Nonlinear Dynamic Analysis of Structures Using Modal Superposition

K. K. F. Wong[1]

[1]Engineering Laboratory, National Institute of Standards and Technology, MS 7302, 100 Bureau Dr., Bldg. 226, Room B-264, Gaithersburg, MD 20899-7302, USA; PH (301) 975-6361; FAX (301) 975-4032; email: kfwong@nist.gov

ABSTRACT

A fast nonlinear dynamic analysis algorithm based on modal superposition of structural response incorporating both material and geometric nonlinearities is proposed. Because linear modal superposition has found great acceptances in performance-based seismic engineering, it is here extended to the nonlinear domain by using the force analogy method to address material nonlinearity, where the stiffness force is quantified by a change in displacement instead of using the traditional way of changing stiffness. Geometric nonlinearity is incorporated into the analysis using stability functions. State space method is used to explicitly calculate the dynamic responses of each modal single-degree-of-freedom system. Through the combination of force analogy method, stability functions, state space method, and modal superposition, numerical simulation is performed and results are demonstrated to contain both accuracy and efficiency.

INTRODUCTION

Buildings constructed in seismic regions are vulnerable to earthquake activities and are required to be designed such that its seismic capacity must exceed the corresponding seismic demand with sufficient ductility. While design codes and standards have been developed to establish the seismic capacity in design, estimating the seismic demand in nonlinear structures has always been a challenging task mainly because of the lack of scientific measurement tools. Since the time when Newmark (1959) introduced the linear response history analysis for studying seismic effects on civil engineering structures, the development of current seismic design procedures relying on this type of linear analysis technique has advanced significantly because of its fundamental simplicity in the calculation procedure. However, while linear dynamic analysis is so popular, it has a shortcoming because all structures are designed with the anticipation that they will behave nonlinearly when a major earthquake occurs.

Significant research works have been attempted to extend the linear dynamic analysis to the nonlinear domain (Liu 2003 and 2005, Au and Yan 2008), but presently nonlinear dynamic analysis techniques were employed in seismic design only in some special occasions mainly because the analysis itself is a time-consuming process. Current nonlinear dynamic numerical algorithms require significant computation time to update the stiffness matrix to accommodate for nonlinearity in structures. In addition, many simulations need to be considered due to the uncertainties in earthquake ground motions. Even though nonlinear response history analysis is believed to be the most accurate method of computing the seismic demand, it requires a huge computational effort and making the analysis impractical in the design process. Therefore, a fast analysis method with high accuracy in predicting the maximum seismic demand is needed, yet the development of such nonlinear dynamic analysis algorithm remains as a challenge without deeper understanding of the theory of nonlinear structural analysis and dynamics.

In this research, linear dynamic analysis based on modal superposition is extended to analyze structural responses in the nonlinear domain – known here as the nonlinear modal analysis (NMA). While the force analogy method is a simple nonlinear analysis tool based on initial stiffness and treats material nonlinearity as an equivalent force, the state space method is a dynamic algorithm that can perform time-stepping calculations explicitly to progress the effects of the equivalent force on structural responses to the next time step. Although some results through the combination of these methods have been published (Yang et al. 2004, Zhao and Wong 2006, Zhang et al. 2007, Chao and Loh 2007), none of these works studied the nonlinear structural responses in the modal coordinate system. In addition, it is the intent of this research to develop a rigorous approach for performing dynamic analysis through the use of stability functions as the closed-form solution to treating geometric nonlinearities. Accuracy and efficiency are demonstrated through applying the algorithm on an example 10-story frame by comparing output results with existing commercial software packages including Perform-3D and SAP2000.

FORCE ANALOGY METHOD

The detailed derivation of the force analogy method has been presented in Wong and Yang (1999). Consider a moment-resisting frame with n degrees of freedom (DOFs) and m plastic hinge locations (PHLs), let the total displacement $\mathbf{x}(t)$ at each DOF be represented as the summation of the elastic displacement $\mathbf{x}'(t)$ and the inelastic displacement $\mathbf{x}''(t)$:

$$\mathbf{x}(t) = \mathbf{x}'(t) + \mathbf{x}''(t) \tag{1}$$

Similarly, let the total moment $\mathbf{m}(t)$ at the PHLs of a moment-resisting frame be separated into elastic moment $\mathbf{m}'(t)$ and inelastic moment $\mathbf{m}''(t)$:

$$\mathbf{m}(t) = \mathbf{m}'(t) + \mathbf{m}''(t) \tag{2}$$

The displacements in Eq. (1) and the moments in Eq. (2) are related by the equations:

$$\mathbf{m}'(t) = \mathbf{K}'(t)^T \mathbf{x}'(t) \quad , \qquad \mathbf{m}''(t) = -\left[\mathbf{K}''(t) - \mathbf{K}'(t)^T \mathbf{K}(t)^{-1} \mathbf{K}'(t)\right]\mathbf{\Theta}''(t) \tag{3}$$

where $\mathbf{\Theta}''(t)$ is the plastic rotation at the PHLs, $\mathbf{K}(t)$ is the $n \times n$ global stiffness matrix, $\mathbf{K}'(t)$ is the $n \times m$ stiffness matrix that relates the plastic rotations at the PHLs and the forces at the DOFs, and $\mathbf{K}''(t)$ is the $m \times m$ stiffness matrix that relates the plastic rotations with the corresponding moments at the PHLs. These stiffness matrices are functions of time when geometric nonlinearity with varying axial load is incorporated in the dynamic analysis, and they can be formed by assembling the individual element stiffness matrices in the appropriate DOFs and PHLs. For the ith column member of the structure with plastic hinges at both ends, the axially-loaded element stiffness matrices related to shear and bending can be written as:

$$\mathbf{K}_i(t)=\left[\begin{array}{cc|cc} s'EI/L^3 & \bar{s}EI/L^2 & -s'EI/L^3 & \bar{s}EI/L^2 \\ \bar{s}EI/L^2 & sEI/L & -\bar{s}EI/L^2 & scEI/L \\ \hline -s'EI/L^3 & -\bar{s}EI/L^2 & s'EI/L^3 & -\bar{s}EI/L^2 \\ \bar{s}EI/L^2 & scEI/L & -\bar{s}EI/L^2 & sEI/L \end{array}\right]$$

$$\mathbf{K}'_i(t)^T=\left[\begin{array}{cc|cc} \bar{s}EI/L^2 & sEI/L & -\bar{s}EI/L^2 & scEI/L \\ \bar{s}EI/L^2 & scEI/L & -\bar{s}EI/L^2 & sEI/L \end{array}\right] \qquad \mathbf{K}''_i(t)=\begin{bmatrix} sEI/L & scEI/L \\ scEI/L & sEI/L \end{bmatrix}$$

where

$$s=\frac{\lambda(\sin\lambda-\lambda\cos\lambda)}{2-2\cos\lambda-\lambda\sin\lambda} \qquad c=\frac{\lambda-\sin\lambda}{\sin\lambda-\lambda\cos\lambda}$$

$$\bar{s}=s+sc=s(1+c)=\frac{\lambda^2(1-\cos\lambda)}{2-2\cos\lambda-\lambda\sin\lambda} \qquad s'=2\bar{s}-\lambda^2=\frac{\lambda^3\sin\lambda}{2-2\cos\lambda-\lambda\sin\lambda}$$

$\lambda=(P(t)/EI)^{0.5}\times L$, $P(t)$ is the time-varying axial compressive force in the column which causes the stiffness matrices to be also time-varying, and E, I, and L are the elastic modulus, moment of inertia, and length of the column member, respectively. The relationship between plastic rotation $\mathbf{\Theta}''(t)$ and inelastic displacement $\mathbf{x}''(t)$ is:

$$\mathbf{x}''(t)=\mathbf{K}(t)^{-1}\mathbf{K}'(t)\mathbf{\Theta}''(t) \tag{4}$$

Substituting Eq. (3) into Eq. (2) and making use of Eqs. (1) and (4), then rearranging the terms gives the governing equation of the force analogy method:

$$\mathbf{m}(t)+\mathbf{K}''(t)\mathbf{\Theta}''(t)=\mathbf{K}'(t)^T\,\mathbf{x}(t) \tag{5}$$

NONLINEAR MODAL ANALYSIS

When the force analogy method is used, the stiffness force is calculated by multiplying the global stiffness matrix $\mathbf{K}(t)$ with the elastic displacement $\mathbf{x}'(t)$. For an n-DOF system with plastic hinges at both ends of each member subjected to earthquake ground motion, the equation of motion can therefore be written as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}(t)\mathbf{x}'(t)=-\mathbf{M}\ddot{\mathbf{g}}(t)+\mathbf{F}_h(t) \tag{6}$$

where $\mathbf{M}$ is the $n\times n$ mass matrix, $\mathbf{C}$ is the $n\times n$ damping matrix, $\dot{\mathbf{x}}(t)$ is the $n\times 1$ velocity vector at each DOF, $\ddot{\mathbf{x}}(t)$ is the $n\times 1$ acceleration vector at each DOF, $\ddot{\mathbf{g}}(t)$ is the $n\times 1$ ground acceleration vector corresponding to each DOF, and $\mathbf{F}_h(t)$ is an $n\times 1$ force vector that takes into consideration the geometric nonlinearity of the entire structure (i.e., the P-Δ effect), such as the force imposed on the structure using a P-Δ column. In two-dimensional modeling, this force is related to the displacement of the structure, and it can be computed by the formula $\mathbf{F}_h(t)=\mathbf{K}_h\mathbf{x}(t)$, where $\mathbf{K}_h$ is the $n\times n$ stiffness matrix computed using the column heights and axial gravity forces of the

P-Δ column members. Note that the P-δ effect has been included in $\mathbf{K}(t)$.

Let $\mathbf{K}(t) = \mathbf{K}_o - \mathbf{K}_G(t)$, where $\mathbf{K}_o$ represents the initial stiffness due to gravity load only, and $\mathbf{K}_G(t)$ represents the change in stiffness due to the variation of axial forces while subjected to the earthquake ground motion. Now replacing the elastic displacement $\mathbf{x}'(t)$ in Eq. (6) by the difference of total displacement $\mathbf{x}(t)$ and inelastic displacement $\mathbf{x}''(t)$ that is rewritten using Eq. (1), it follows that

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}_o\mathbf{x}(t) = -\mathbf{M}\ddot{\mathbf{g}}(t) + \mathbf{K}_h\mathbf{x}(t) - \mathbf{K}_G(t)\mathbf{x}(t) + \mathbf{K}(t)\mathbf{x}''(t) \tag{7}$$

Following a similar procedure for the treatment of P-Δ effects discussed in Wilson (2002), define $\mathbf{K}_e = \mathbf{K}_o - \mathbf{K}_h$, where $\mathbf{K}_e$ represents the elastic stiffness of the structure. Now substituting Eq. (4) into Eq. (7) gives

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}_e\mathbf{x}(t) = -\mathbf{M}\ddot{\mathbf{g}}(t) - \mathbf{K}_G(t)\mathbf{x}(t) + \mathbf{K}'(t)\mathbf{\Theta}''(t) \tag{8}$$

Let the modal displacement response $\vec{\mathbf{q}}(t)$ be the $r \times 1$ vector of the form

$$\mathbf{x}(t) = \mathbf{\Phi}\vec{\mathbf{q}}(t) = \begin{bmatrix}\boldsymbol{\varphi}_1 & \boldsymbol{\varphi}_2 & \cdots & \boldsymbol{\varphi}_r\end{bmatrix}\begin{bmatrix}\vec{q}_1(t) & \vec{q}_2(t) & \cdots & \vec{q}_r(t)\end{bmatrix}^T \tag{9}$$

where $\mathbf{\Phi}$ is the $n \times r$ modal matrix – a collection of the first r mode shapes $\boldsymbol{\varphi}_i$ in column form, and r is the number of modes to be considered in the analysis, where $r \le n$. Differentiating the modal displacement $\vec{\mathbf{q}}(t)$ with respect to time gives the modal velocity $\dot{\vec{\mathbf{q}}}(t)$, and differentiating it once again with respect to time gives the modal acceleration $\ddot{\vec{\mathbf{q}}}(t)$. Now substituting Eq. (9) into Eq. (7):

$$\mathbf{M\Phi}\ddot{\vec{\mathbf{q}}}(t) + \mathbf{C\Phi}\dot{\vec{\mathbf{q}}}(t) + \mathbf{K}_e\mathbf{\Phi}\vec{\mathbf{q}}(t) = -\mathbf{M}\ddot{\mathbf{g}}(t) - \mathbf{K}_G(t)\mathbf{x}(t) + \mathbf{K}'(t)\mathbf{\Theta}''(t) \tag{10}$$

Pre-multiplying Eq. (10) by $\mathbf{\Phi}^T$, it follows that

$$\begin{aligned}&\mathbf{\Phi}^T\mathbf{M\Phi}\ddot{\vec{\mathbf{q}}}(t) + \mathbf{\Phi}^T\mathbf{C\Phi}\dot{\vec{\mathbf{q}}}(t) + \mathbf{\Phi}^T\mathbf{K}_e\mathbf{\Phi}\vec{\mathbf{q}}(t)\\ &\qquad = -\mathbf{\Phi}^T\mathbf{M}\ddot{\mathbf{g}}(t) - \mathbf{\Phi}^T\mathbf{K}_G(t)\mathbf{x}(t) + \mathbf{\Phi}^T\mathbf{K}'(t)\mathbf{\Theta}''(t)\end{aligned} \tag{11}$$

While assuming that the damping matrix $\mathbf{C}$ exhibits proportional damping property, let $\vec{\mathbf{M}} = \mathbf{\Phi}^T\mathbf{M\Phi}$, $\vec{\mathbf{C}} = \mathbf{\Phi}^T\mathbf{C\Phi}$, and $\vec{\mathbf{K}} = \mathbf{\Phi}^T\mathbf{K\Phi}$ be the diagonal modal mass, modal damping, and modal stiffness matrices, respectively. This gives r modal equations:

$$\vec{m}_i\ddot{\vec{q}}_i(t) + \vec{c}_i\dot{\vec{q}}_i(t) + \vec{k}_i\vec{q}_i(t) = -\boldsymbol{\varphi}_i^T\mathbf{M}\ddot{\mathbf{g}}(t) - \boldsymbol{\varphi}_i^T\mathbf{K}_G(t)\mathbf{x}(t) + \boldsymbol{\varphi}_i^T\mathbf{K}'(t)\mathbf{\Theta}''(t) \quad i = 1,\ldots,r \tag{12}$$

where $\vec{m}_i$, $\vec{c}_i$, and $\vec{k}_i$ are the i<u>th</u> modal mass, damping, and stiffness, respectively.

STATE SPACE METHOD

For all r modes given in Eq. (12), let the three terms on the right hand side of the equation be treated as the equivalent modal forces:

$$\vec{m}_i\vec{a}_i(t) = \boldsymbol{\varphi}_i^T\mathbf{M}\ddot{\mathbf{g}}(t) \quad i = 1,\ldots,r \tag{13}$$

$$\vec{f}_{mi}(t) = \boldsymbol{\varphi}_i^T\mathbf{K}'(t)\mathbf{\Theta}''(t) \quad i = 1,\ldots,r \tag{14}$$

$$\vec{f}_{Gi}(t) = \boldsymbol{\varphi}_i^T\mathbf{K}_G(t)\mathbf{x}(t) \quad i = 1,\ldots,r \tag{15}$$

where $\vec{a}_i(t)$ is the modal ground acceleration, $\vec{f}_{mi}(t)$ is the equivalent modal force due to material nonlinearity, and $\vec{f}_{Gi}(t)$ is the equivalent modal force due to geometric nonlinearity. Representing each modal equation in Eq. (12) in the state space form:

$$\dot{\vec{\mathbf{z}}}_i(t) = \mathbf{A}_i \vec{\mathbf{z}}_i(t) + \mathbf{G}_i \vec{f}_{mi}(t) - \mathbf{G}_i \vec{f}_{Gi}(t) + \mathbf{H}_i \vec{a}_i(t) \qquad i = 1,\ldots,r \tag{16}$$

where

$$\vec{\mathbf{z}}_i(t) = \begin{Bmatrix} \vec{q}_i(t) \\ \dot{\vec{q}}_i(t) \end{Bmatrix}, \quad \mathbf{A}_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}, \quad \mathbf{G}_i = \begin{bmatrix} 0 \\ 1/\vec{m}_i \end{bmatrix}, \quad \mathbf{H}_i = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

and ω_i and ζ_i are the natural frequency and damping ratio of the ith mode, respectively.

The solution to the first order linear differential equation given in Eq. (16) is

$$\vec{\mathbf{z}}_i(t) = \mathbf{e}^{\mathbf{A}_i(t-t_0)} \vec{\mathbf{z}}_i(t_0) + \mathbf{e}^{\mathbf{A}_i t} \int_{t_0}^{t} \mathbf{e}^{-\mathbf{A}_i s} \left[\mathbf{G}_i \vec{f}_{mi}(s) - \mathbf{G}_i \vec{f}_{Gi}(s) + \mathbf{H}_i \vec{a}_i(s) \right] ds \quad i = 1,\ldots,r \tag{17}$$

where t_0 is the initial time of integration. The state transition matrix $\mathbf{e}^{\mathbf{A}_i t}$ of a SDOF system is

$$\mathbf{e}^{\mathbf{A}_i t} = e^{-\zeta_i \omega_i t} \begin{bmatrix} \cos\omega_{di} t + \dfrac{\zeta_i}{\sqrt{1-\zeta_i^2}} \sin\omega_{di} t & \dfrac{1}{\omega_i \sqrt{1-\zeta_i^2}} \sin\omega_{di} t \\ -\dfrac{\omega_i}{\sqrt{1-\zeta_i^2}} \sin\omega_{di} t & \cos\omega_{di} t - \dfrac{\zeta_i}{\sqrt{1-\zeta_i^2}} \sin\omega_{di} t \end{bmatrix}$$

and ω_{di} is the damped natural frequency of the ith mode computed by the formula $\omega_{di} = \omega_i (1-\zeta_i^2)^{0.5}$. Define $t_{k+1} = t$, $t_k = t_0$, and $\Delta t = t - t_0$, it follows that Eq. (17) can be discretized and integrated as

$$\vec{\mathbf{z}}_{k+1}^{(i)} = \mathbf{F}_d^{(i)} \vec{\mathbf{z}}_k^{(i)} + \mathbf{G}_d^{(i)} \vec{f}_{mk}^{(i)} - \mathbf{G}_d^{(i)} \vec{f}_{Gk}^{(i)} + \mathbf{H}_d^{(i)} \vec{a}_k^{(i)} \qquad i = 1,\ldots,r \tag{18}$$

where

$$\mathbf{F}_d^{(i)} = \mathbf{e}^{\mathbf{A}_i \Delta t} \quad , \qquad \mathbf{G}_d^{(i)} = \mathbf{e}^{\mathbf{A}_i \Delta t} \mathbf{G}_i \Delta t \quad , \qquad \mathbf{H}_d^{(i)} = \mathbf{e}^{\mathbf{A}_i \Delta t} \mathbf{H}_i \Delta t$$

$\vec{\mathbf{z}}_k^{(i)}$, $\vec{f}_{mk}^{(i)}$, $\vec{f}_{Gk}^{(i)}$, and $\vec{a}_k^{(i)}$ are the discretized forms of $\vec{\mathbf{z}}_i(t)$, $\vec{f}_{mi}(t)$, $\vec{f}_{Gi}(t)$, and $\vec{a}_i(t)$, respectively, and the superscript *i* in parenthesis denotes the calculation is based on the ith mode. Once the modal displacement and velocity responses embedded in $\vec{\mathbf{z}}_k^{(i)}$ are obtained, the modal relative acceleration response can be calculated using Eq. (12):

$$\ddot{\vec{q}}_k^{(i)} = \vec{f}_{mk}^{(i)} / \vec{m}_i - \vec{f}_{Gk}^{(i)} / \vec{m}_i - \vec{a}_k^{(i)} - \vec{c}_i \dot{\vec{q}}_k^{(i)} / \vec{m}_i - \vec{k}_i \vec{q}_k^{(i)} / \vec{m}_i \qquad i = 1,\ldots,r \tag{19}$$

The absolute acceleration of the ith mode, $\ddot{\vec{q}}_{a,k}^{(i)}$, can be calculated by summing the relative acceleration in Eq. (19) with the modal ground acceleration defined in Eq. (13):

$$\ddot{\vec{q}}_{a,k}^{(i)} = \ddot{\vec{q}}_k^{(i)} + \vec{a}_k^{(i)} \qquad i = 1,\ldots,r \tag{20}$$

Finally, the absolute acceleration of the structure $\ddot{\mathbf{y}}(t)$, or $\ddot{\mathbf{y}}_k$ in discretized form, is computed as:

$$\ddot{\mathbf{y}}_k = \ddot{\mathbf{x}}_k + \ddot{\mathbf{g}}_k = \mathbf{\Phi}\ddot{\vec{\mathbf{q}}}_k + \mathbf{\Phi}\vec{\mathbf{a}}_k = \mathbf{\Phi}\ddot{\vec{\mathbf{q}}}_{a,k} = \mathbf{\Phi}\left[\ddot{\vec{q}}^{\,(1)}_{a,k} \quad \ddot{\vec{q}}^{\,(2)}_{a,k} \quad \cdots \quad \ddot{\vec{q}}^{\,(r)}_{a,k}\right]^T \tag{21}$$

Figure 1 shows the flowchart of the proposed calculation procedure for performing the nonlinear response history analysis. Note that nonlinearity causes coupling between modes, as shown in Figure 1. Therefore, all *r* modes are required to be run simultaneously in order to perform modal combination at each time step.

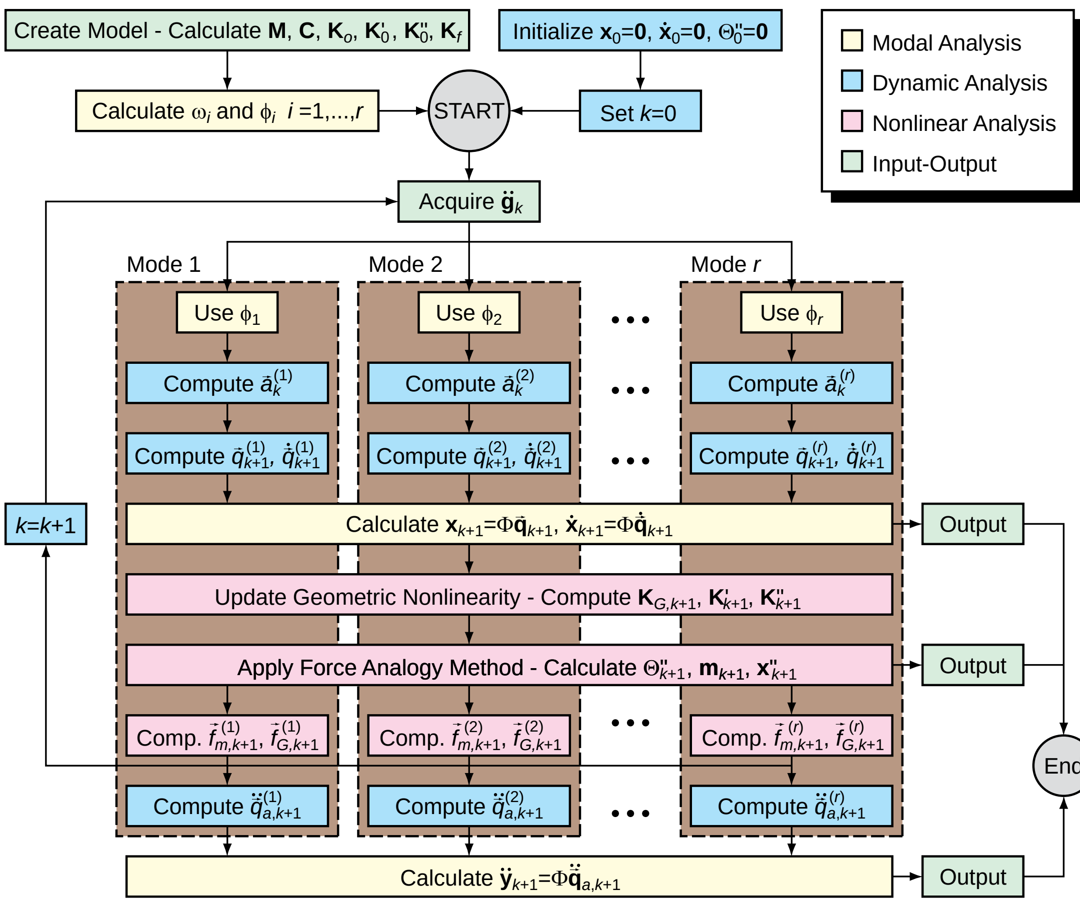
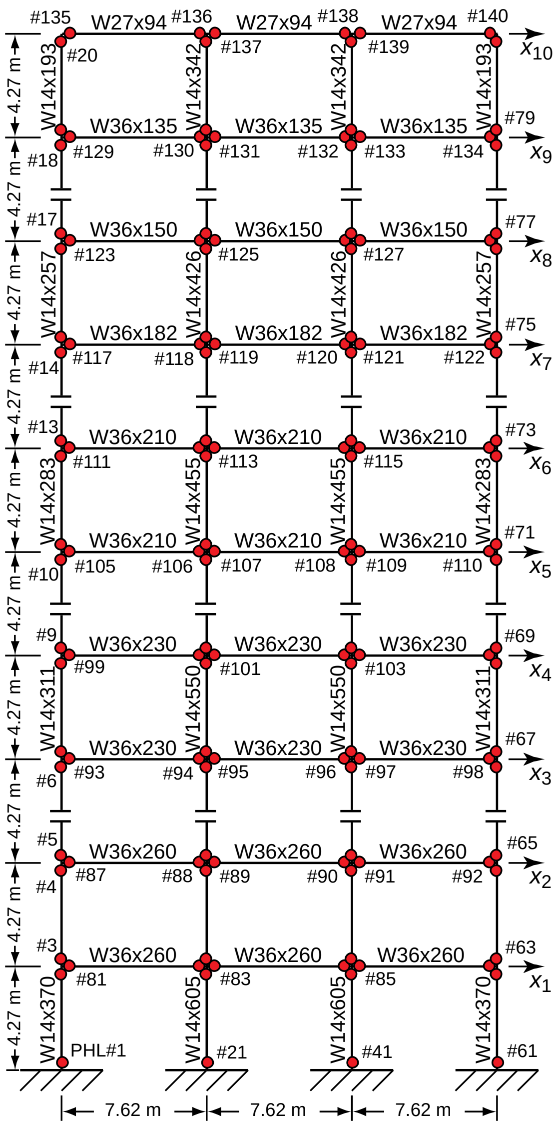
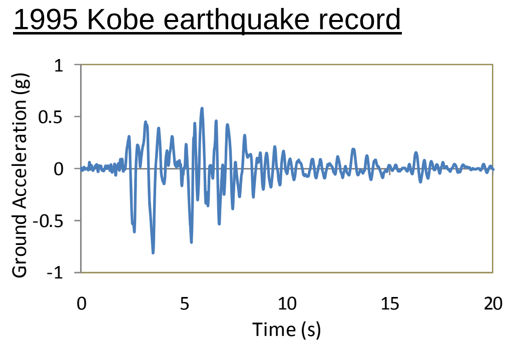


Figure 1. Flowchart for deriving the fully nonlinear response history analysis.

NUMERICAL SIMULATION

Consider a 10-story moment-resisting steel frame as shown in Figure 2. Let the mass be 218.9 Mg on each floor. This gives 10 elastic periods of vibration based on $\mathbf{K}_e$ as summarized in the figure. The damping is assumed to be 3% in all modes for the purpose of this study, even though larger damping ratio may be used in the higher modes which can be easily adjusted in any modal analysis that uses modal coordinates as the mathematical representation. A total of 140 PHLs as shown in Figure 2 are used in the analysis, all of which are assumed to exhibit elastic-plastic behavior with moment capacity $m_c = \sigma_Y Z$, where σ_Y is the yield stress of steel, or 248.2 MPa, and Z is the plastic section modulus of the member. All beams are subjected to a 21.89 kN/m uniform gravity load, while interaction between axial force and moment capacity is ignored in the columns. Gravity loads of 5,978 kN are applied on each floor of the *P*-Δ column to simulate the geometric nonlinearity of the global structure.





Mass: 218.9 Mg per floor

Dead Load: 21.89 kN/m per beam

P-Δ Column: 5,978 kN per floor

Mode	Period (s)	Damping
1	1.546	3%
2	0.559	3%
3	0.339	3%
4	0.237	3%
5	0.181	3%
6	0.148	3%
7	0.123	3%
8	0.105	3%
9	0.094	3%
10	0.082	3%

Figure 2. Ten-story moment-resisting steel frame model with earthquake record.

To demonstrate the accuracy and efficiency of the current NMA algorithm, the NMA structural responses using all 10 modes subjected to the 1995 Kobe earthquake ground motion as shown in Figure 2 are compared to those obtained using Perform-3D (P3D) and SAP2000 (SAP). Since no interaction between axial force and moment capacity is assumed in this study, only moment hinges are used to model all the PHLs for both beams and columns in P3D. Nonlinear link elements are used to model the plastic hinges in SAP, since the computation will be very time-consuming if the plastic hinges in the beam element are explicitly modeled in the program. The global displacement, velocity, and absolute acceleration responses at the selected floors between various analysis methods are compared in Figures 3, 4, and 5, respectively. The local plastic rotations at selected beam and column PHLs using both NMA and P3D are also compared in Figure 6. Table 1 summarizes the computation time of each analysis method.

Table 1. Computing time of various programs of fully nonlinear dynamic analysis.

	NMA	P3D	SAP-DI	NMA(v)
Computing Time* (1.0 × Kobe)	107 s	78 s	940 s	19 s
Computing Time* (1.3 × Kobe)	109 s	85 s	1,146 s	22 s

* Based on 32-bit desktop computer with 2.4 GHz Intel processor and 3 GB RAM.

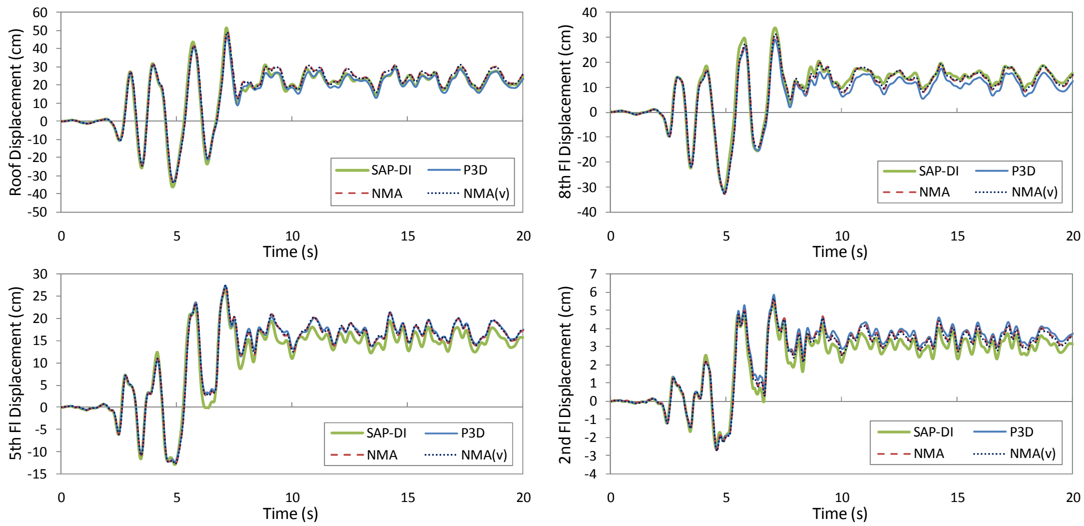


Figure 3. Comparison of displacement responses at selected floors due to 1.0 × Kobe.

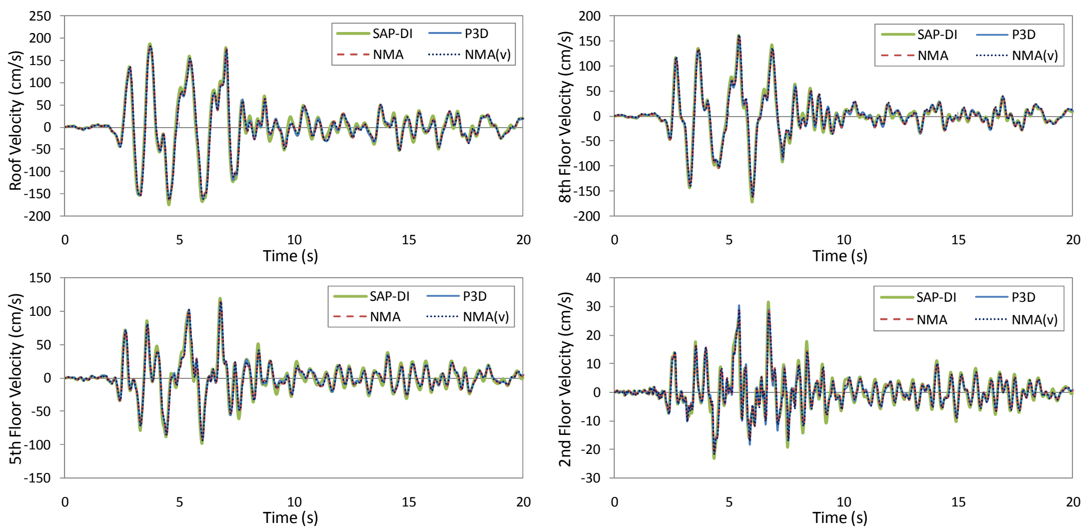


Figure 4. Comparison of velocity responses at selected floors due to 1.0 × Kobe.

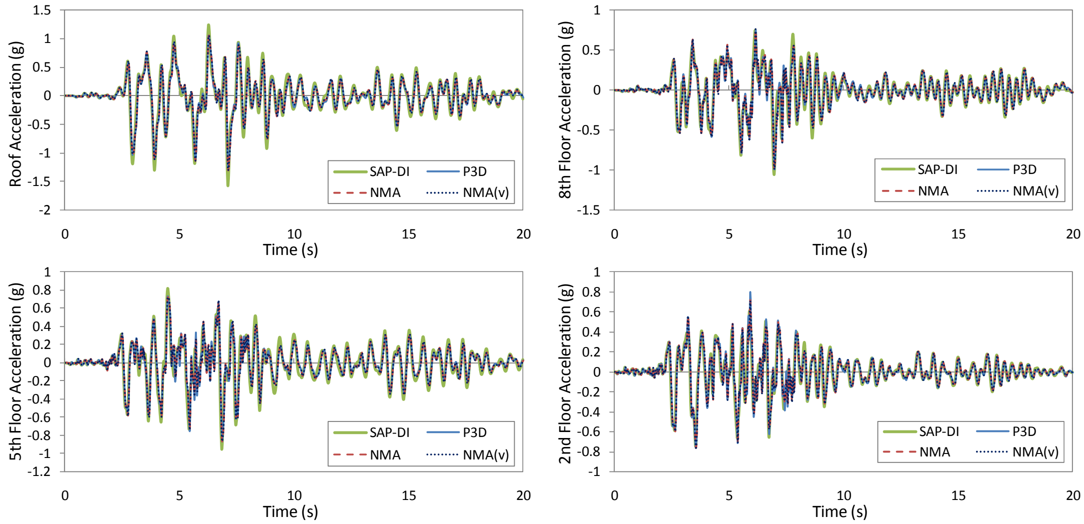


Figure 5. Comparison of acceleration responses at selected floors due to 1.0 × Kobe.

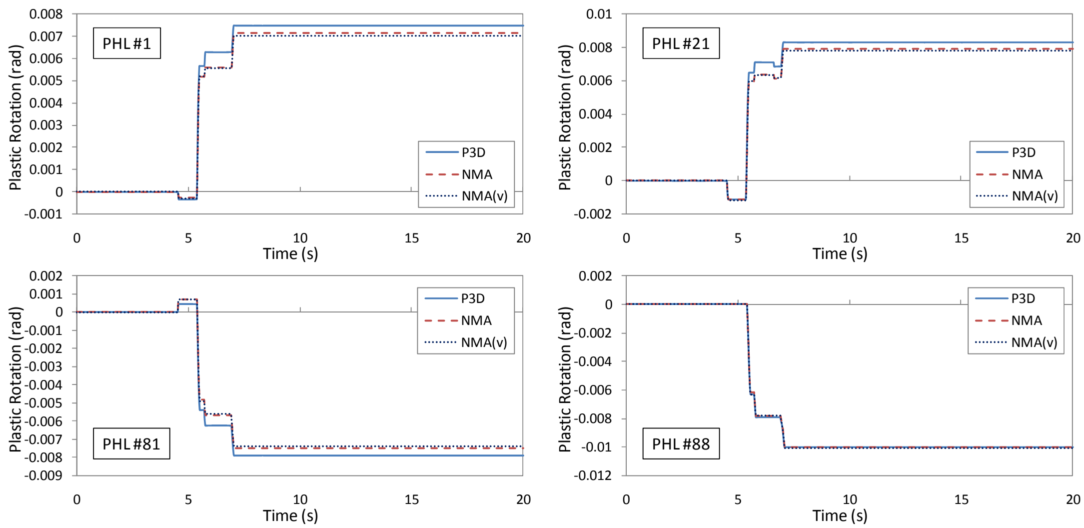


Figure 6. Comparison of plastic rotation at selected PHLs due to 1.0 × Kobe.

Comparison of Table 1 suggests that the computation using the NMA algorithm is quite time-consuming when the change in stiffness for $\mathbf{K}_G(t)$ matrix is updated at every time step. Since the elastic stiffness matrix $\mathbf{K}_e$ has already taken into consideration the local *P*-δ effects due to gravity loads and also the global *P*-Δ effects due to both gravity and earthquake loads in the proposed algorithm, it is sensible to consider another case of NMA where no update is performed, i.e., set $\mathbf{K}_G(t) = \mathbf{0}$ – denoted here as NMA(ν). This gives $\vec{f}_{Gi}(t) = 0$ for all modes according to Eq. (15), which eliminates the need to update the local *P*-δ effects due to earthquake loads. Results using NMA(ν) are also shown in Figures 3 to 6 and Table 1 for comparisons.

Figures 3 to 6 show that results from both NMA (i.e., full update) and NMA(ν) (i.e., no update) compare well with other methods of analysis, demonstrating the accuracy and efficiency of the proposed NMA algorithms. It is therefore quite motivating to study the structural responses with greater nonlinear effects, such as at a limit state of near collapse, by magnifying the ground motion. At this collapse prevention limit state, significant geometric nonlinearity comes into play, which may cause greater differences in the results between NMA and NMA(ν). Similarly, greater material nonlinearity is expected for larger earthquake while causing soft stories in the structure to become more apparent, which can give an indication as to whether the structure is properly designed according to the modern building code.

By subjecting the 10-story frame to the 1995 Kobe earthquake ground motion (all shown in Figure 2) with an amplification factor of 1.3, the global displacement, velocity, and absolute acceleration responses between various analysis methods are compared in Figures 7, 8, and 9, respectively. The local plastic rotations at selected beam and column PHLs using both NMA and P3D are compared and shown in Figure 10. In addition, the computation time of each analysis method due to the Kobe earthquake record with an amplification factor of 1.3 has been summarized in Table 1.

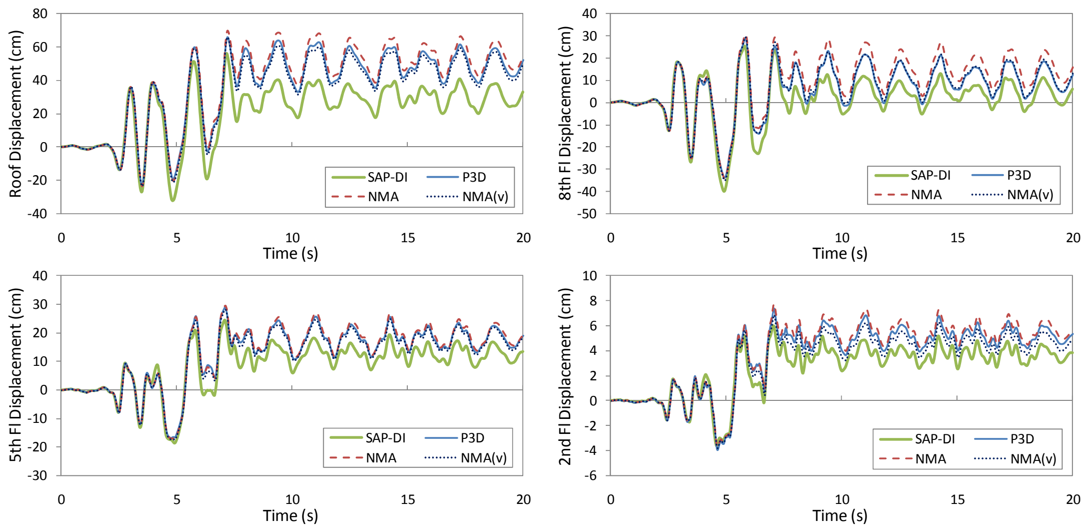


Figure 7. Comparison of displacement responses at selected floors due to 1.3 × Kobe.

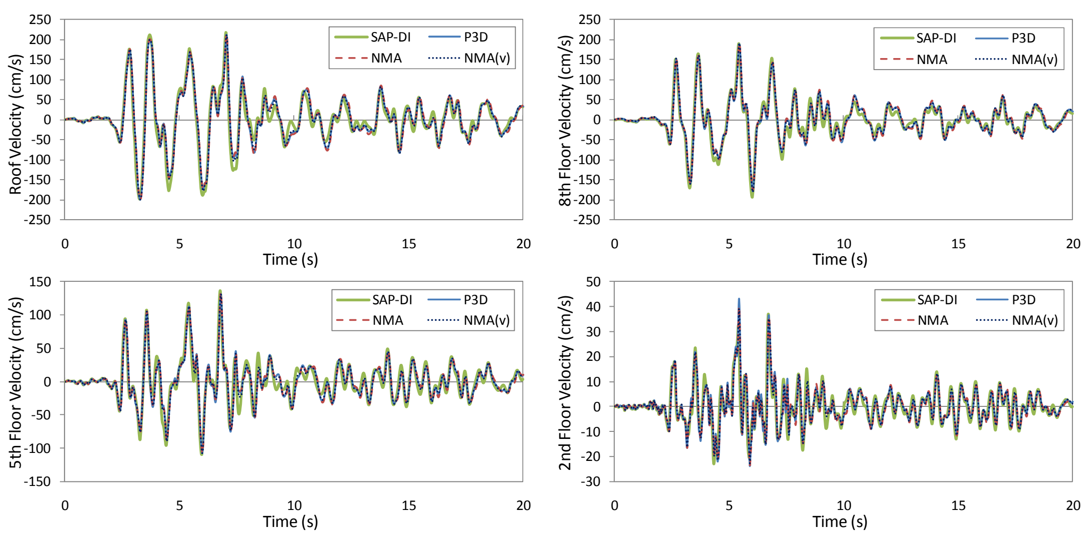


Figure 8. Comparison of velocity responses at selected floors due to 1.3 × Kobe.

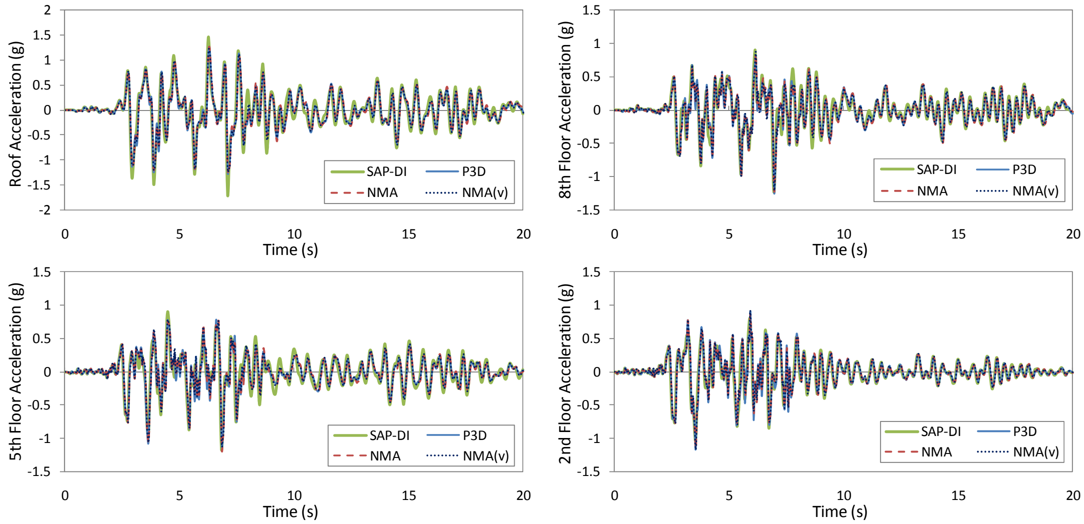


Figure 9. Comparison of acceleration responses at selected floors due to 1.3 × Kobe.

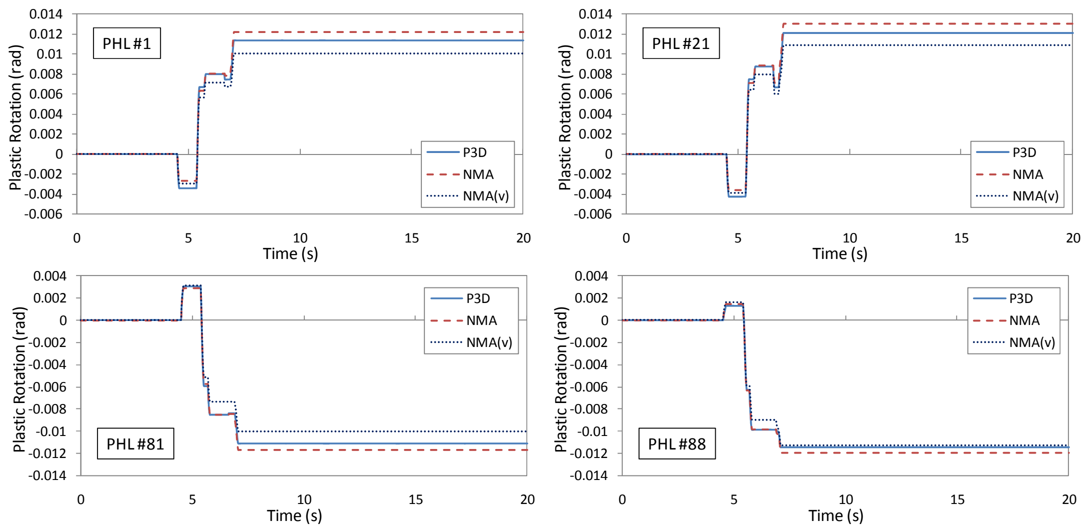


Figure 10. Comparison of plastic rotation at selected PHLs due to 1.3 × Kobe.

Comparison of results in Figures 7 to 10 again demonstrates the accuracy and efficiency of the proposed NMA algorithms. In particular, NMA(ν) results seem to be very close to the NMA results and those from Perform-3D, indicating that updating the time-varying nonlinear stiffness matrices due to geometric nonlinearity may not be an important concern in performing nonlinear response history analysis. However, further study, particularly using taller structural models loaded up to near collapse, is necessary to prove this point.

CONCLUSIONS

A nonlinear dynamic analysis algorithm for computing the dynamic response of structures subjected to earthquake excitation based on modal superposition was presented. This algorithm combined the force analogy method, state space method, and stability functions while transforming the analysis into the modal coordinates. Although modal coupling occurs because of the material and geometric nonlinearities in the analysis, this coupling is decomposed back into each mode using equivalent modal forces. By using the proposed algorithm, numerically simulated results were compared with those obtained from Perform-3D and SAP2000, and excellent correlations were obtained. These results demonstrate that the proposed algorithm has the potential of becoming an excellent scientific measurement tool that contains high accuracy and efficiency. More importantly, it has the potential of expanding the scope to nonlinear response spectra analysis that may have a greater impact to seismic design.

DISCLAIMER

Certain commercial software may be identified in this paper in order to specify the analytical procedure adequately. Such identification is not intended to imply recommendation or endorsement by the National Institute of Standards and Technology (NIST), nor is it intended to imply that the software identified are necessarily the best available for the purpose.

REFERENCES

Au, F. T. K. and Yan, Z. H. (2008). "Dynamic analysis of frames with material and geometric nonlinearities based on semi-rigid technique." *International Journal of Structural Stability and Dynamics*, 8(3), 415-438.

Chao, S. H. and Loh, C. H. (2007). "Inelastic response analysis of reinforced concrete structures using modified force analogy method." *Earthquake Engineering and Structural Dynamics*, 36(12), 1659-1683.

Liu, J. L. (2003). "Exact solution for dynamic response of multi-degree-of-freedom bilinear hysteretic systems." *Journal of Engineering Mechanics*, ASCE, 129(11), 1342-1350.

Liu, J. L. (2005). "Exact solution of nonlinear hysteretic responses using complex mode superposition method and its application to base-isolated structures." *Journal of Engineering Mechanics*, ASCE, 131(3), 282-289.

Newmark, N. M. (1959). "A method of computation for structural dynamics." *Journal of Engineering Mechanics Division*, ASCE, 85, 67-94.

Wilson E. (2002). *Three-Dimensional Static and Dynamic Analysis of Structures - A Physical Approach with Emphasis on Earthquake Engineering*, 3rd Edition, Computer and Structures Inc., Berkeley, CA.

Wong, K. K. F. and Yang, R. (1999). "Inelastic dynamic response of structures using force analogy method." *Journal of Engineering Mechanics*, ASCE, 125(10), 1190-1199.

Yang, R., Wong, K. K. F., and Pan, T. C. (2004). "Predictive inelastic state control of modern structures during earthquakes." *Journal of Structural Control and Health Monitoring*, 11(4), 291-309.

Zhang, X., Wong, K. K. F., and Wang, Y. (2007). "Performance assessment of moment resisting frames during earthquakes based on force analogy method." *Engineering Structures*, 29(10), 2792-2802.

Zhao, D. F. and Wong, K. K. F. (2006). "New approach for seismic nonlinear analysis of inelastic framed structures." *Journal of Engineering Mechanics*, ASCE, 132(9), 959-966.