Why do we care about large covariance matrices?

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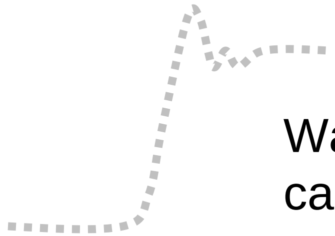
Propagate directly to parameter



Waveform is not calculated explicitly

Temperature

Contact resistance

Geometrical effects

Scaling

M sources of uncertainty α

$$\sigma_P^2 = \frac{\partial P}{\partial \boldsymbol{\alpha}} \boldsymbol{\Sigma}_{\boldsymbol{\alpha}} \frac{\partial P}{\partial \boldsymbol{\alpha}}^{\mathrm{T}}$$

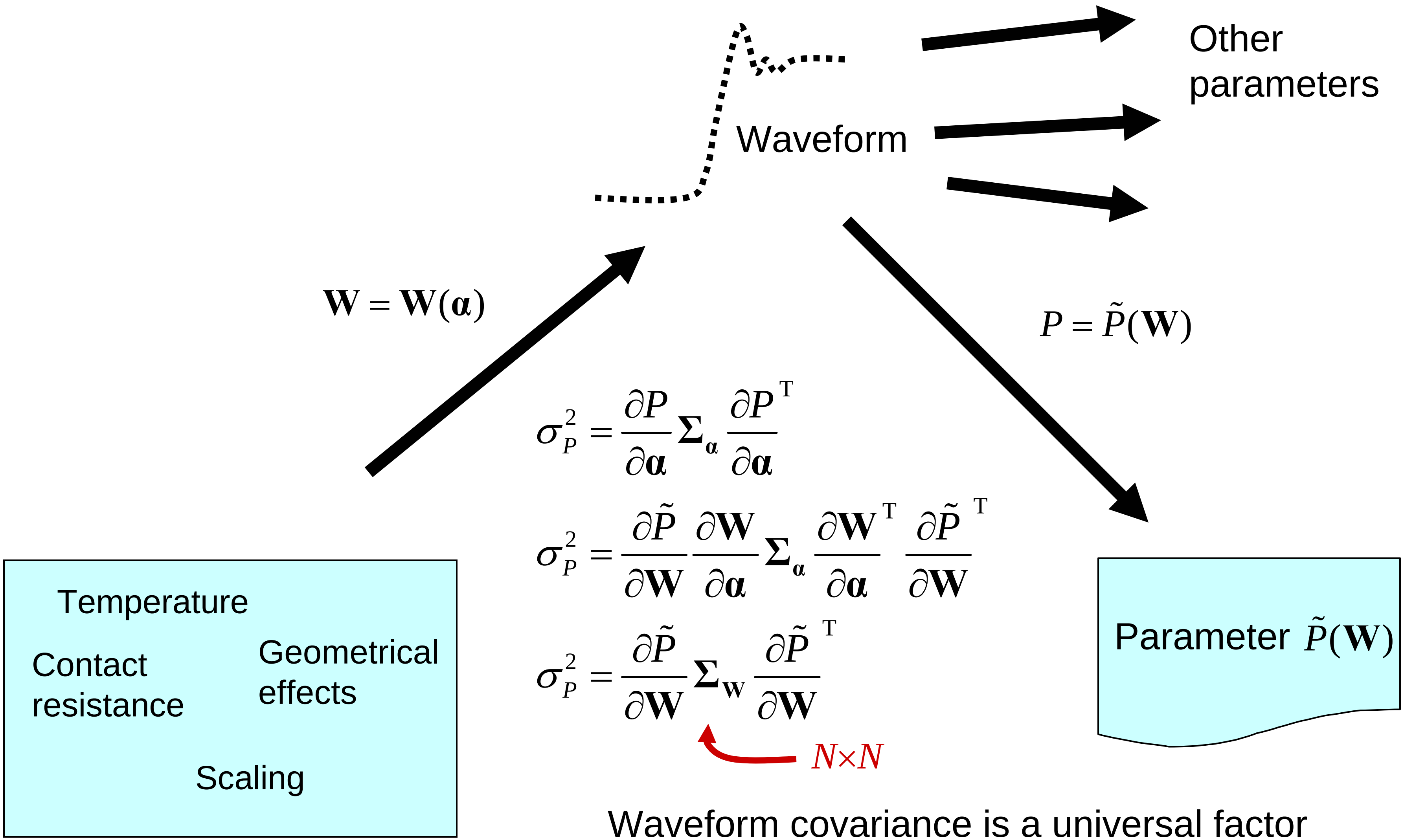
Parameter

$P = P(\boldsymbol{\alpha})$

$$\sigma_P^2 = \frac{\partial P}{\partial \boldsymbol{\alpha}} \begin{bmatrix} \sigma_{\alpha_1}^2 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \sigma_{\alpha_M}^2 \end{bmatrix} \frac{\partial P}{\partial \boldsymbol{\alpha}}^{\mathrm{T}}$$

Needs to be recalculated for each parameter

Waveform as universal intermediary



$$\sigma_P^2 = \frac{\partial P}{\partial \boldsymbol{\alpha}} \boldsymbol{\Sigma}_{\boldsymbol{\alpha}} \frac{\partial P}{\partial \boldsymbol{\alpha}}^{\mathrm{T}}$$

$$\sigma_P^2 = \frac{\partial \tilde{P}}{\partial \mathbf{W}} \frac{\partial \mathbf{W}}{\partial \boldsymbol{\alpha}} \boldsymbol{\Sigma}_{\boldsymbol{\alpha}} \frac{\partial \mathbf{W}}{\partial \boldsymbol{\alpha}}^{\mathrm{T}} \frac{\partial \tilde{P}}{\partial \mathbf{W}}^{\mathrm{T}}$$

$$\sigma_P^2 = \frac{\partial \tilde{P}}{\partial \mathbf{W}} \boldsymbol{\Sigma}_{\mathbf{W}} \frac{\partial \tilde{P}}{\partial \mathbf{W}}^{\mathrm{T}}$$

Waveform covariance is a universal factor of the uncertainty of any parameter P

1- Computational intractability

- If waveform has N elements, covariance matrix has N^2 elements
- N ~ 2000 works OK for $\mathbf{J\Sigma J}^\mathsf{T}$
- N ~ 4000 crashes my computer
- Want N ~ 50,000 to 1,000,000 for some problems

2 – Singularity of Σ

- We have a high dimensional problem with a low number of measured waveform vectors: rank deficient
- How do we make inferences regarding correlations in this under-determined problem when Σ^{-1} does not exist?
- Joint estimation of off diagonal elements
  - Note that diagonal elements are not a problem
  - Correlations appear that are not there

3 – Covariance modeling, parameterization, or compression

- Model large covariance matrix with a small number of the most significant parameters
- How do we test the model if we only have a few measurements?