waveforms on the accuracy of voltmeters, ammeters, and phase meters, a number of commercial instruments were tested under the following conditions:

a. sine wave frequency-response (10 Hz to 10 kHz),

b. square wave frequency-response (10 Hz to 400 Hz),

c. 60 Hz chopped sine wave at different firing angles,

d. 60-Hz sine wave plus 40% 3rd harmonic at different phase angles.

e. 60-Hz sine wave plus 3% 3rd harmonic at different phase angles (phase meter test only).

The waveforms were obtained from a dual-channel digital waveform synthesizer [3] which has amplitude uncertainties of < 0.01% and phase angle uncertainty of < 0.01°. The source was verified using thermal rms-responding instruments for voltage and current, and using an impedance bridge for phase angle.

Voltmeters and Ammeters

Test results for voltmeters and ammeters are plotted in Figures 11–18 where the following abbreviations indicate the operating principle of the instruments tested:

TM	- transconductance multiplier,
Log	- log/antilog multiplier,
Avg	- aav-responding calibrated in rms,
TDM	- time-division-multiplier,
Samp	- digital sampling
TVC	- thermal voltage converter.

The results of tests "a"(sine wave) and "b" (square wave) were normalized to the 60-Hz data point to show the relative frequency response rather than the absolute error. It should be noted that for tests "b" the aav-responding (Avg) voltmeter and ammeter were in error, as expected, by the ratio of the form factors.

The results of test "c" (chopped sine wave which simulates a thyristor-controlled circuit) were normalized to the 0 degree firing angle point (sine wave), while those of test "d" (the 3rd harmonic test) were normalized to the 0 degree phase angle point. For these tests, the dashed lines, that go off the graph, represent the error of the aav-responding instruments which, as expected, follow the change in form factor rather than the rms value.

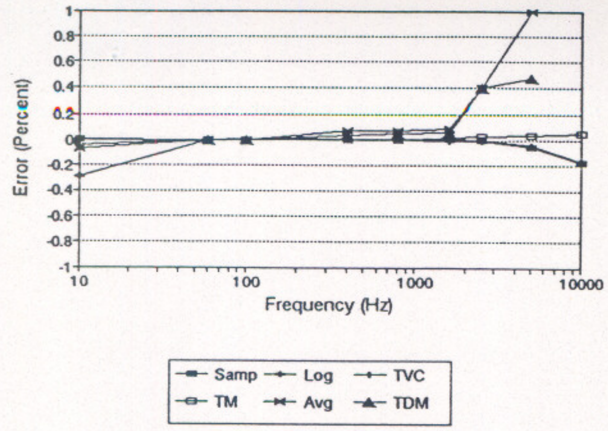

Fig. 11. Errors vs frequency of voltmeters based on six different operating principles (test signal = 120-V sine wave).

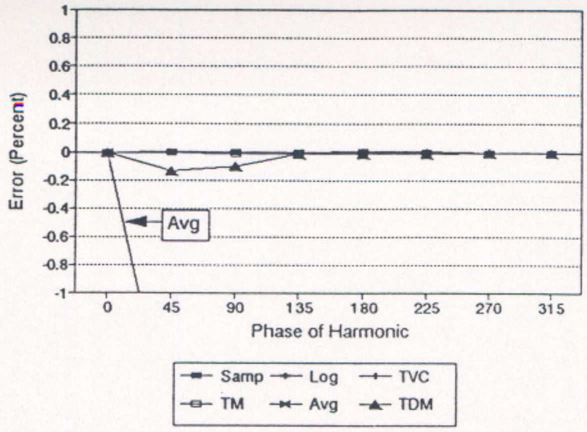

Fig. 14. Errors vs harmonic phase angle of the six voltmeters (test signal = sine wave plus 40% 3rd harmonic).

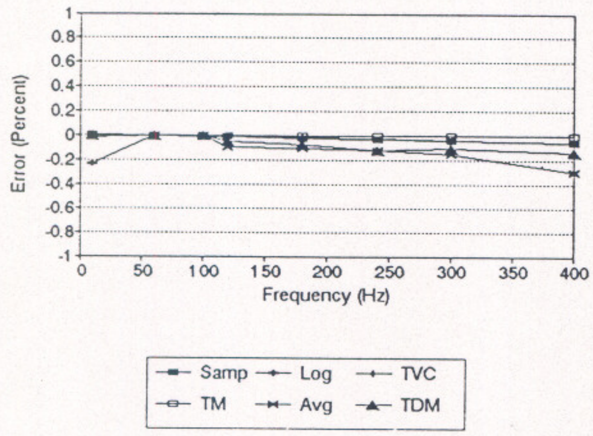

Fig. 12. Errors vs frequency of the six voltmeters (test signal = 120-V square wave).

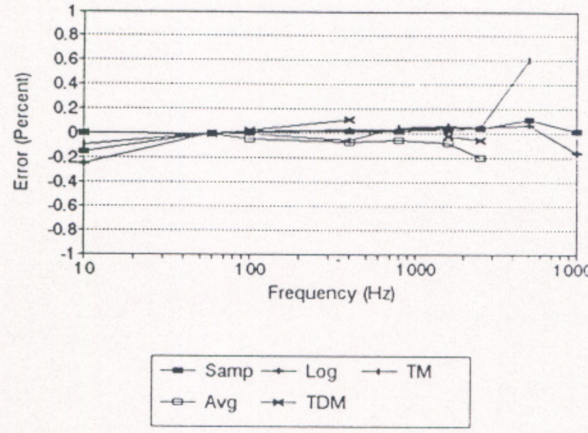

Fig. 15. Errors vs frequency of ammeters based on five different operating principles (test signal = 1-A sine wave).

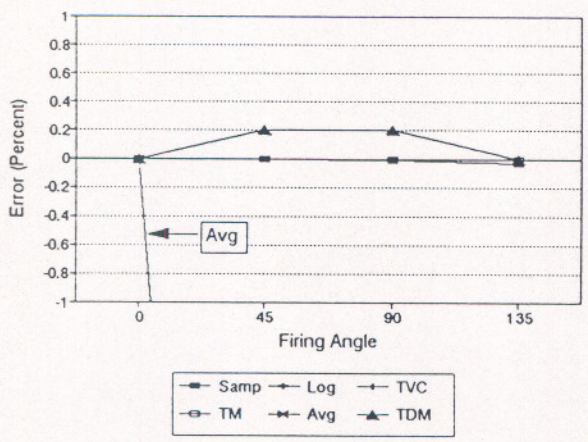

Fig. 13. Errors vs firing angle of the six voltmeters (test signal = 60-V chopped sine wave).

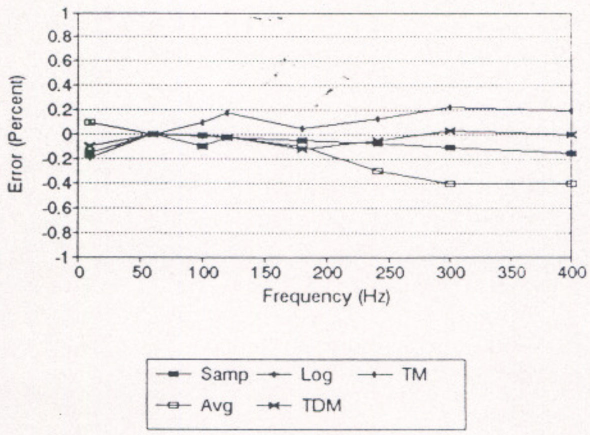

Fig. 16. Errors vs frequency of the five ammeters (test signal = 1-A square wave).

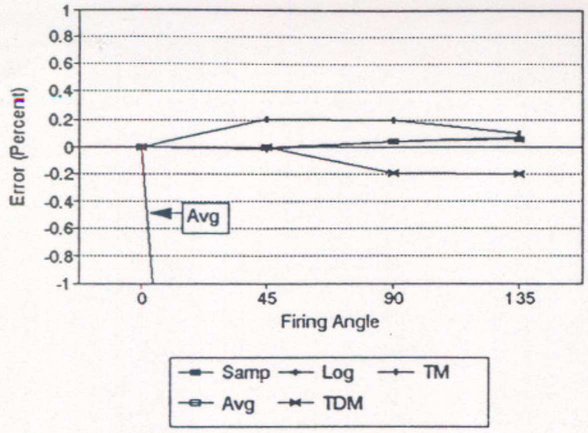

Fig. 17. Errors vs firing angle of the five ammeters (test signal = 0.5-A chopped sine wave).

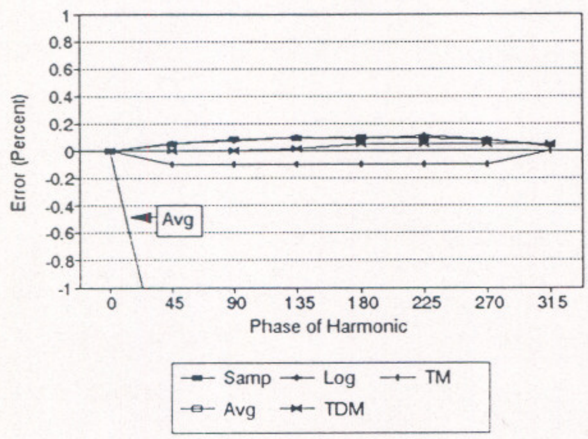

Fig. 18. Errors vs harmonic phase angle of the five ammeters (test signal = sine wave plus 40% 3rd harmonic).

Phase Meters

Two commercial phase meters, labelled "P1" and "P2" were tested using waveforms "a" through "e" described above with each of the waveforms adjusted to approximately 3-V_{rms}. It should be noted that the phase angle between complex waveforms is not defined [1] and the desired result of these tests is the phase angle between the fundamental components of the various waveforms. With sine waves applied to both channels, the two phase meters were within ±0.05° out to 1 kHz, degrading somewhat at 10 kHz. Similarly, with square waves applied to both channels, errors were within ±0.1° from 10 to 400 Hz. Various configurations of the other waveforms led to the phase meter results shown in Figs. 19–22.

The results shown in Fig. 19 were obtained by applying a sine wave to channel 1 and a chopped sine wave (at various firing angles) to channel 2. The phase angle between the fundamental waveforms (channel 1 – channel 2) was measured at 0°, 60°, and 90°. Phase meter P2 responded with no significant error while P1 had a maximum error of 6° at a 90° firing angle.

The results shown in Fig. 20 were obtained by replacing the chopped waveform in channel 2 with a sine wave distorted with 40% 3rd harmonic. The phase relationship between the 3rd harmonic and the fundamental was adjusted between 0° and 315° (see examples in Figs. 8–10), while the phase angle of the fundamental between the two channels was measured at 0°, 60°, and 90°. The errors for both meters are within the generalized odd-harmonic specification for "timing" phase meters (i.e. the maximum error in % radians = % odd harmonic) except for one point on phase meter P2 where the error exceeded 100°.

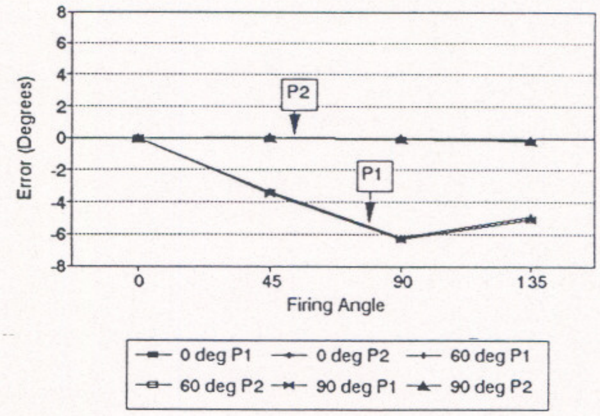

Fig. 19. Errors vs firing angle of phase meters P1 and P2 with a sine wave applied to channel 1 and a chopped sine wave applied to channel 2.

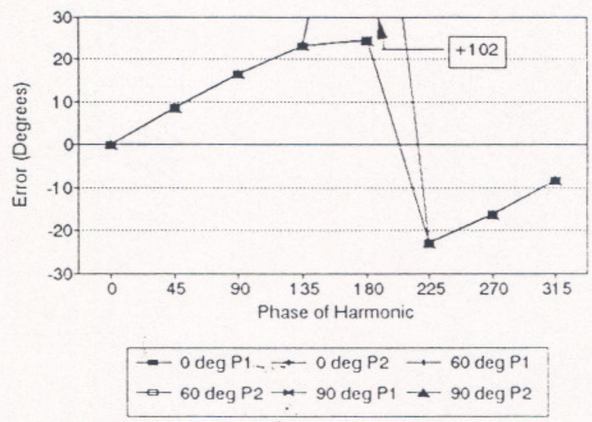

Fig. 20. Errors vs harmonic phase angle of the two phase meters with a sine wave applied to channel 1 and a sine wave plus 40% 3rd harmonic applied to channel 2.

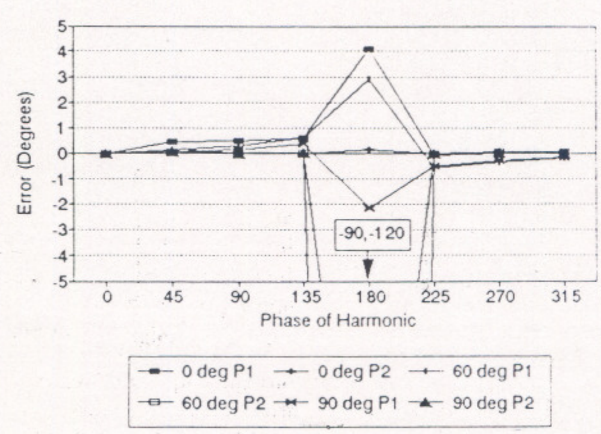

Fig. 21. Errors vs harmonic phase angle of the two phase meters with a sine wave plus 40% 3rd harmonic applied to both channels.

When the signals applied to both channels have the same waveshape, the harmonic errors described above should not occur. This premise was tested with the sine wave distorted with 40% 3rd harmonic. The phase of the harmonic was adjusted as before (with the same waveshape applied to both channels) and the fundamental phase between channels was measured at 0°, 60°, and 90°. In general, both phase meters were insensitive to these highly distorted waveforms (see Fig. 21), except at a harmonic phase of 180° where there are multiple zero-crossings (see Fig. 10). Both meters had difficulties at this point; however, the P2 readings were in error by 90° and 120° at fundamental phase angles of 60° and 90°, respectively.

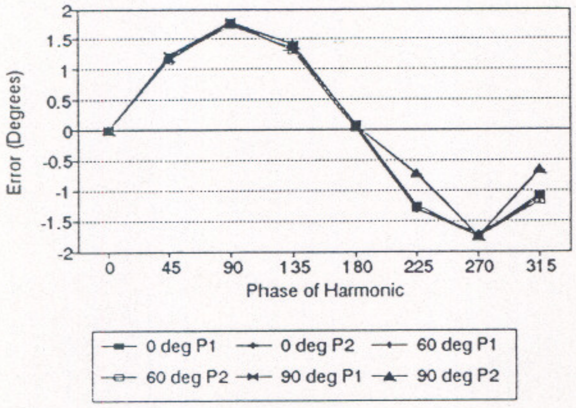

Fig. 22. Errors vs harmonic phase angle of the two phase meters with a sine wave applied to channel 1 and a sine wave plus 3% 3rd harmonic applied to channel 2.

Finally, tests were performed using a sine wave in channel 1 and a sine wave distorted with 3% 3rd harmonic in channel 2. This represents a more realistic condition, and at this level of distortion the waveform looks nearly sinusoidal on an oscilloscope so one might expect the phase meters to operate properly. The results of this test, given in Fig. 22, show that the errors are dependent on the phase relationship of the harmonic to the fundamental and independent of the phase angle between channels. Here both phase meters fall within the specified uncertainty (maximum error= $\pm 0.03 \times 57° = \pm 1.7°$). However, unlike the results shown in Fig. 20, the maximum errors occur at 90° and 270° and not at 180°. The actual and specified errors, for a level of distortion that is quite common in a power system, are more than 30 times the errors specified for sinusoidal waveforms. Therefore, when making critical phase measurements using this type of phase meter, it is very important to accurately measure the odd harmonic distortion.

CONCLUSIONS

The overall results indicate that the rms-responding voltmeters and ammeters performed quite well under highly distorted conditions; errors were typically within ±0.25%. It is clear, however, that rms-responding as opposed to aav-responding rms-indicating instruments must be used under nonsinusoidal conditions.

The two phase meters tested were very susceptible to distortion. One of the largest errors (102°) was observed with a sine wave applied to one channel and the test waveform applied to the other - a condition that might exist when measuring the phase angle between the voltage applied to a nonlinear load and a voltage proportional to the load current. It is clear that this type of phase meter cannot be relied upon under highly distorted conditions. For critical applications, a phase meter based on waveform sampling may be required.

REFERENCES

[1] *IEEE Standard Dictionary of Electrical and Electronic Terms*, ANSI/IEEE Std. 100-1988, (IEEE New York, NY).

[2] *Nonlinear Circuits Handbook*, (Analog Devices, Inc., Norwood, MA).

[3] N.M. Oldham, O.B. Laug, and B.C. Waltrip, "Digitally Synthesized Power Calibration Source," IEEE Trans. Instrum. Meas., vol. IM-36, no. 2, pp. 341-346, June 1987.