Simultaneous Solutions of Coupled Thermal Airflow Problem for Natural Ventilation in Buildings

ABSTRACT

Natural and hybrid ventilation are sustainable building ventilation strategies, where airflow is driven naturally by thermal buoyancy and/or wind forces other than pure mechanical means. The simulation and design of these systems thus need to consider mutual impact of the thermal and the air behaviors. A numerical solution of such thermal airflow problems often follows a segregate and iterative manner. Either the air temperature in the thermal problem or the air pressure in the airflow problem is solved separately with the other parameter known from a previous iteration. The newly solved parameter is then substituted successively in the other problem. For highly coupled thermal airflow problems, the segregate method could cause solution fluctuation or even divergence when relaxation factors are not carefully selected to avoid abrupt changes of air parameters in the successive substitution procedure. This paper investigated two non-segregate methods to solve thermal and airflow problems simultaneously. In the fully-simultaneous method, air temperatures and pressures for all rooms of a building are solved simultaneously by a single Jacobian matrix. In the semi-simultaneous method, a Jacobian matrix for the air temperature and pressure of one room is solved when air temperatures and pressures of other rooms are kept as constants. The same procedure is then repeated for each room of a building. In both cases, relaxations are not required. The simultaneous solution methods are demonstrated by a two-zone building with thermal buoyancy-driven flows, and validated by an experimental study of combined wind and buoyancy forces in a light well. It was shown that the simultaneous solvers provide stable solutions without using any relaxation in both cases. The predicted results also agree reasonably well with the experimental data.

NOMENCLATURE

C_i	Heat capacity of zone i, J/(kg·°C)
C_{ij}	Flow coefficient of path ij, kg/(s·Pa^n)
$\mathbf{f}$	Air mass or energy balance function
F_i	Air mass source per unit time in zone i, kg/s
F_{ij}	Airflow rate of airflow path ij, kg/s
g	Acceleration of gravity, m/s^2
h	Relative height of airflow path
h_k	Overall heat transfer coefficient at the wall surface node k, W/°C
$\mathbf{J}$	Jacobian matrix
k	Thermal conductivity, W/(m·°C)
m_i	Air mass of zone i, kg
n	Flow exponent; the nth iteration
N	Number of zones in a building
n_{ij}	Flow exponent of path ij
P_i	Pressure of zone i, Pa
P_j	Pressure of zone j, Pa
$P_{ij,s}$	Stack pressure at path ij, Pa
$P_{ij,w}$	Wind pressure at path ij, Pa
ΔP_{ij}	Pressure difference across airflow path ij, Pa
q''	Heat flux for wall conduction, W/m^2
Q_i	Heat source in zone i, W
r_i	Relaxation factor of zone i
T_i	Air temperature of zone i, °C
V_i	Volume of zone i, m^3
x_i	Unknown air parameter (temperature or pressure) of zone i

$\mathbf{X_i}$	Vector of zone state variables
z_i	Floor elevation of zone i, m
z_j	Floor elevation of zone j, m
Greek letters	
Δ	Difference
ρ	Air density, kg/m^3

INTRODUCTION

Natural and hybrid ventilations are important strategies of sustainable building designs. Depending on wind pressure force and/or buoyancy effect created from indoor and outdoor temperature difference, natural and hybrid ventilations reduce the usage of mechanical power for ventilation and improve indoor air quality by introducing outdoor air directly when it is suitable for use. Their applications include but not limited to passive cooling ventilation system such as night cooling, double skin façade, and use of large vertical space such as atria and light wells. The design of these systems and spaces includes the determination of the size, location and number of the ventilation openings, and the height and width of the atria and light wells so the use of buoyancy and wind effects is optimal for ventilation. This involves the prediction of airflow rates through ventilation openings and air temperatures in each room of a building by solving a coupled thermal and airflow problem.

Numerical solutions of airflow ventilation rates, and temperatures of room air and wall assemblies are often treated separately. Multizone airflow network models, such as CONTAM (Walton and Dols 2008) and COMIS (Feustel 1999), predict building airflows without solving energy equation. As a result, a known air temperature is required to be provided for each room. On the other hand, building energy analysis software tools are often not designed for inter-zonal airflows as a multizone network model does. To solve a coupled thermal airflow problem, many previous efforts have been conducted to integrate a building airflow network model with a building energy analysis tool in the past twenty years. Huang et al. (1999) linked the COMIS 3.0 with the EnergyPlus building energy simulation program (Crawley et al. 2001). Axley et al. (2002) developed a coupled thermal and airflow simulation tool, and demonstrated it for modeling a naturally ventilated commercial building. McDowell et al. (2003) integrated CONTAM with the TRNSYS building energy simulation software package (Beckman et al. 1994). Gu (2007) added an airflow network model to EnergyPlus and showed some encouraging results. Another popular energy simulation package with its own airflow models is the Environmental Systems Performance, Research version (ESP-r) building simulation program (Clarke 1985). These software tools and studies solve a coupled thermal airflow problem in a similar manner. Either the air temperature in the energy equation or the air pressure in the airflow mass balance equation is solved first with the other parameter kept at a constant. The newly solved parameter is then substituted successively in the other equation. This segregate and successive solution of energy and airflow equations requires relaxation factors to be selected carefully to avoid abrupt changes of room air temperature or pressure in the successive substitution procedure. For highly coupled thermal airflow problems, the segregate method could cause solution fluctuation or even divergence for poor relaxations. Ketkar (1993a) and Schneider et al. (1995) found that numerical instabilities can be caused when the airflow balance equation and the energy equation are of quite different orders of magnitude. For example, a slight change of air temperature as determined by the energy equation may cause a huge variation of airflow results in the airflow balance equation. In such case, a strong relaxation factor has to be used in the segregate solver, which however may slow down the overall convergence of the solution. Weber et al. (2003) developed an integrated thermal airflow simulation tool, TRNFlow, by linking COMIS and TRNSYS. They found that a single relaxation factor might not be possible to guarantee convergence at each time step during the whole simulation period. A method to generate adapted relaxations automatically was proposed and demonstrated by a two-zone case with buoyancy-driven airflow. This method, however, has not been verified by other studies. Another group of methods for solving highly coupled thermal airflow problems are simultaneous solutions, one of which is to assemble the airflow and energy balance equations for a whole building in a single matrix (Ketkar 1993b; Schneider et al. 1995). It was showed that the simultaneous solvers provided a suitable alternative to the segregate methods when numerical instabilities or slow convergence occur. However, for large problems, simultaneous solvers were found to require more computer power and memory due to increased numbers of iterations.

This study investigates simultaneous solvers for modeling coupled thermal airflow problems. In a fully-simultaneous solver, air temperatures and pressures for all rooms of a building are solved simultaneously by a single Jacobian matrix. A semi-simultaneous method is also proposed, in which a Jacobian matrix for the unknown air temperature and pressure of one room is solved when keeping air temperatures and pressures of other rooms as constants. Then the same procedure is repeated for each room of a building one after another. As a comparison, the segregate solvers with fixed and adapted relaxations are also studied and verified. All the four solvers are compared in the simulation of a two-zone building with buoyancy-driven flows. The fully-simultaneous and semi-simultaneous solvers are then validated by an experimental study of natural ventilation under combined wind and buoyancy forces in a light well.

THEORY

The formulation of a coupled thermal airflow problem of natural ventilation can be shown by the two-zone building in Figure 1, which is based on the case from Weber et al. (2003). The air is assumed to be well-mixed and static so the unknown air parameters of each zone are temperature (T) and pressure (P). The air pressure is also assumed to vary hydrostatically relative to the zone pressure, which is located at the floor of each room.

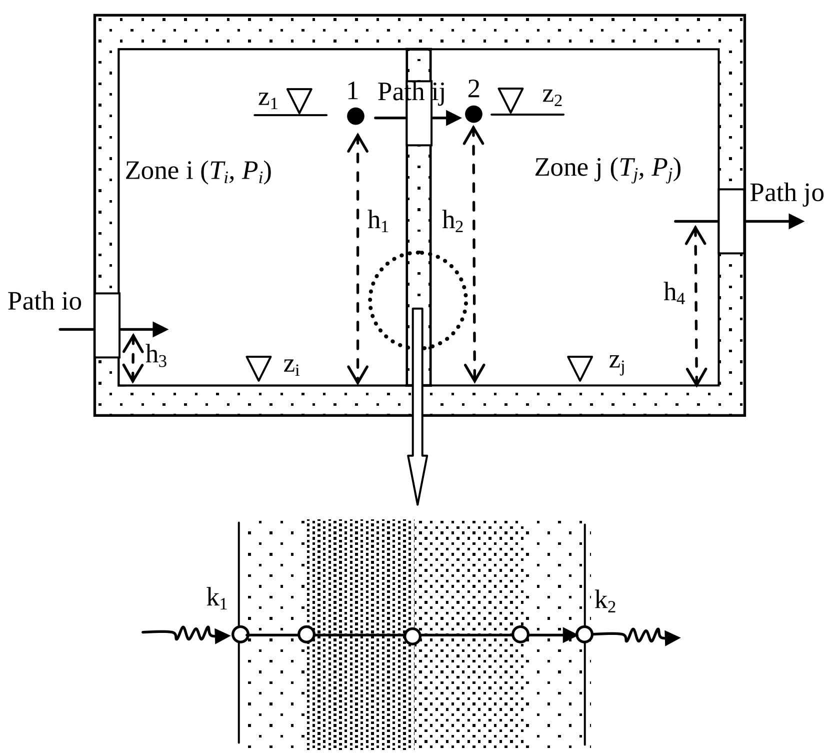

Figure 1. Schematic of airflow in a two-zone building and enlarged heat transfer at the surface and through an internal wall assembly.

The airflow from zone i to zone j through path ij (F_{ij}), is often modeled by a power law function of the pressure drop across the path, ΔP_{ij}. Assuming F_{ij} is positive for $\Delta P_{ij} > 0$:

$$F_{ij} = C_{ij}(\Delta P_{ij})^{n_{ij}} \tag{1}$$

$$\Delta P_{ij} = P_{ij,w} + P_{ij,s} + P_i - P_j \tag{2}$$

The stack pressure for path ij

$$P_{ij,s} = \frac{g}{2}[(\rho_i + \rho_j)(z_i - z_j) + (\rho_j - \rho_i)(h_1 + h_2)] \tag{3}$$

Based on the Fourier's law of conduction, the heat flux from surface thermal node k_1 to k_2 of the wall separating zone i and j

$$q^{"} = -k\frac{\partial T}{\partial y} \tag{4}$$

For zone i, the air mass and energy conservation equations can be written as

$$\frac{dm_i}{dt} = \sum_j F_{ji} - \sum_j F_{ij} + F_i \tag{5}$$

$$\frac{d(C_i T_i m_i)}{dt} = \sum_j C_j T_j F_{ji} - \sum_j C_i T_i F_{ij} + Q_i + \sum_k h_k (T_k - T_i) \tag{6}$$

Apply Euler method for the transient terms and assume the ideal gas law to obtain air density, Eqs. (5) and (6) can be discretized respectively as

$$\frac{V_i}{\Delta t}(\frac{P_i}{R_i T_i})_t - \sum_j C_{ji}(\Delta P_{ji})^{n_{ji}} + \sum_j C_{ij}(\Delta P_{ij})^{n_{ij}} - \frac{1}{\Delta t}(m_i)_{t-\Delta t} - F_i = 0 \tag{7}$$

$$\frac{V_i}{\Delta t}(\frac{C_i P_i}{R_i})_t - \sum_j C_j C_{ji} T_j (\Delta P_{ji})^{n_{ji}} + \sum_j C_i C_{ij} T_i (\Delta P_{ij})^{n_{ij}} - \frac{1}{\Delta t}(C_i m_i T_i)_{t-\Delta t} - Q_i - \sum_k h_k (T_k - T_i) = 0 \tag{8}$$

Eqs. (7) and (8) can be combined to a matrix formation for zone i, where a vector for the zone state variables is defined as $\mathbf{X_i} = (P_i, T_i)^T$

$$\mathbf{f}(\mathbf{X_i}) = 0 \tag{9}$$

Eq. (9) is nonlinear due to the power-law terms. Applying the Newton–Raphson method, we can linearize the equations to obtain a Jacobian matrix.

$$\mathbf{J}(\mathbf{X}_i{}^{(n)})(\Delta \mathbf{X_i}{}^{(n)}) = \mathbf{J}(\mathbf{X}_i{}^{(n)})(\mathbf{X}_i{}^{(n)} - \mathbf{X}_i{}^{(n+1)}) = \mathbf{f}(\mathbf{X}_i{}^{(n)}) \tag{10}$$

where $J^{(n)} = (\frac{\partial f}{\partial x})^{(n)}$. The elements of the Jacobian matrix can be found by applying the derivative of Eqs. (7) and (8) over either air temperature or pressure.

After Eq. (10) is solved, the zone state variables can be corrected by Eq. (11).

$$\mathbf{X}_i{}^{(n+1)} = \mathbf{X}_i{}^{(n)} - \Delta \mathbf{X_i}{}^{(n)} \tag{11}$$

Due to the nonlinearity of the problem, the Jacobian matrix needs to be updated after the correction so iterations are often needed.

In a fully-simultaneous solver, Eq. (10) is applied to all zones of a building to assemble a single linear matrix.

$$\mathbf{J}(\mathbf{X}^{(n)})(\Delta\mathbf{X}^{(n)}) = \mathbf{J}(\mathbf{X}^{(n)})(\mathbf{X}^{(n)} - \mathbf{X}^{(n+1)}) = \mathbf{f}(\mathbf{X}^{(n)}) \tag{12}$$

where $\mathbf{X} = (\mathbf{X}_i, ... \mathbf{X}_j ... \mathbf{X}_N)^T$ for a building with N zones.

The size of the matrix $\mathbf{J}(\mathbf{X}^{(n)})$ is proportional to the product of the number of zones by the number of zone state variables. For the example in Figure 1, given two zones and two unknown zone state variables, T and P, the matrix of the fully simultaneous method will be 4 × 4. A building with N zones will create a matrix of $2N \times 2N$. Since all parameters are solved simultaneously at the iteration of n and $n+1$, huge computer memory is required for storing the information. The calculation of the Jacobian matrix also needs extensive data processing efforts. The matrix can be solved directly by a direct matrix solver such as the LU factorization method.

As a comparison, a segregate solver calculates Eq. (7) for air pressures for all zones or Eq. (8) for all air temperatures separately when keeping the other parameters as constants at every iteration. Then the solved air parameters in one equation are substituted successively in the other equation. The resultant matrix will be with the size of $N \times N$ for a building with N zones, and can be solved iteratively so that less computer storage and processing is needed than a simultaneous solver. However, relaxation factors are required to avoid abrupt change of air parameters during the correction and substitution process. In such case, Eq. (11) for zone i becomes

$$x_i^{(n+1)} = x_i^{(n)} - r_i^{(n)} \cdot \Delta x_i^{(n)} \tag{13}$$

where x_i can be the air temperature and pressure. For example, a segregate solver can start with the input air temperatures, $T_i^{(n)}$, and solve the air mass balance equation for all zones to obtain air pressures and airflows. Then, these solved airflows are used in the energy balance equation to calculate the correction term, $\Delta T_i^{(n)}$, which updates $T_i^{(n+1)}$ based on Eq. (13). The updated air temperature is then used as new input for the air mass balance equation at the next iteration *n+1*. It was found that a fixed relaxation factor, which ensures convergence at some time step, may slow down the iteration at other time steps where a relaxation is unnecessary. A poorly selected relaxation factor may cause numerical instabilities for highly coupled thermal airflow problems. Weber et al. (2003) suggested an adapted relaxation method to avoid numerical instabilities.

- $r_i^{(n)} = 0.5 r_i^{(n-1)}$, when $\Delta T_i^{(n)} \Delta T_i^{(n-1)} \le 0$ (14a)

- $r_i^{(n)} = 1.5 r_i^{(n-1)}$, when $\Delta T_i^{(n)} \Delta T_i^{(n-1)} \ge 0$ (14b)

This study proposes a semi-simultaneous solver, which have some characteristics between fully-simultaneous and segregate solvers. The semi-simultaneous method solves air temperature and pressure of one zone, e.g. $\mathbf{X}_i^{(n)}$, simultaneously by Eq. (10) while air temperatures and pressures of other zones are kept as constants. $\mathbf{X}_i^{(n)}$ is then substituted in the Eq. (10) for the next zone, e.g. zone j, to calculate $\mathbf{X}_j^{(n)}$. The same procedure is repeated for each of the rest zones until a full sweep of iterations for all zones is completed. More sweeps are necessary before Eqs. (10) for all zones converge. Furthermore, an external iteration loop is needed to consider the impact of heat transfer through building walls on the zone thermal airflow model. In this method, the Jacobian matrix is only 2 × 2, which is significantly less than the fully-simultaneous method. As a simultaneous solution, relaxation factor is also avoided in this solver.

In details, the semi-simultaneous solver follows the following procedure:

1. Initialize zone pressures, temperatures, and zone mass and heat capacity
2. Calculate $P_{ij,w}$ and $P_{ji,w}$ for airflow paths
3. Start internal iteration for a single zone
    a. Calculate $P_{ij,s}$ and $P_{ji,s}$ fore airflow paths

b. Calculate ΔP_{ji}, ΔP_{ij}, $\dfrac{\partial F_{ji}}{\partial P_i}$ and $\dfrac{\partial F_{ij}}{\partial P_i}$

c. Fill $\mathbf{J}^{(\mathbf{n})}$ and calculate $\mathbf{f}^{(\mathbf{n})}$

d. Solve $\mathbf{J}^{(\mathbf{n})}\mathbf{\Delta X}^{(\mathbf{n})} = \mathbf{f}^{(\mathbf{n})}$ directly by the LU factorization method

e. Correct $\mathbf{X}_i^{(n+1)} = \mathbf{X}_i^{(n)} - \Delta\mathbf{X}_\mathbf{i}^{(n)}$

f. Update zone properties, e.g. density and viscosity, and zone mass

g. Check convergence of internal iteration. If it is not convergent or the total number of internal iteration is less than the maxtium iteration, repeat Step a – f. If the internal iteration is convergent, move to a next zone and repeat Step 3.

4. After calculating thermal airflows for all zones, solve 1-D heat conduction equations for all walls in the building

5. Check overall convergence. The solution is convergent if $\mathbf{f}^{(\mathbf{n}+1)}$ is less than a certain criterion or the maximum number of external iterations is reached. If it is not convergent, repeat Steps 3-4. If convergent, move to the next time step.

COMPARISON AND VERIFICATION

The fully-simultaneous, semi-simultaneous and segregate solvers with fixed and adpated relaxations are implemented in CONTAM97R, a multizone aiflow network model with energy analysis capabilities in the CONTAM family. To compare these four methods, a two-zone building with buoyancy-driven airflows from Weber et al. (2003) is used. Figure 2 shows the plan view of the two-zone building in CONTAM97R. The room size is 45 m^3 each. The air temperature is maintained as constant in room 2 at 20 °C and initially in room 1. The outdoor wind speed is zero with a constant temperature of 10 °C. The height of airflow path (af's) is 1 m for af 1, 5 m for af 2, and 3 m for af 3, which are relative to the floor of the corresponding room. The flow coefficient and exponent is 1.0 $kg/s/Pa^n$ and 0.6 respectively for all airflow paths. The wall thickness is 152 mm with a conductivity of 0.047 W/(m·K) for all the heat transfer path (ht's). A transient simulation is conducted for a period of 24 hours with a time step of five minutes.

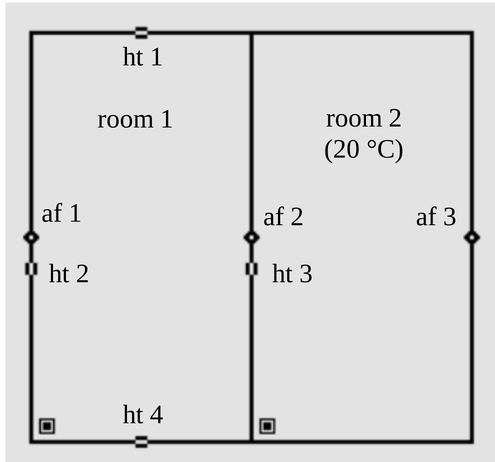

Figure 2. CONTAM97R model of the two-zone building with three airflow paths (af's) and four heat transfer paths (ht's).

Figure 3 compares the predicted airflow rates through path 1 for the four solvers. Given the airflow from the ambient to room 1 is positive, the airflow through af 1 is an inflow to room 1 in the beginning of the simulation, which causes the temperature of room 1 to decrease. The airflow from the ambient into the house continues to drop with the decrease of the room temperature until it reaches around 14.914 °C. At this point, the stack effect at af 1 and af 3 is balanced so the airflow through the building is zero. However, the heat loss through the external walls (ht 1, 2 and 4) keeps driving the temperature of room 1 down so that the airflow of af 1 reverses to outflow after this point. Figure 3 shows that a slight variation of air temperature of room 1 (in the order of 1×10^{-3} °C) changes the airflow characteristics significantly (from inflow to outflow) near this critical point. For such a highly coupled thermal airflow problem, the segregate solver with fixed relaxation of 0.1 causes numerical instabilities and eventually produced erroneous results as shown in Figure 3. By using the adapted relaxation method in Eq. (14), the segregate solver reaches a convergent result, although there is a slight fluctuation near the critical point. In comparison, both simultaneous solvers produce identical and convergent results without using any relaxation factor.

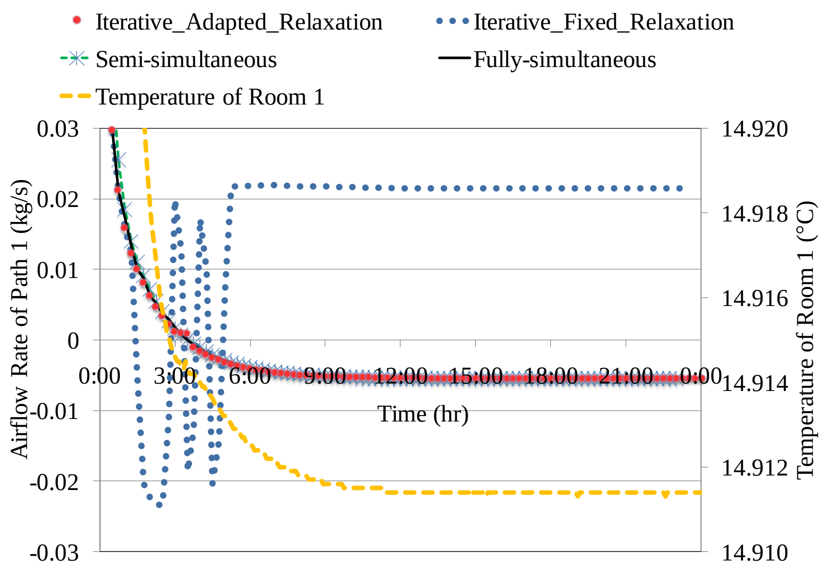

Figure 3. Comparison of predicted airflow rates through the airflow path, af 1, and temperature of room 1 by the segregate iterative solvers with adapted and fixed relaxations, and the semi-simultaneous and fully-simultaneous solvers.

The minor fluctuation of the adapted relaxation method can be explained in Figure 4. Near the critical point, the maximum iteration, which is set to be 100 in this case, is reached even for a relaxation of 0.013. An increase of the maximum iteration number will not improve the situation. However, this minor fluctuation does not affect the overall convergence. Figure 4 also shows that heavy relaxations and many iterations are needed for highly coupled thermal airflows, for which the simultaneous solvers provide good alternative methods without using any relaxation. In the meanwhile, this case verifies the adapted relaxation method of Weber et al. (2003). Note that due to incompleteness of data from their studies, this case is not set up exactly as the one from the Weber's study. However, similar trend of results are observed in their analysis.

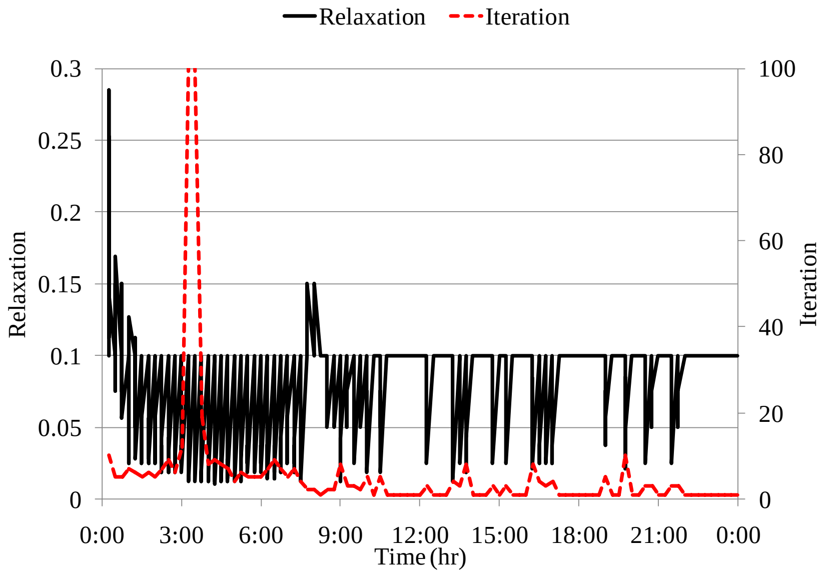

Figure 4. Adapted relaxation factors and number of iterations at each time step for the segregate solver.

VALIDATION

The case of the two-zone building only provides pure simulation results for comparison purpose. This study selects another case with experimental data to validate the simultaneous solvers. Figure 5 shows the experimental study of a light well in a wind tunnel with airflows driven by combined wind and buoyancy forces (Kotani et al. 2003). The size of the light well is 144 mm × 144 mm × 480 mm with an internal void of 72 mm × 72 mm × 480 mm, which corresponds to a 41-story building for a scale of 1/250. The size of the opening is 12 mm × 72 mm for the lower one and 72 mm × 72 mm for the top. The heat in the void is generated by Nichrome wires for a range of 10 to 40 W. The wind speed is adjusted to be from 0 to 1.5 m/s with the wind direction perpendicular to the lower opening. The air in the wind tunnel is kept at 12 °C. In this study, the light well is divided into eight vertical zones with same size as shown in Figure 5. An orifice airflow equation is used for the lower, inter-zonal and top openings with a flow exponent of 0.5 and a discharge coefficient provided from this experimental study. The experiment neglects heat transfer through the light well structures so this study does not consider heat conduction in walls.

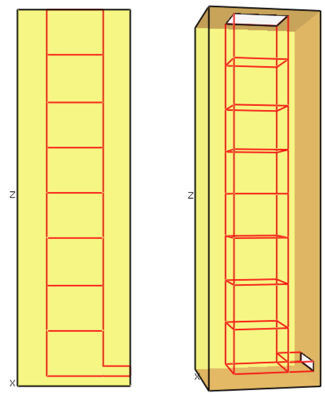

Figure 5. Airflows driven by combined wind and buoyancy effects in a light well, which is partitioned into eight sub-zones.

Figure 6 compares the predicted airflow rates through the light well at steady state by two previous simulation studies (Kotani et al. 2003; Zhai et al. 2009) and the current study with the measured data. Both simultaneous solvers predicted almost identical results, which agree well with the measured airflow rates. It seems the current study predicts slightly better results than the other studies, which should not be conclusive without further confirmation by more validation studies. Figure 6 also shows that all predictions overestimate the airflow rates, which was attributed to the inaccurate measurement of heat generation rate used in the simulations (Kotani et al. 2003).

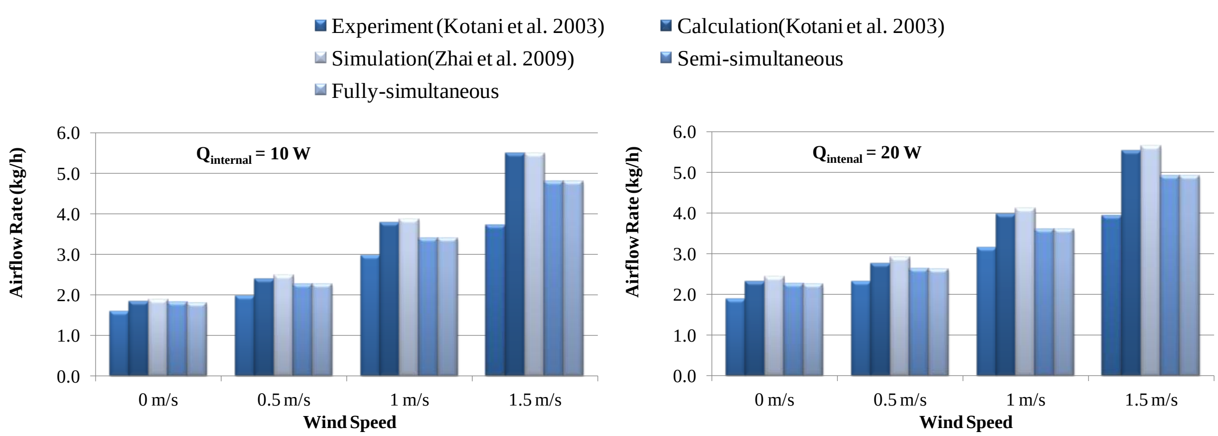

Figure 6(a)

Figure 6(b)

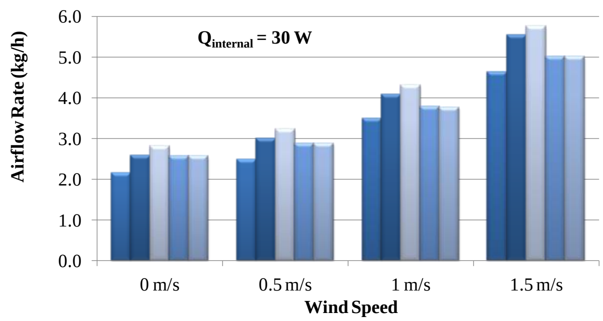

Figure 6(c)

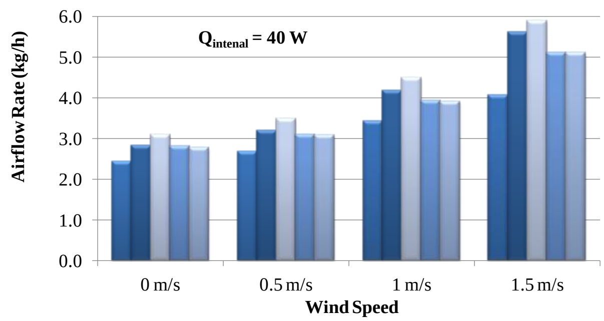

Figure 6(d)

Figure 6. Comparison of predicted airflow rates through the light well by previous and current studies with measured data for different wind speeds and internal heat sources.

Another reason for the airflow overestimation can be the use of eight zones to model the light well. The actual temperature gradient may not be predicted appropriately as illustrated in Figure 7. For different scenarios of wind speeds and heat generation rates, the temperature predictions by the simultaneous solvers are generally greater than the measured data, which contributes to the overestimation of airflow rates. On the other hand, the overall trend of the prediction agrees resonably well with the measured temperatures with some major exceptions in the lower zones of the light well, where the simultaneous solvers underestimate the temperatures. Similar problems are observed in the calculations of Kotani et al. (2003). It is possible that a better estimation can be obtained by other simulation models, such as computational fluid dynamics (CFD) with more accurate inputs of the heat generation rates. However, as computationally economic methods, the simultaneous solvers provide predictions of airflow rates and temperatures in the light well with certain accuracy. Note that both simultaneous solvers calculate almost identical results in Figure 7 so only one simulation line is shown for each scenario.

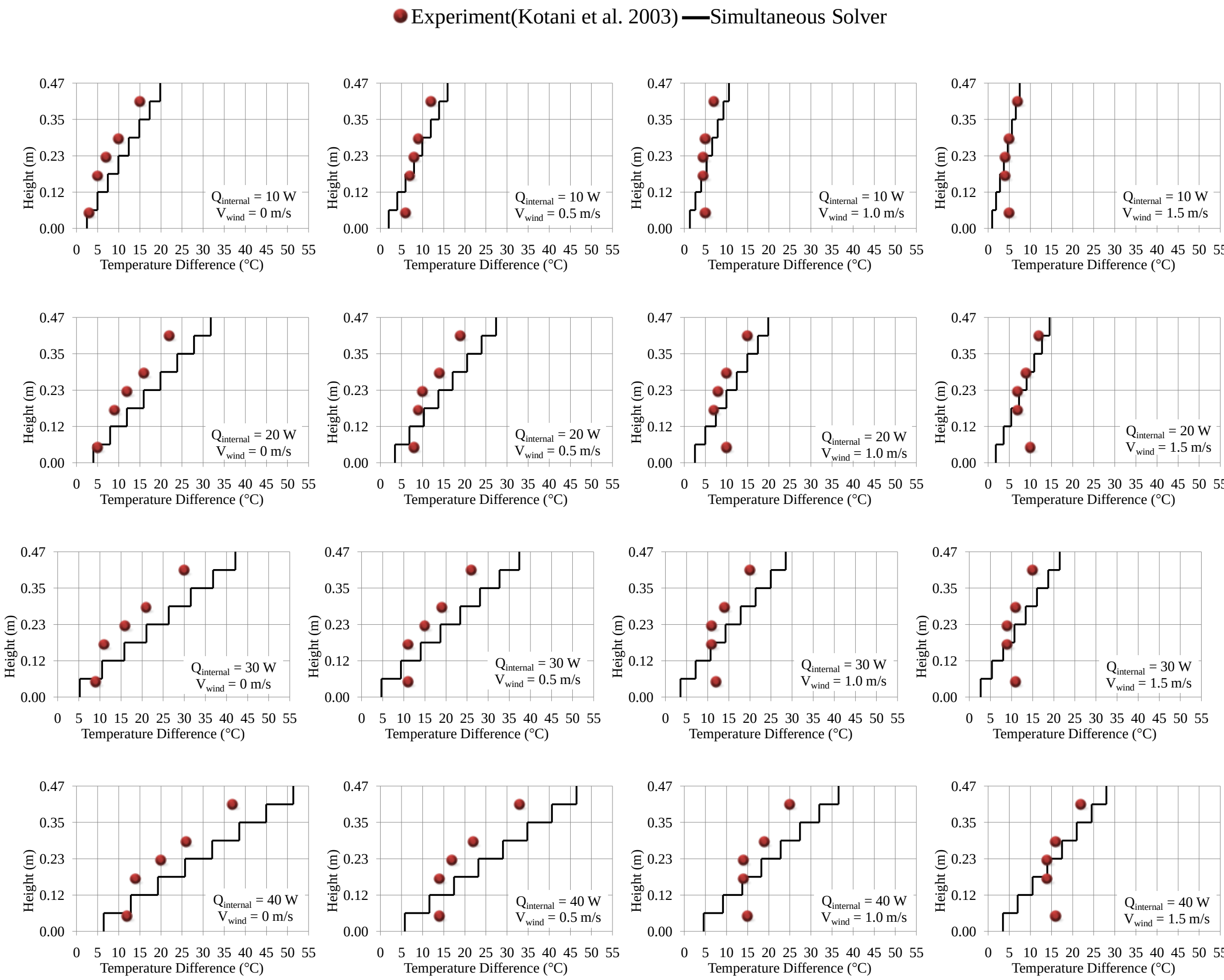

Figure 7. Comparison of predicted temperature distribution in the light well by the simultaneous solvers with measured data for different wind speeds and internal heat sources.

CONCLUSION

This study investigates simultaneous solutions of coupled thermal airflow problems in natural ventilations by comparing to the segregate solvers for solving same problems. It is showed that the segregate iterative solver with fixed relaxations can cause numerical instabilities for highly coupled problems. This study verified that the adapted relaxation method from a previous study is able to avoid numerical convergence problems, although there may be minor fluctuation near the critical point, where the airflow direction is reversed in a two-zone building case. Two simultaneous solvers are proposed and studied. The fully-simultaneous method solves a single Jacobian matrix for air temperatures and pressures of all zones simultaneously, which requires more computer memory than other methods. The semi-simultaneous method solves a matrix of air temperature and pressure for each zone one after another and requires much less computer storage. The common benefit of both methods is that solution convergence can be obtained without using any relaxation even for highly coupled thermal airflow problems. Both methods are also validated for the predictions of airflow rates and temperatures in a light well with combined wind and buoyancy effects. It is showed that the simultaneous solvers predict the results resonably well, considering that their computatinal cost is much lower than more advanced models, such as CFD. Further studies of the simultaneous solvers are necessary for more general or complicated applications of natural and hybrid ventilations. With the fast development of computer power and memory storage recently and in the projected future, the simultaneous solvers may become an important and popular technique for solving multiply coupled building physics. This study is conducted with the hope to peek into this potential trend for the near future.

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