Changes

The saga of the interaction between historians of mathematics and historians of digital computing's origins is long and complicated, with many strongly held and opposing opinions. Most would agree that at least *some* of the modern era's pioneers were mathematicians (think von Neumann and Turing). The debates over computer *use* in mathematics have been either more fierce or less so, depending on your viewpoint. If you're interested in any of the vast areas related to computational modeling—including differential equations, optimization, signal processing, and machine learning—computation's central role is obvious: it's the way you get the answer! Another well-studied connection is the one between computing's mathematical theory and actual computation. Complexity results provide guidance as to what we should try to compute, while real-world computational problems suggest important questions about complexity.

This is all well and good, but there's another set of issues lurking in the shadows—namely, questions about the use of computation in teaching and doing mathematics. My last column included some remarks on the doing; the topic is healthy and growing. The real battlefield has been computation's use in teaching mathematics.

Almost everyone accepts the idea of using computers to produce useful graphics and, of course, the use of typesetting tools such as TeX to produce slides and class notes is almost universal. But what about using computing actually to do, say, calculus? Should we insist that mastery of calculus include the ability to differentiate things like **x^x?** Or (much as I enjoy such problems), are they something best left to tools such as Maple and Mathematica?

And what is calculus anyhow? Archimedes almost discovered integration, and Fermat probably knew the fundamental theorem about the relation between differentiation and integration. So what did Leibniz and Newton invent? The answer is calculus! Here "calculus" means a notation and language along with a set of techniques for reasoning about approximations and their limits. The mechanics of calculus, such as how to use the chain rule and integration by parts, are just that: mechanics.

Mathematics recently lost a great contributor: Jerry Uhl, a long-time professor of mathematics at the University of Illinois. //**okay?**// In the first part of his career, Uhl earned distinction as a researcher in functional analysis, with a special emphasis on vector-valued measures. Together with Joe Diestel, he published one of the standard sources on this topic. He was a teacher par excellence, remembered by all who sat in his classes. He also mentored a long string of PhD students, many of whom are active in teaching and research at major universities today.

In 1978, Tony Peressini, Francis Sullivan, and Jerry Uhl published a textbook on optimization. Working on this seemed to awaken Uhl's interest in computation. Then along came Mathematica, and Uhl got very excited (as only he could) about the possibility of using Mathematica to *teach* calculus. Sullivan predicted that there would be lots of resistance in mathematics departments, mainly from those whose idea of what should be taught as calculus is some combination of tricks for finding integrals and subtle arguments from real analysis.

The prospect of a pioneering effort that would incite battles really excited Uhl, as controversy often did. He joined forces with Horatio Porta and Bill Davis to create Calculus&Mathematica, an entirely new method of teaching calculus that relies heavily on symbolic computation and active classroom participation by students. In doing so, those three mathematicians made much progress and, of necessity, fought many wars both within their math departments and elsewhere.

It's much too early to tell if Uhl and his collaborators won their wars. Resistance to change is a powerful force, and long-standing habits multiply that force. But in the end, the new trumps the old so long as the new offers new insights and proves more productive.

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