Full waveform metrology: Characterizing high-speed electrical signals and equipment

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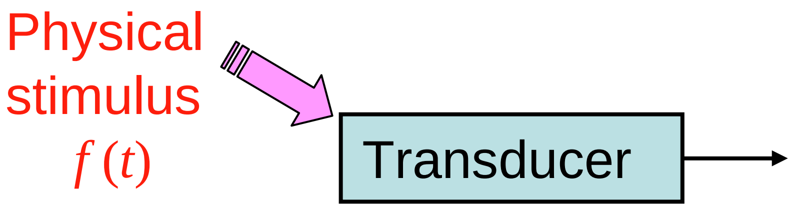
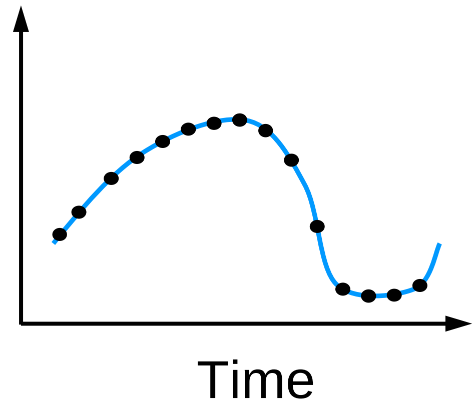
What are waveforms?

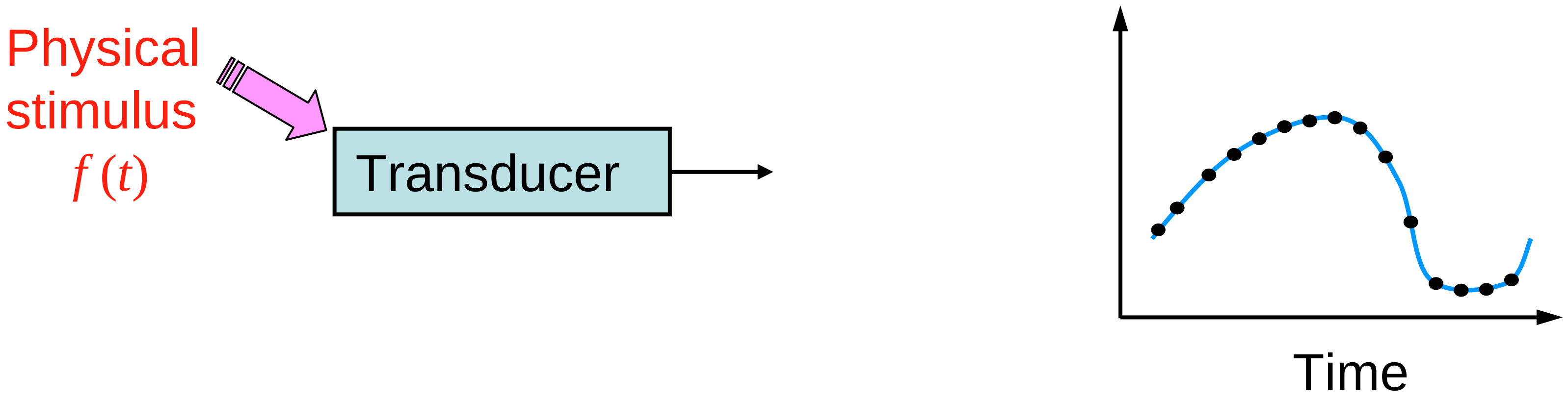




IEEE Standard 181-2003, IEEE Standard on transitions, pulses, and related waveforms

- Signal: A physical phenomenon that is a function of time
- Waveform: A representation of a *signal*
    - For example, a graph, plot, oscilloscope presentation, discrete time series, equation, or table of values.
- A waveform is obtained by some estimation process.

Who cares?



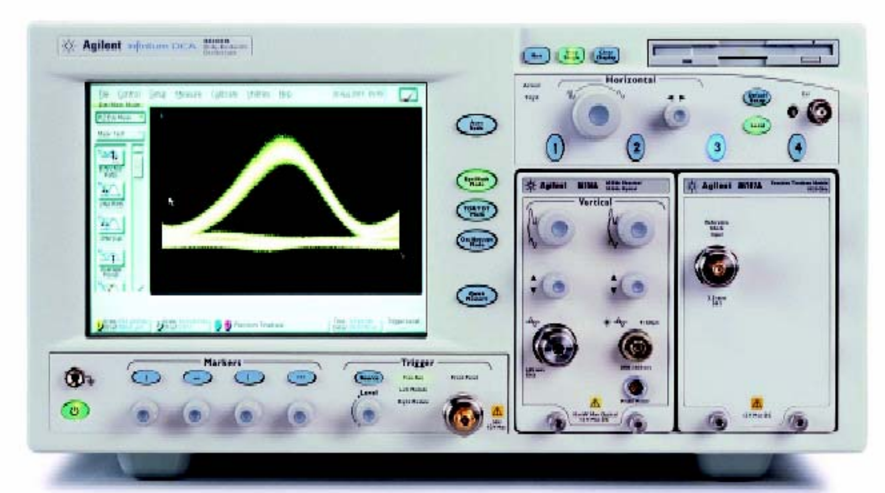
Waveform generation and measurement used in <u>all</u> areas of science and technology

- ~760 oscilloscopes owned by NIST in 2004
  - Wireless, wired, and optical communications
  - Remote sensing
  - Chemical and materials properties
  - ~1 oscilloscope for every 4 NIST staff
- >$1B annual market
- $500M market for high end oscilloscopes

Waveforms in the modern world

- Waveforms constitute the currency of modern, data-intensive communications.
- Digital signals require time-domain measurements

High-speed sampling oscilloscopes

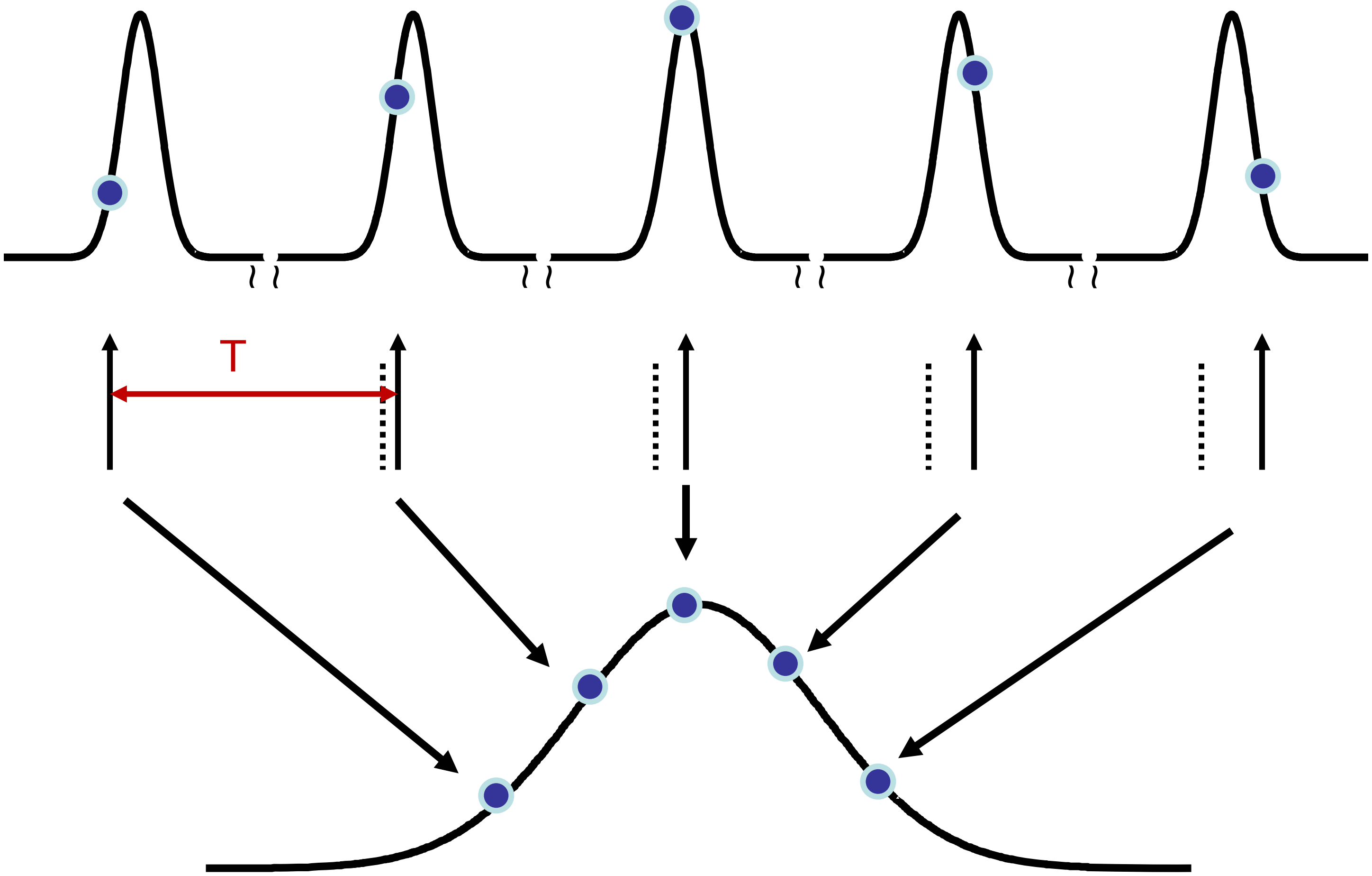


- Bandwidth: 20 to ~100 GHz
- Response pulse width ~ 4 to17 ps
- Need to deconvolve detector/scope response for pulses <16 to 70 ps [3% error, rss]

Equivalent time sampling

Input to sampler:

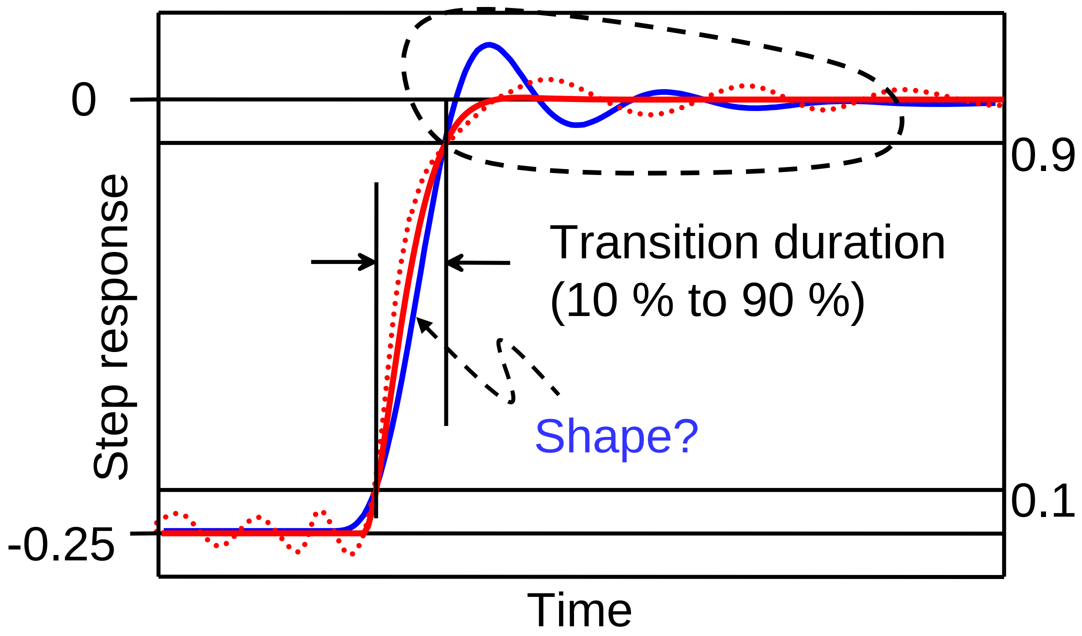
Sampling period $T > 10\ \mu s$



Why full waveform metrology?
Oscilloscope response calibration

Old way: One number (parameter) to describe response

Bandwidth and transition duration



One or only a few parameters do not uniquely describe response function

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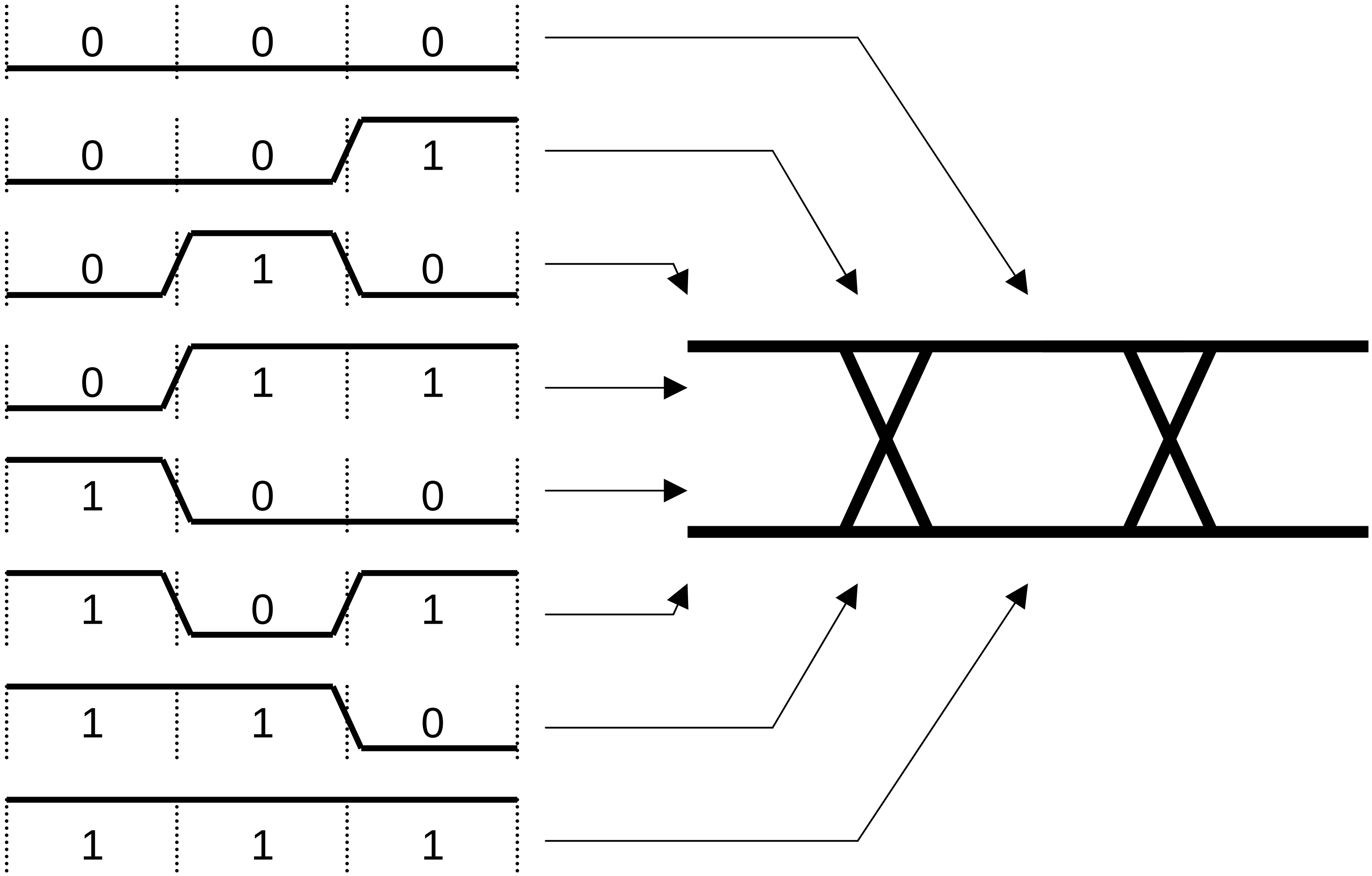
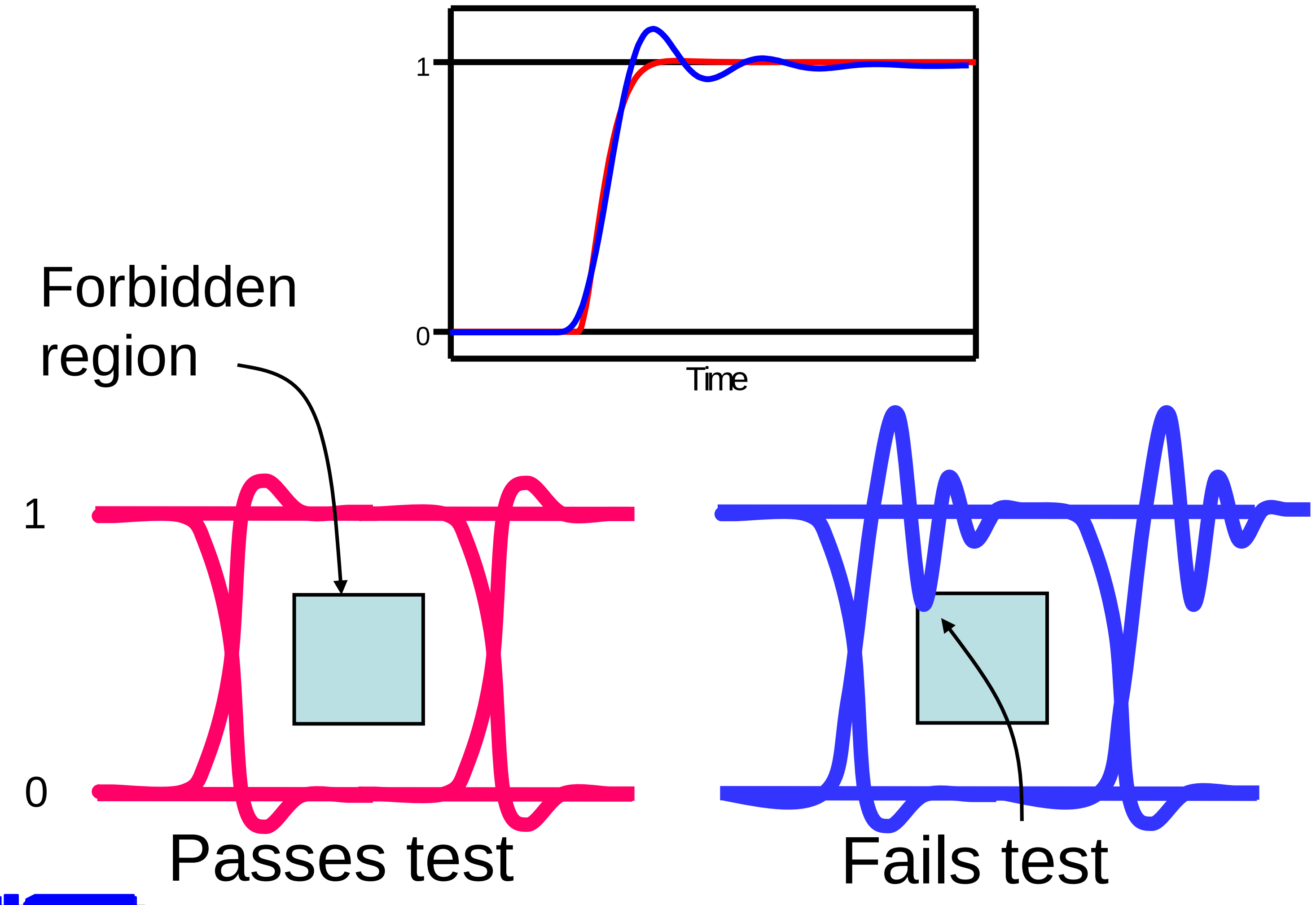
Constructing an ‘eye’ pattern

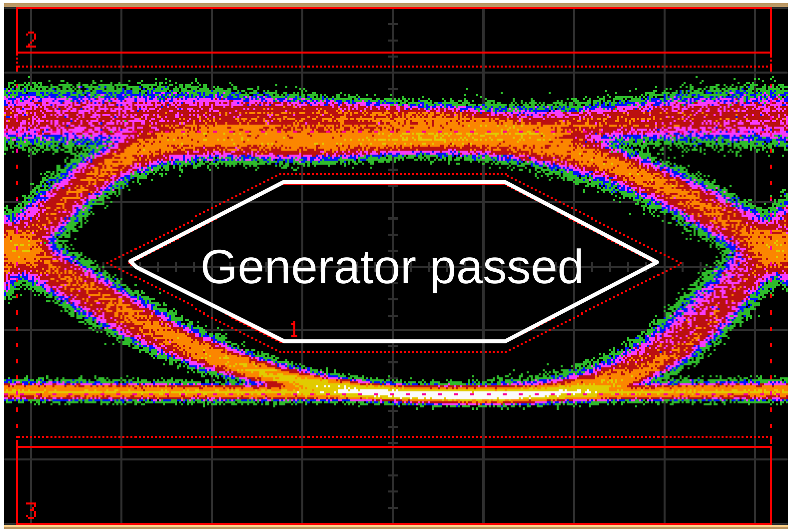


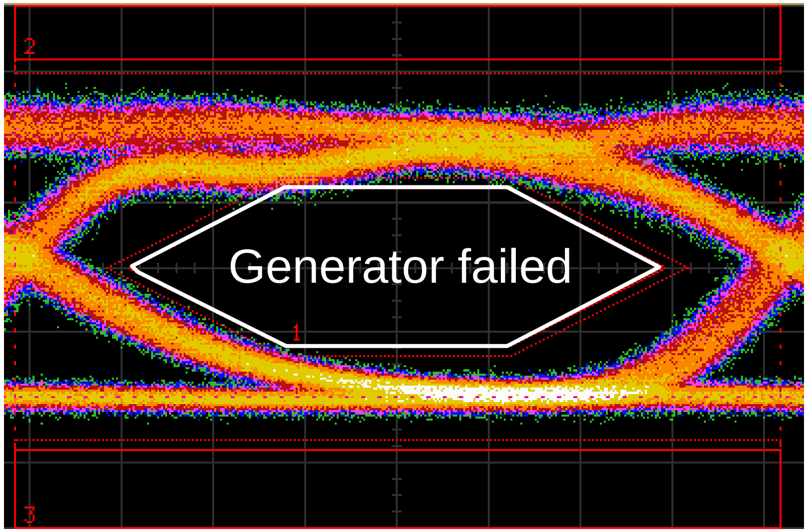
Digital signal test

Different responses yield different results!



A real measurement

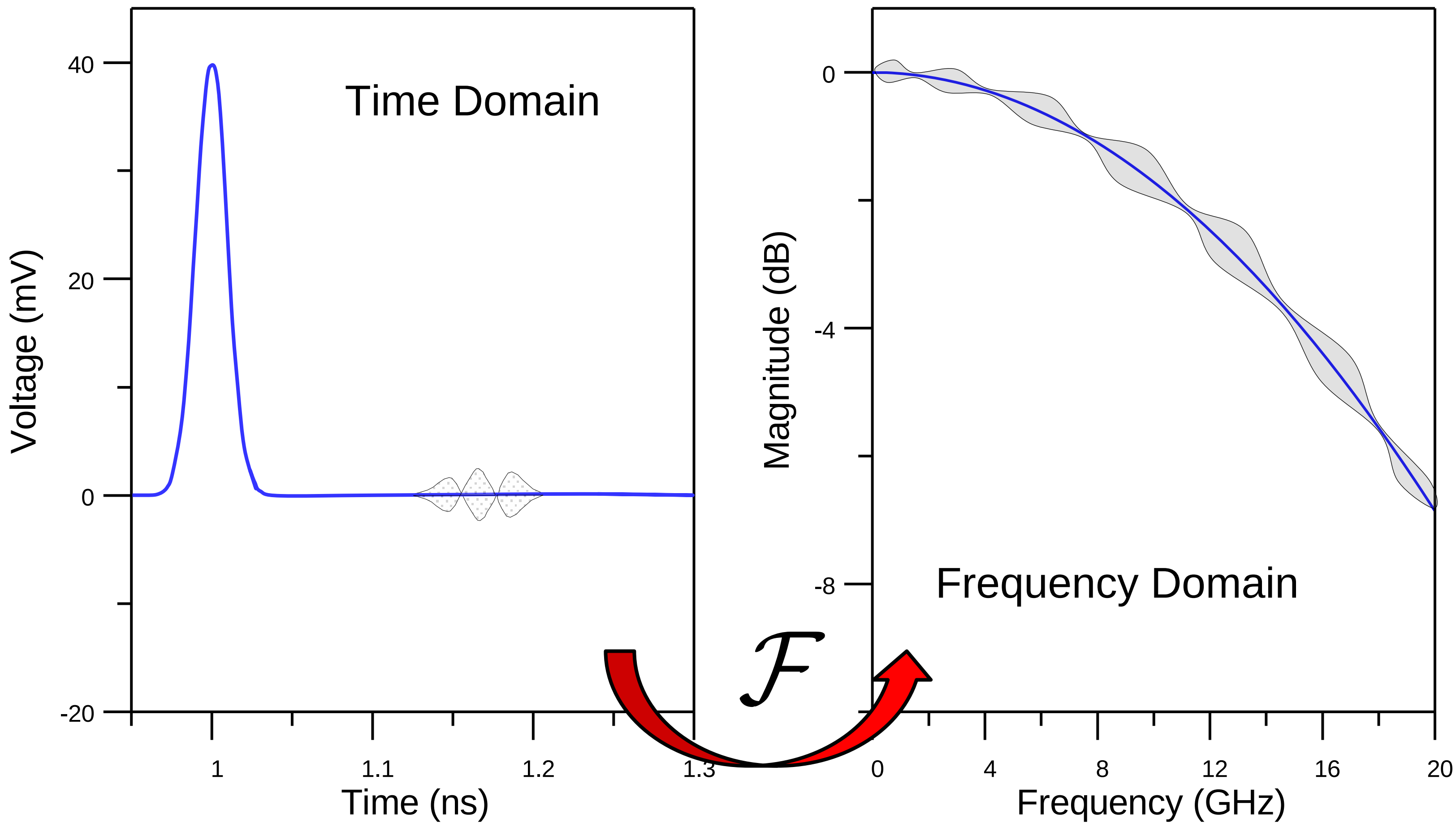




Same source, two different oscilloscopes, same bandwidth

Which measurement is correct?

Full waveform metrology



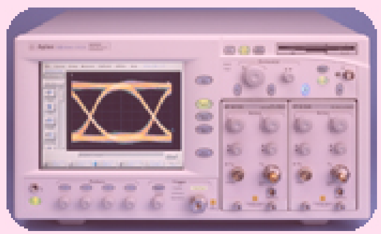
Full waveform metrology

Unified approach to

- *Calibration*
- *Calculating and comparing uncertainty in time and frequency domains*
- *Deliver calibrated sources and waveform measurements*

Whole waveform metrology

Bit Error Ratio Tester (BERT)

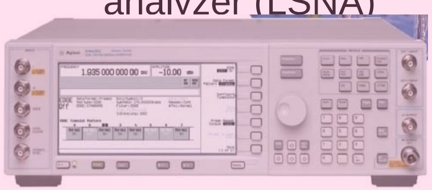


Oscilloscope

Large-signal network analyzer (LSNA)

Vector signal analyzer (VSA)

Pulse generator



Vector signal generator (VSG)

Lightwave component analyzer (LCA)

Considerations for high-speed measurements: Time calibration

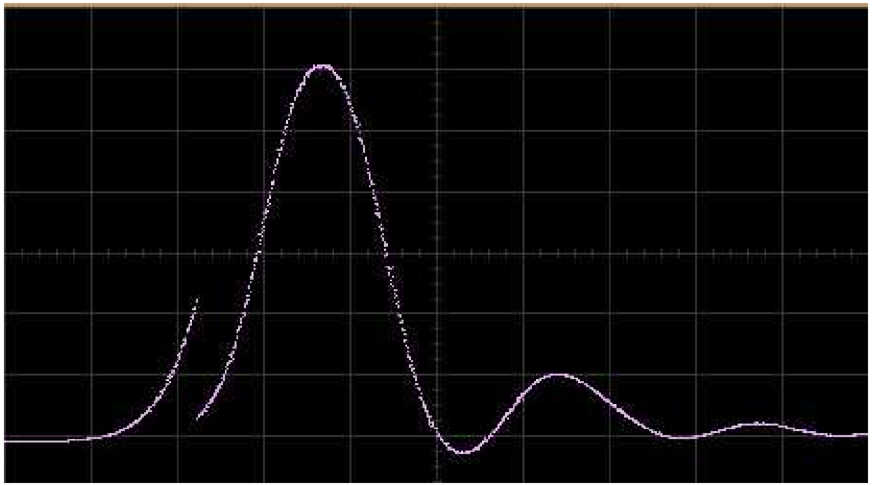
- Sample time errors are significant
- Includes both random and systematic contributions

Timebase distortion

Evenly spaced time samples 1 2 3 4 5 6 7 8 9 10……

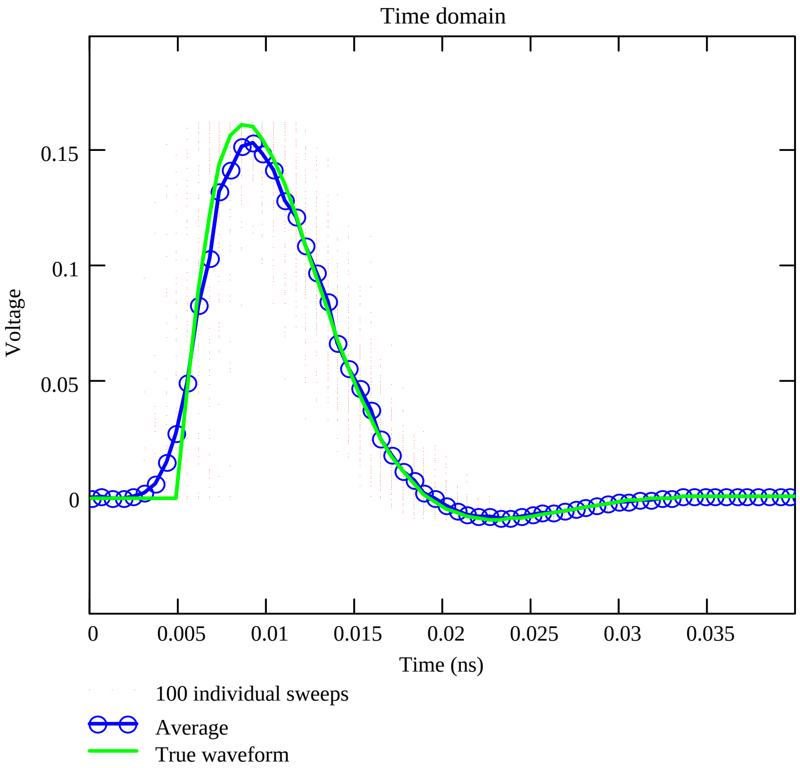
Actual sample timing 1 2 3 45 6 7

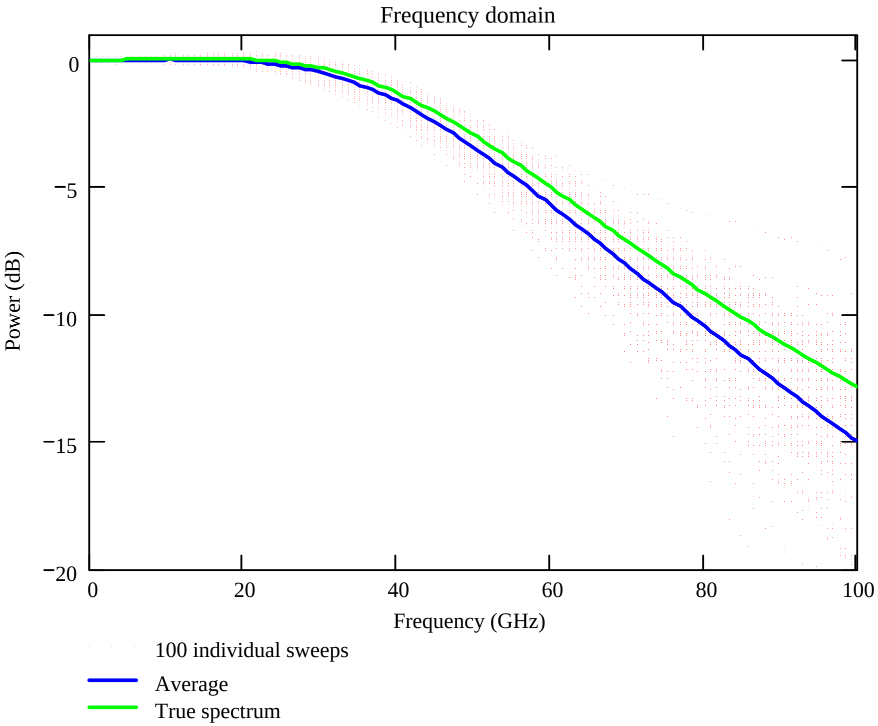
8 9 10 11 12 13



Jitter

Jitter *averaged* acts as lowpass filter$\Rightarrow$ $\quad Loss \sim \exp(-\tfrac{1}{2}\sigma_J{}^2\omega^2)$

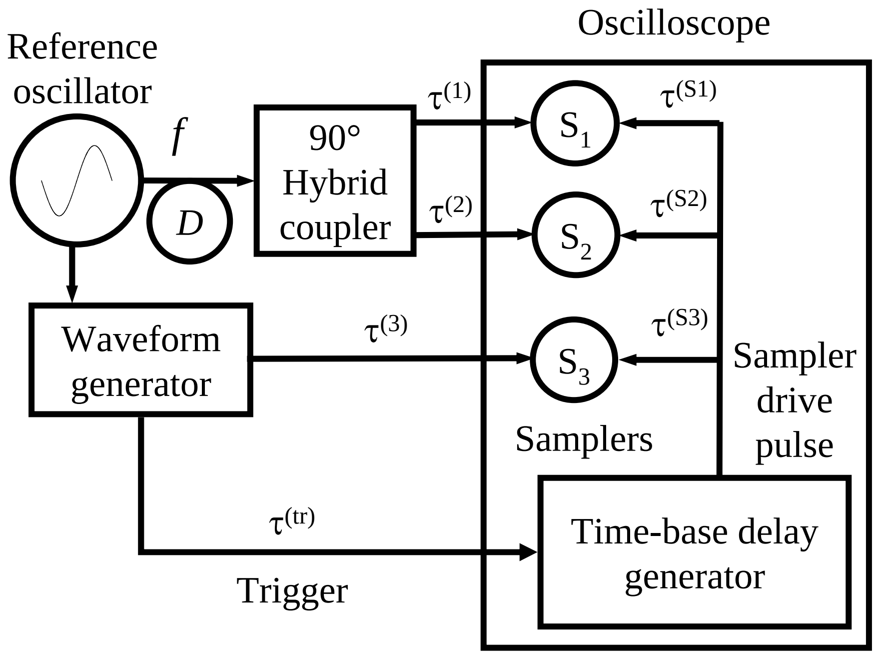


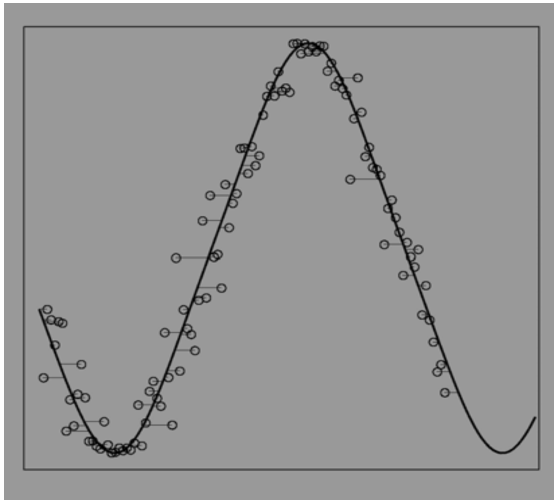


FWHM: 7.4 ps $\rightarrow$ 7.7 ps

Power @ 60 GHz: 0.7 dB error

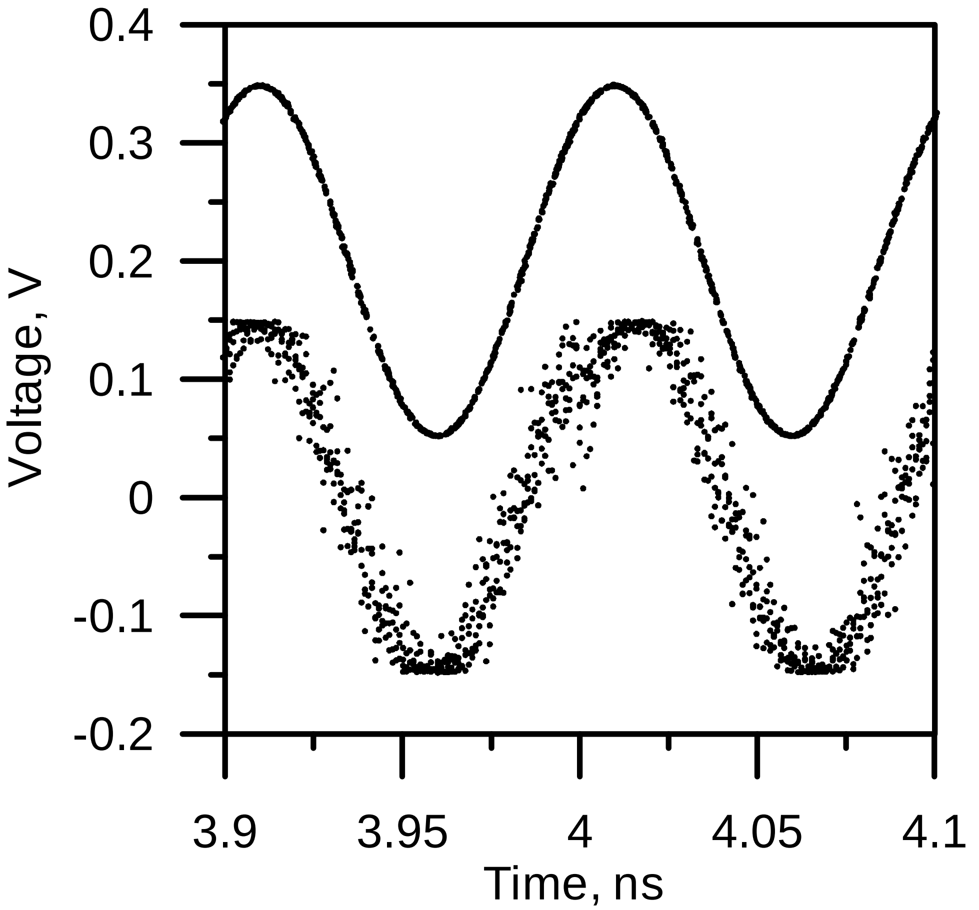
Timebase correction





- Measure reference sine waves simultaneously with signal
- Software fits sine wave data to model
- Find time error at each sample (horizontal lines)
- Correct for time error

Timebase correction in action

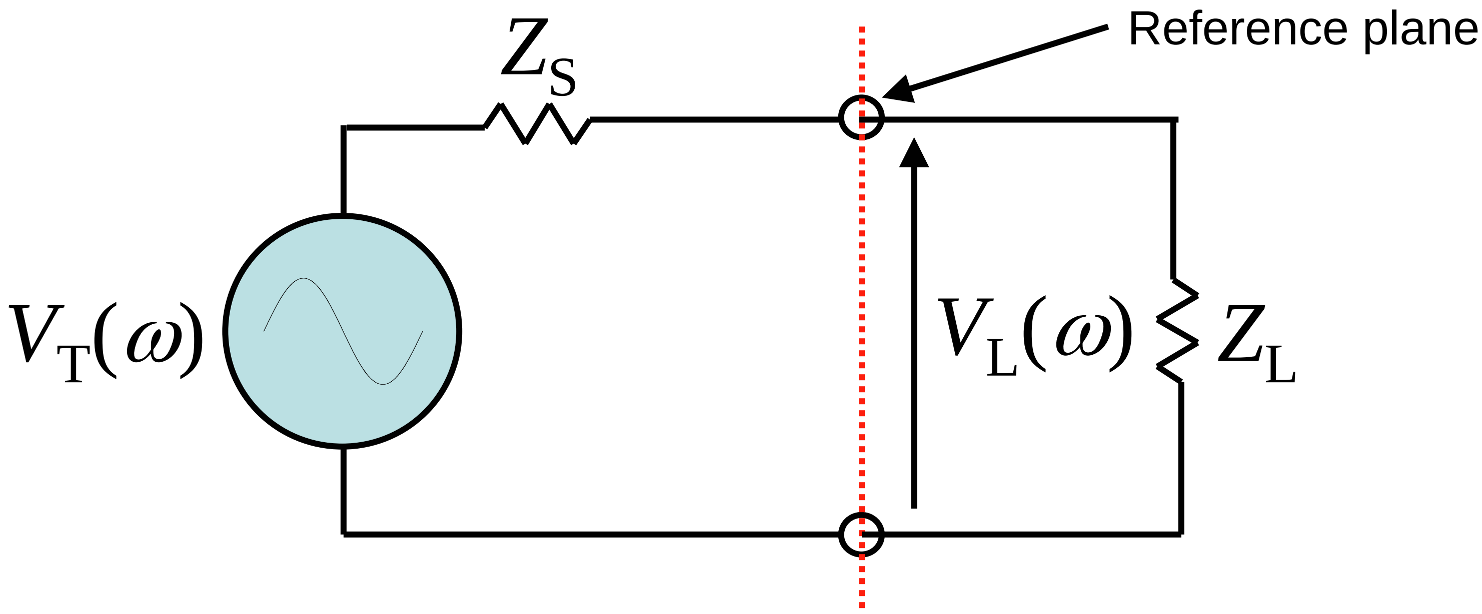


Software available at
http://www.nist.gov/eeel/optoelectronics/sources_detectors/tbc.cfm

Considerations for high-speed measurements: Voltage calibration

- Voltage present at device terminals is a function of the load impedance
  - Dimensions of device are greater than a wavelength
  - Impedance of source and test equipment is not 50 Ω
  - Test fixtures and cables are lossy and may induce reflections

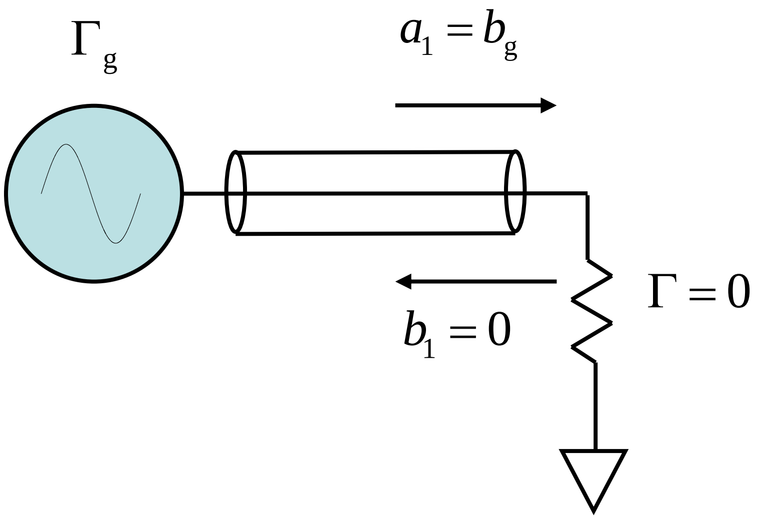
Effect of impedance on measurement



Thévenin equivalent circuit

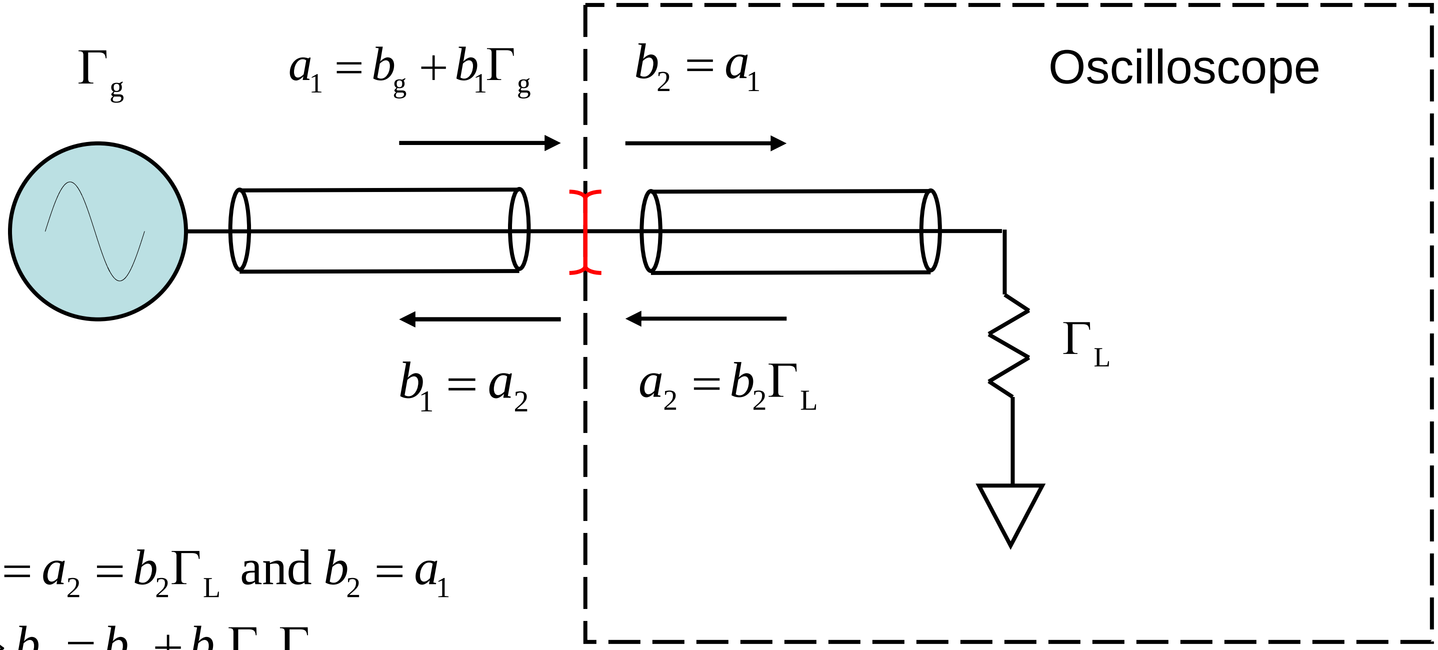
$$V_L(\omega) = V_T(\omega)\left(\frac{Z_L(\omega)}{Z_L(\omega)+Z_S(\omega)}\right)$$

Definition of source wave b_g



- b_g is the amplitude of the wave that the source delivers to a matched load.
- e.g., power meters are calibrated to measure $P = ½|\, b_g|^2$

Estimate b_{g} from measured b_2

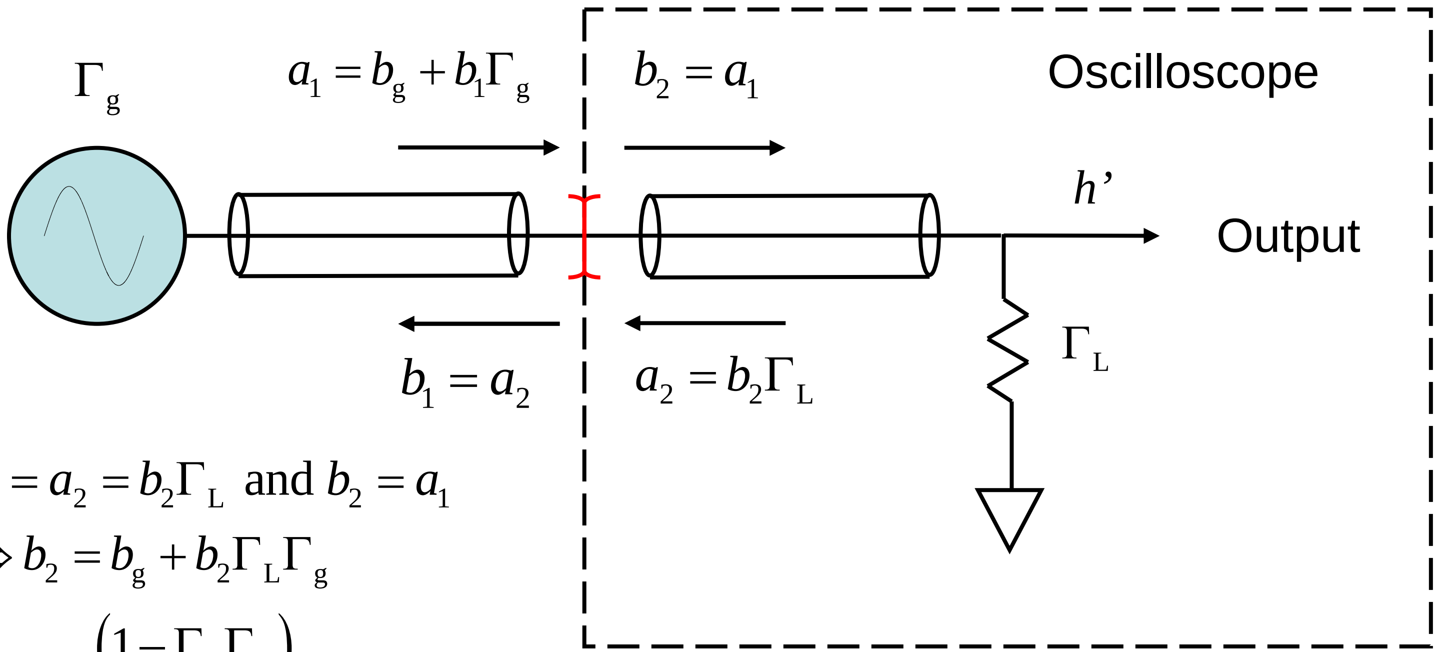


$b_1 = a_2 = b_2\Gamma_{\mathrm{L}}$ and $b_2 = a_1$

$\Rightarrow b_2 = b_{\mathrm{g}} + b_2\Gamma_{\mathrm{L}}\Gamma_{\mathrm{g}}$

$\therefore b_{\mathrm{g}} = \left(1 - \Gamma_{\mathrm{L}}\Gamma_{\mathrm{g}}\right) b_2$

Adding instrument response



$$b_1 = a_2 = b_2\Gamma_L \text{ and } b_2 = a_1$$

$$\Rightarrow b_2 = b_g + b_2\Gamma_L\Gamma_g$$

$$\therefore b_g = \frac{\left(1-\Gamma_L\Gamma_g\right)}{h'} b_2$$

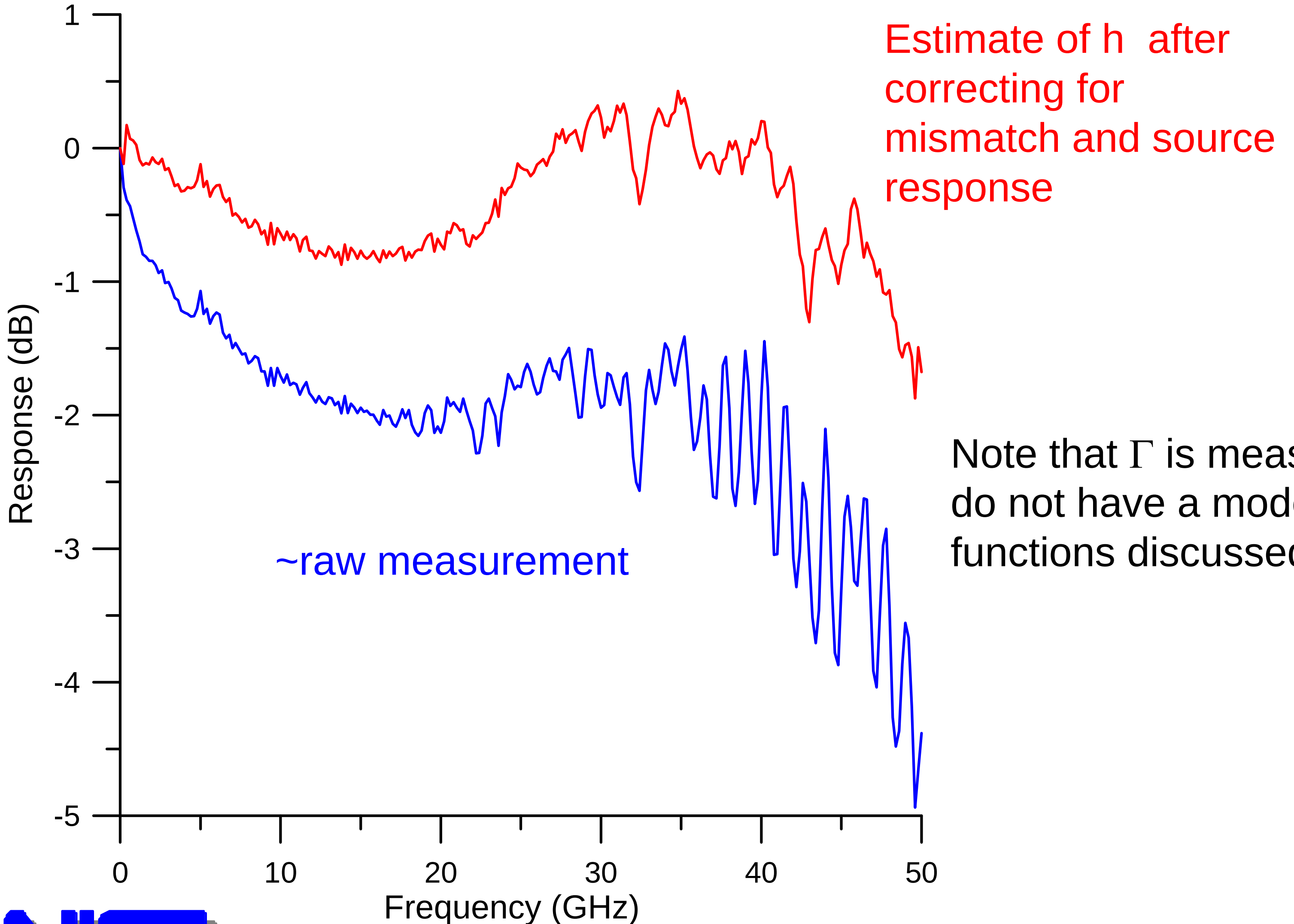
$$\boxed{b_g = h^{-1} b_2 \quad \text{where} \quad h^{-1} = \frac{\left(1-\Gamma_L\Gamma_g\right)}{h'}}$$

$h' \equiv$ oscilloscope response function

$h \equiv$ system response function

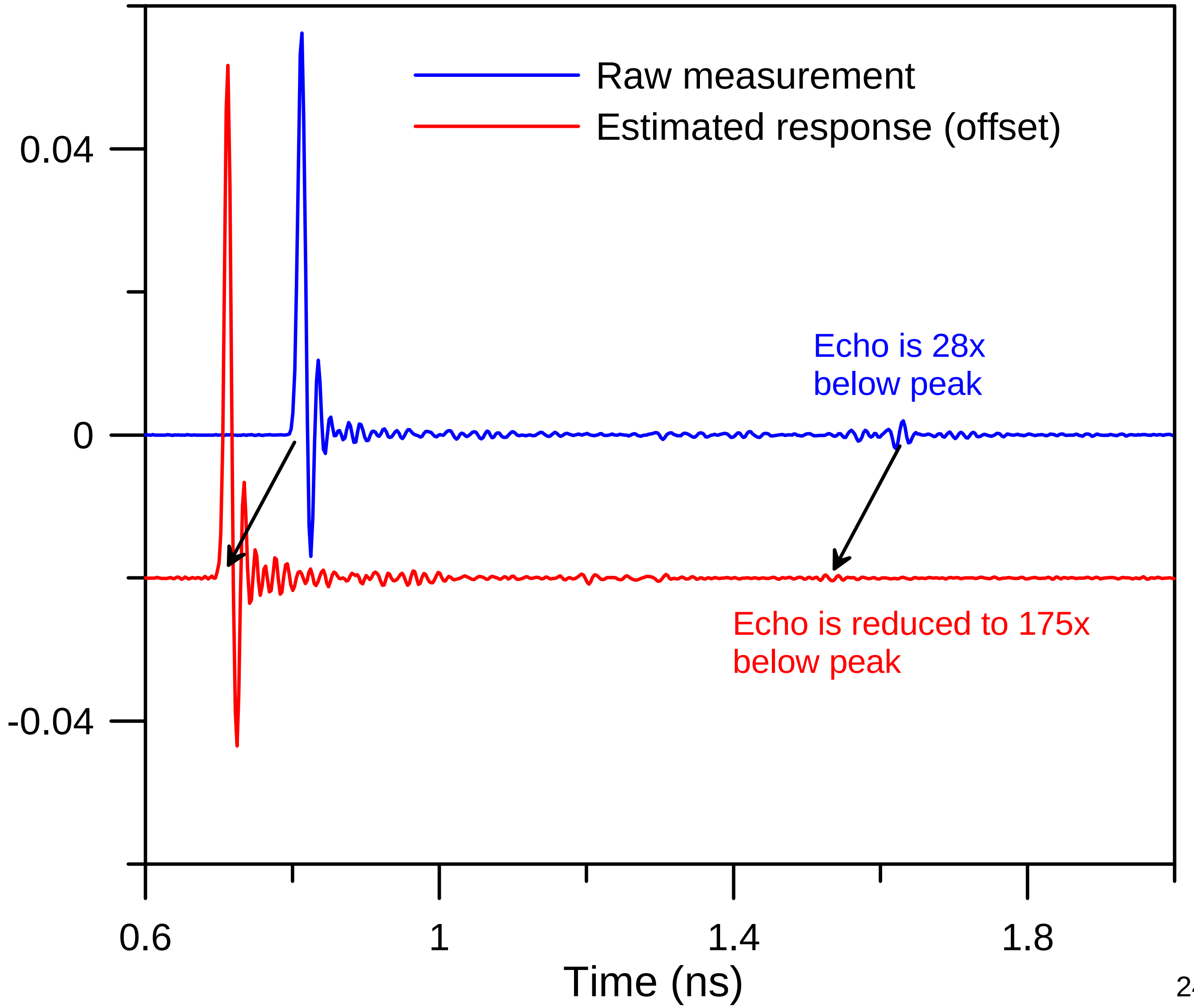
May want to solve for oscilloscope response h instead of b_g

Echo in frequency domain

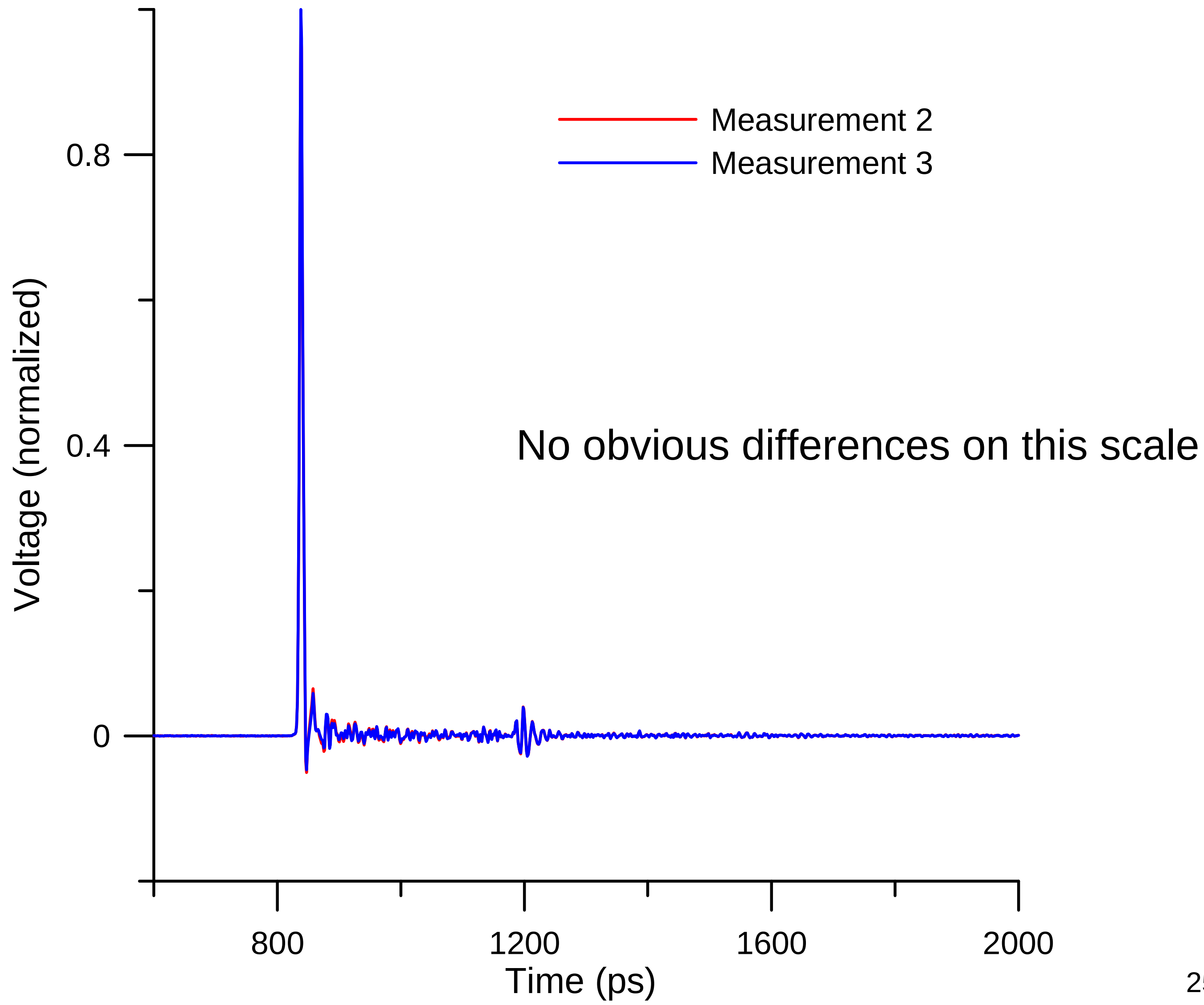


Note that Γ is measured and we do not have a model for any functions discussed in this talk

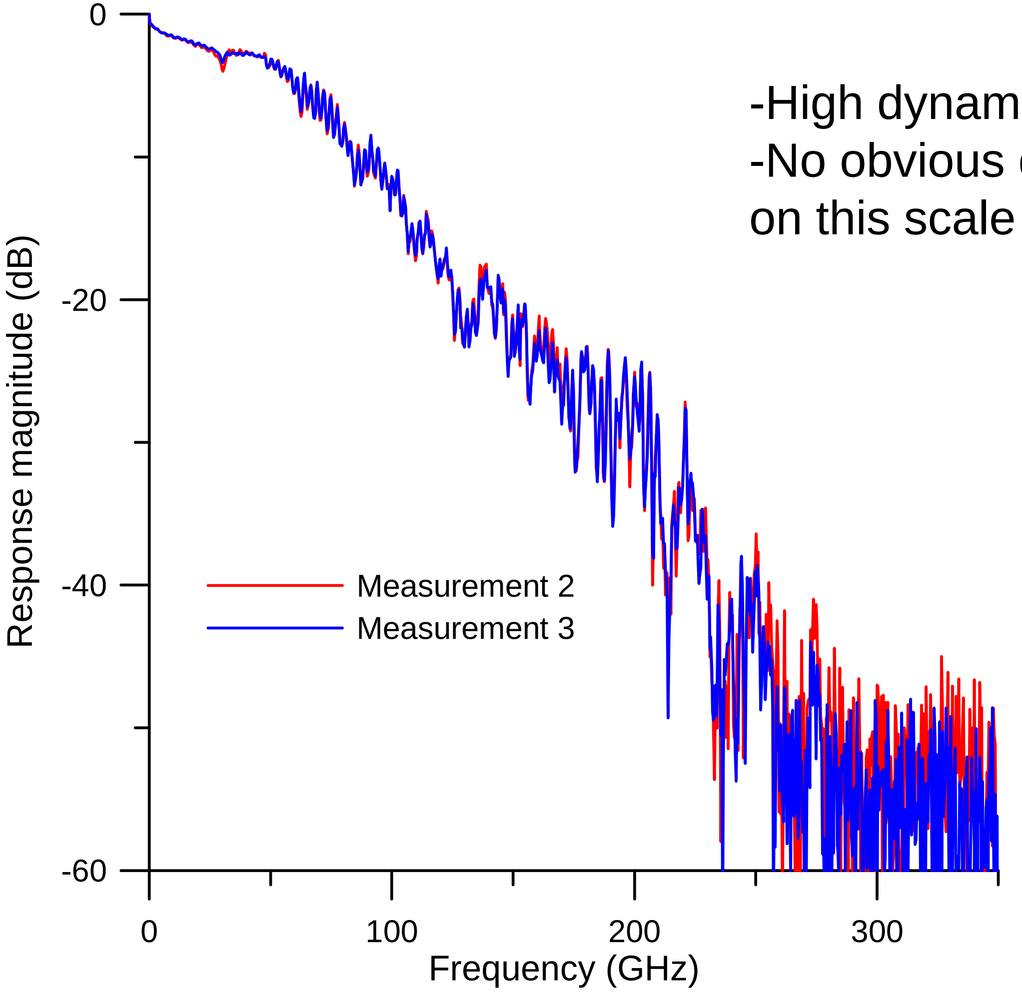
Time-domain echo



Dissect two repeat measurements : Time domain

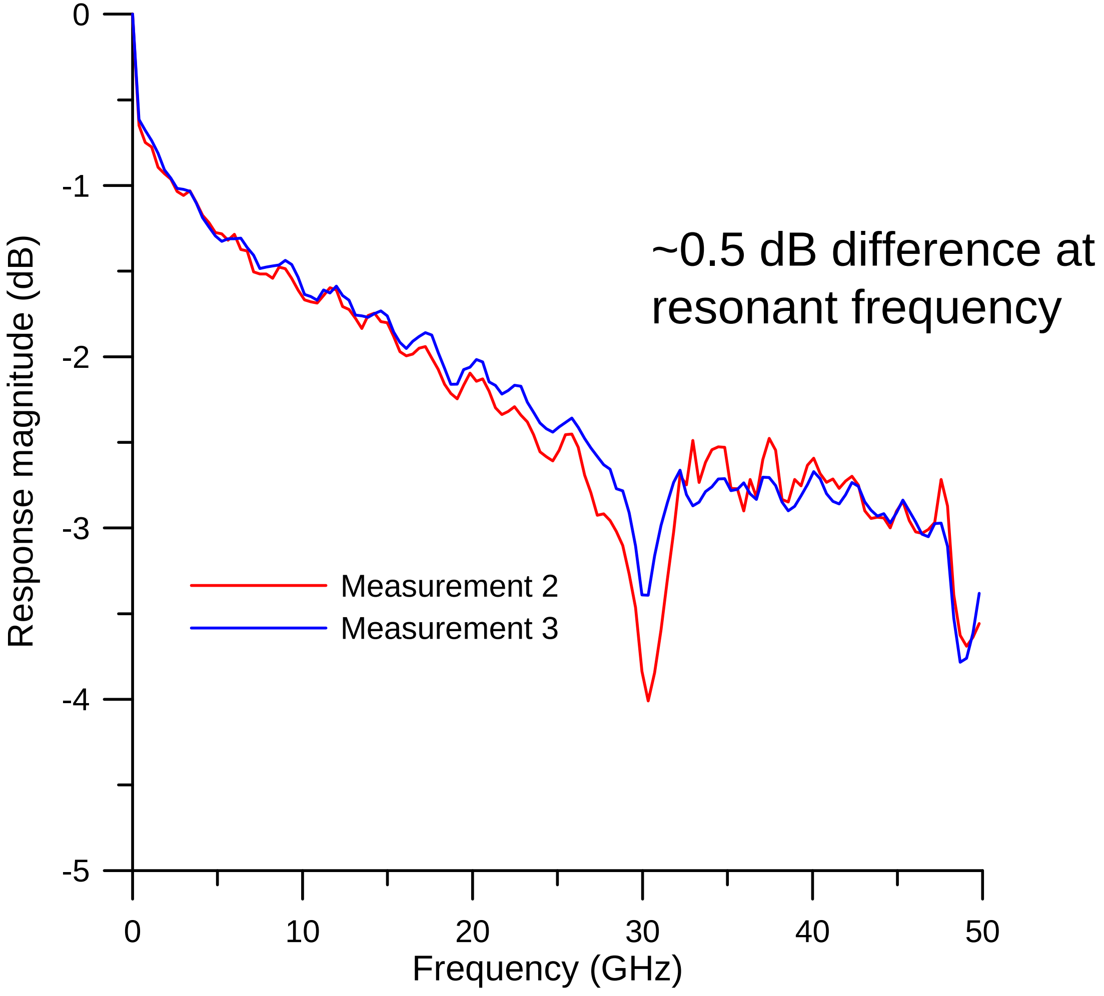


Dissect two measurements in the time and frequency domains

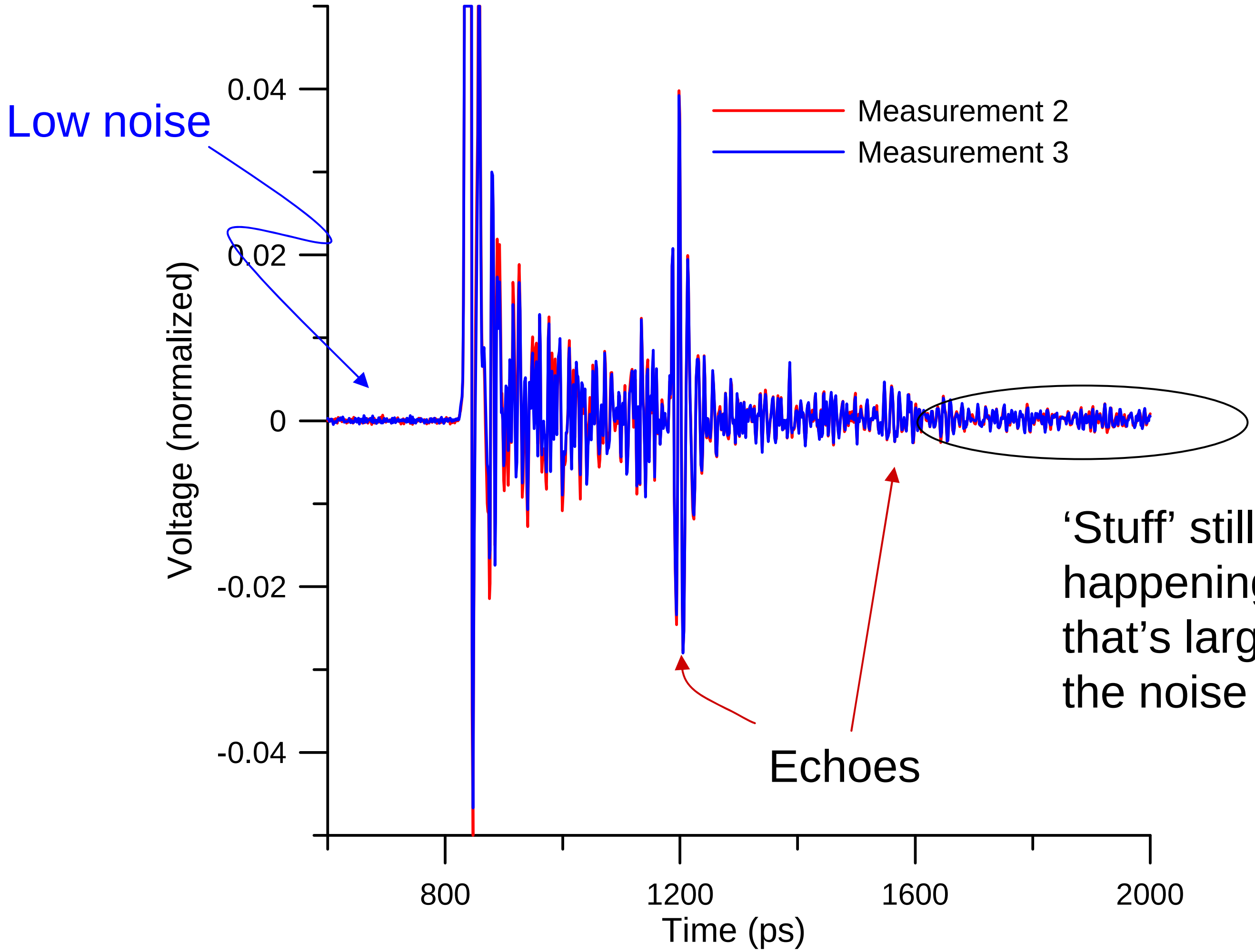


-High dynamic range
-No obvious differences on this scale

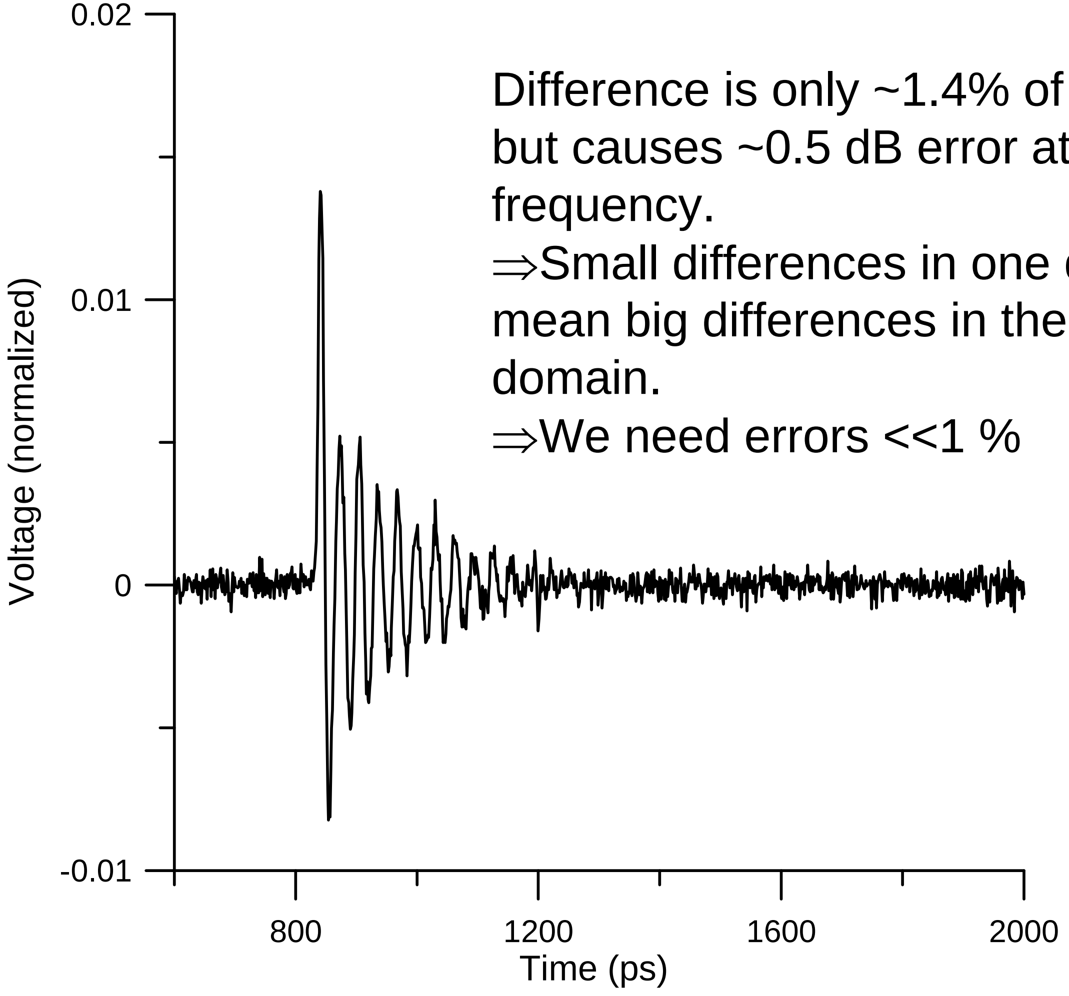
Discrepancy due to contact resistance



Time domain expanded



Difference between measurements



Difference is only ~1.4% of peak value but causes ~0.5 dB error at resonance frequency.

⇒Small differences in one domain may mean big differences in the other domain.

⇒We need errors <<1 %

Considerations for high-speed measurements: Voltage calibration and dynamic response

- Instrument response bandwidth is comparable to the bandwidth of the unknown signal or system
  - $\Delta f_{test}/\Delta f_{signal}$ ~1/2 to 5
- Deconvolution of combined effects of instrument response, loss, and reflections is typicaly required
- Deconvolution often requires regularization

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Linear systems approach

$$y(t) = [x * a](t) + noise$$

$$= \int_0^\infty a(s) x(t-s) ds + noise$$

Using Fourier transform:

$$y(t) = [x * a](t) \Leftrightarrow Y(f) = X(f) A(f)$$

$$\Rightarrow X(f) = Y(f) / A(f)$$

Least squares minimization

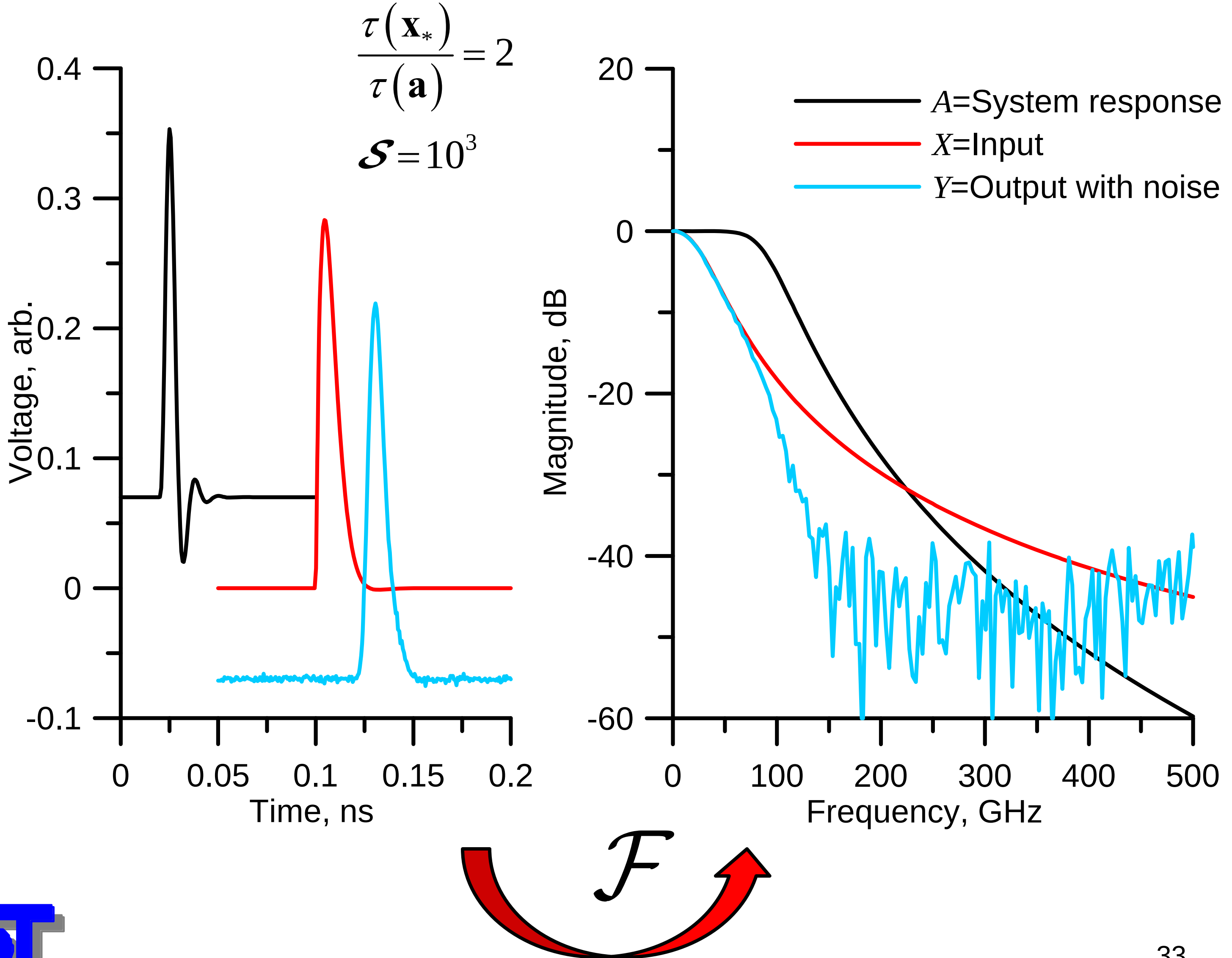
$$\mathbf{y} = \mathbf{A}\mathbf{x} + \sigma\mathbf{n}$$

$$\min_{\{\mathbf{x}\in\Re^n\}} \left\| \mathbf{A}\mathbf{x} - \mathbf{y} \right\|^2$$

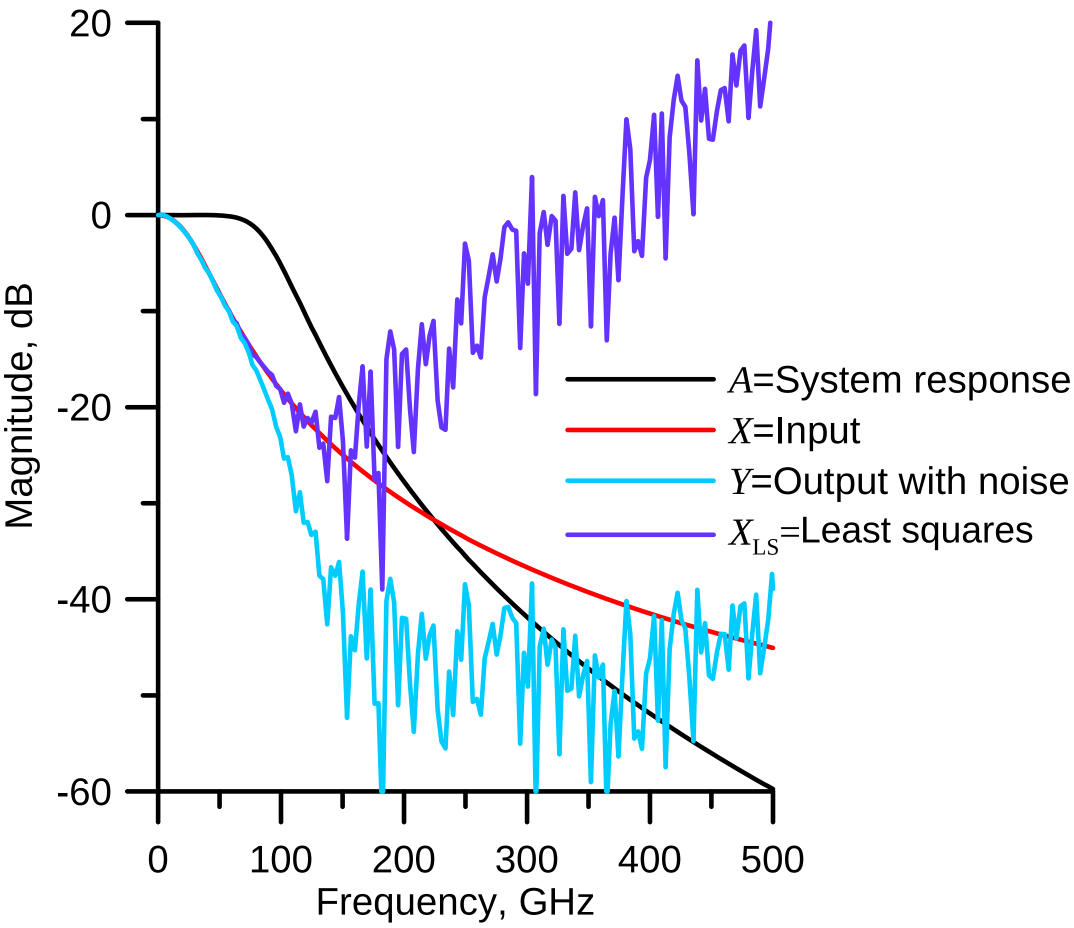
ÿ $$\mathbf{x}_{\mathrm{LS}} = \left(\mathbf{A}^*\mathbf{A}\right)^{-1}\mathbf{A}^*\mathbf{y}$$ “Normal equation”

$$= \mathbf{A}^{-1}\mathbf{y}$$ Because our system is invertible

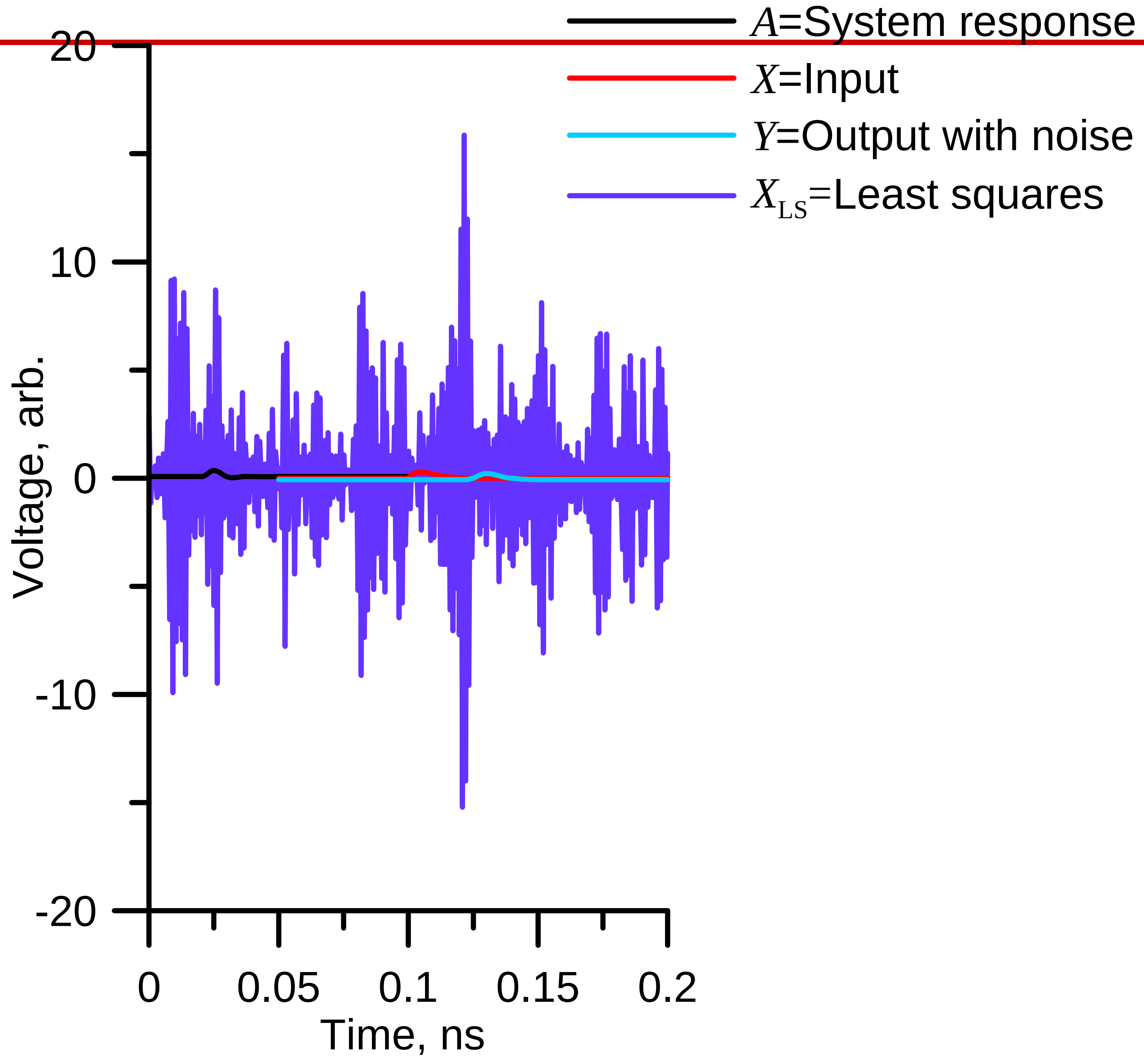
Convolution plus noise



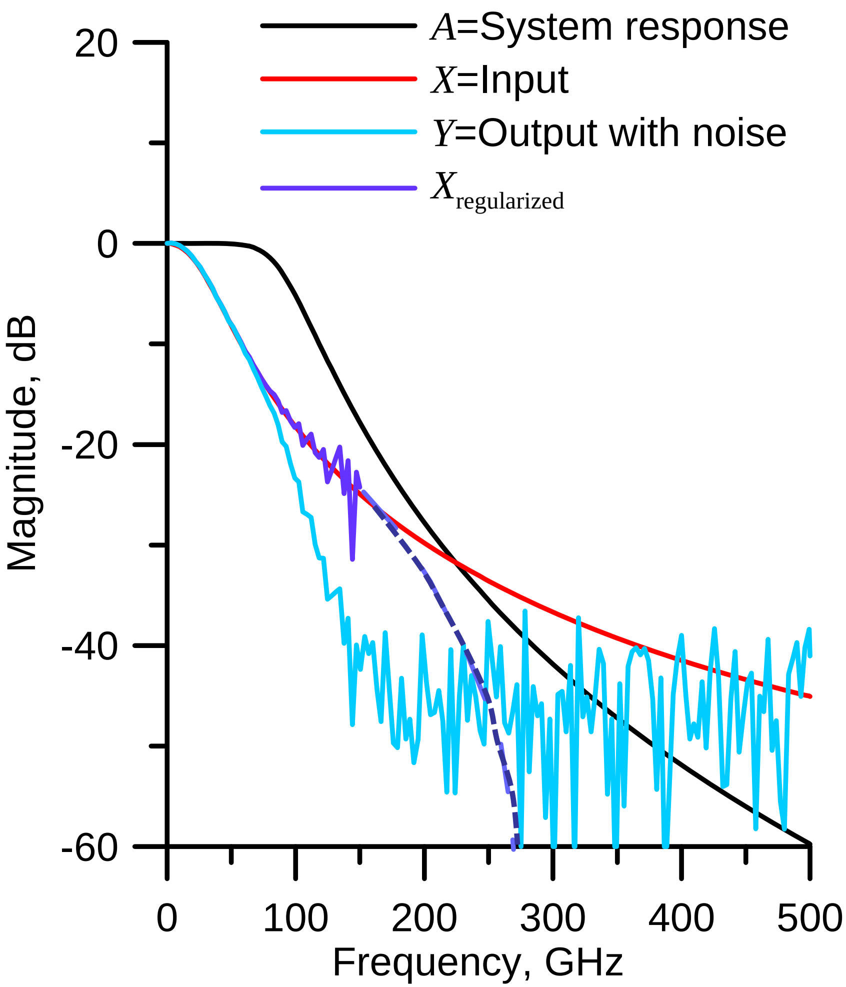
LS solution: Deconvolution without regularization



LS solution is unstable



Control instability



- Introduces free parameter(s)
- Want systematic, rigorous theory
- May want to automate stabilized (i.e., “regularized”) deconvolution

Constrained minimization

Lagrange multiplier

$$\min_{\left\{\mathbf{x}:\|\mathbf{L}\mathbf{x}\|^2 \le M\right\}} \|\mathbf{A}\mathbf{x}-\mathbf{y}\|^2$$

$$\Rightarrow \underbrace{\min_{\left\{\mathbf{x}\in\Re^n\right\}} \left\{\|\mathbf{A}\mathbf{x}-\mathbf{y}\|^2 + \lambda^2\|\mathbf{L}\mathbf{x}\|^2\right\}}_{\text{Tikhonov framework}}$$

Resulting inverse operators

$$\mathbf{L} = \mathbf{I} \Rightarrow \left\|\mathbf{x}\right\|^2 \leq M_1$$

"Energy in $\mathbf{x}(\lambda)$ is bounded"

$$\mathbf{x}(\lambda) = \left(\mathbf{A}^*\mathbf{A} + \lambda^2\right)^{-1}\mathbf{A}^*\mathbf{y}$$

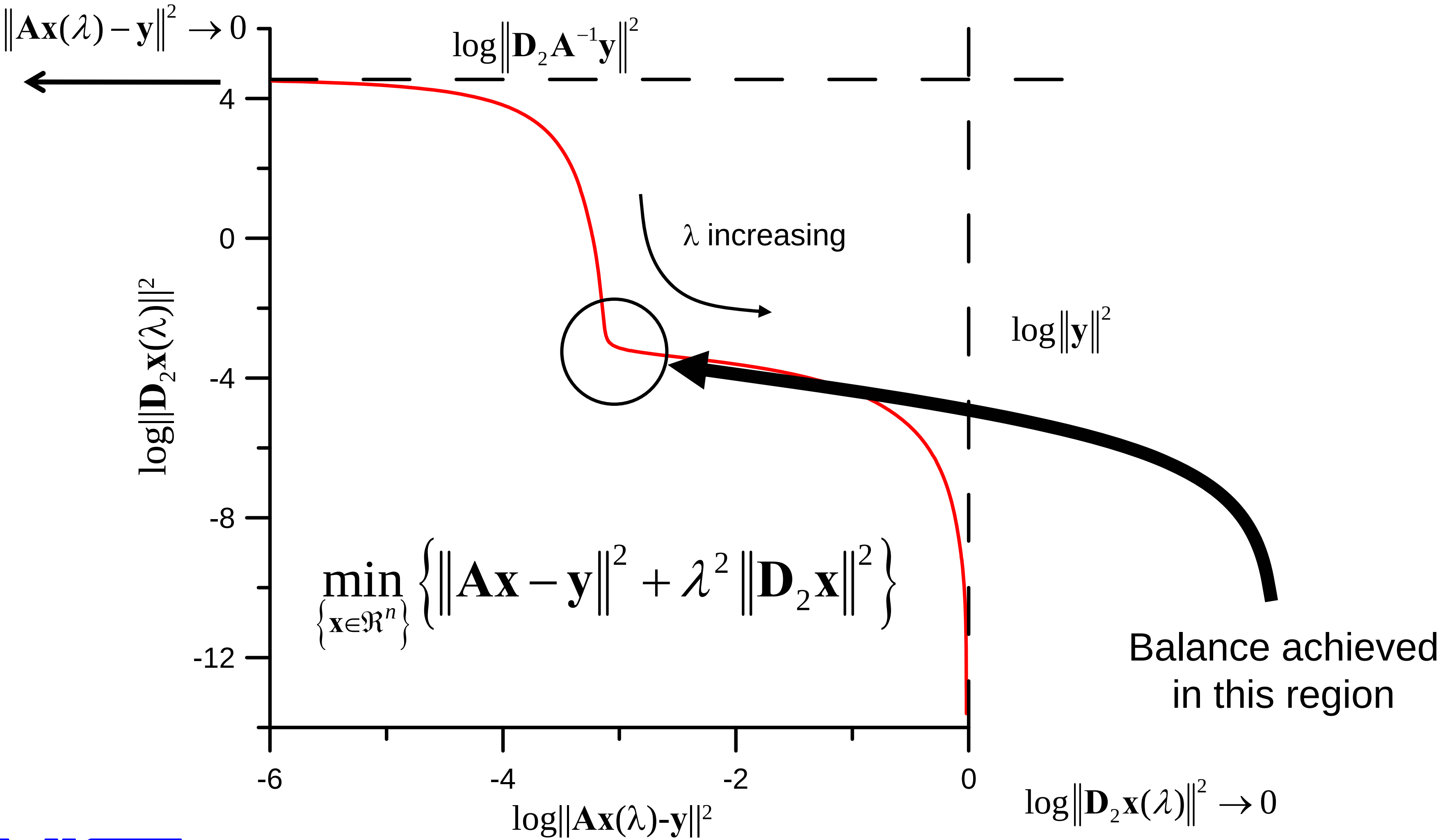
$$\mathbf{L} = \mathbf{D}_2 \Rightarrow \left\|\mathbf{D}_2\mathbf{x}\right\|^2 \leq M_2$$

"Roughness of $\mathbf{x}(\lambda)$ is bounded"

$$\mathbf{x}(\lambda) = \left(\mathbf{A}^*\mathbf{A} + \lambda^2\mathbf{D}_2^*\mathbf{D}_2\right)^{-1}\mathbf{A}^*\mathbf{y}$$

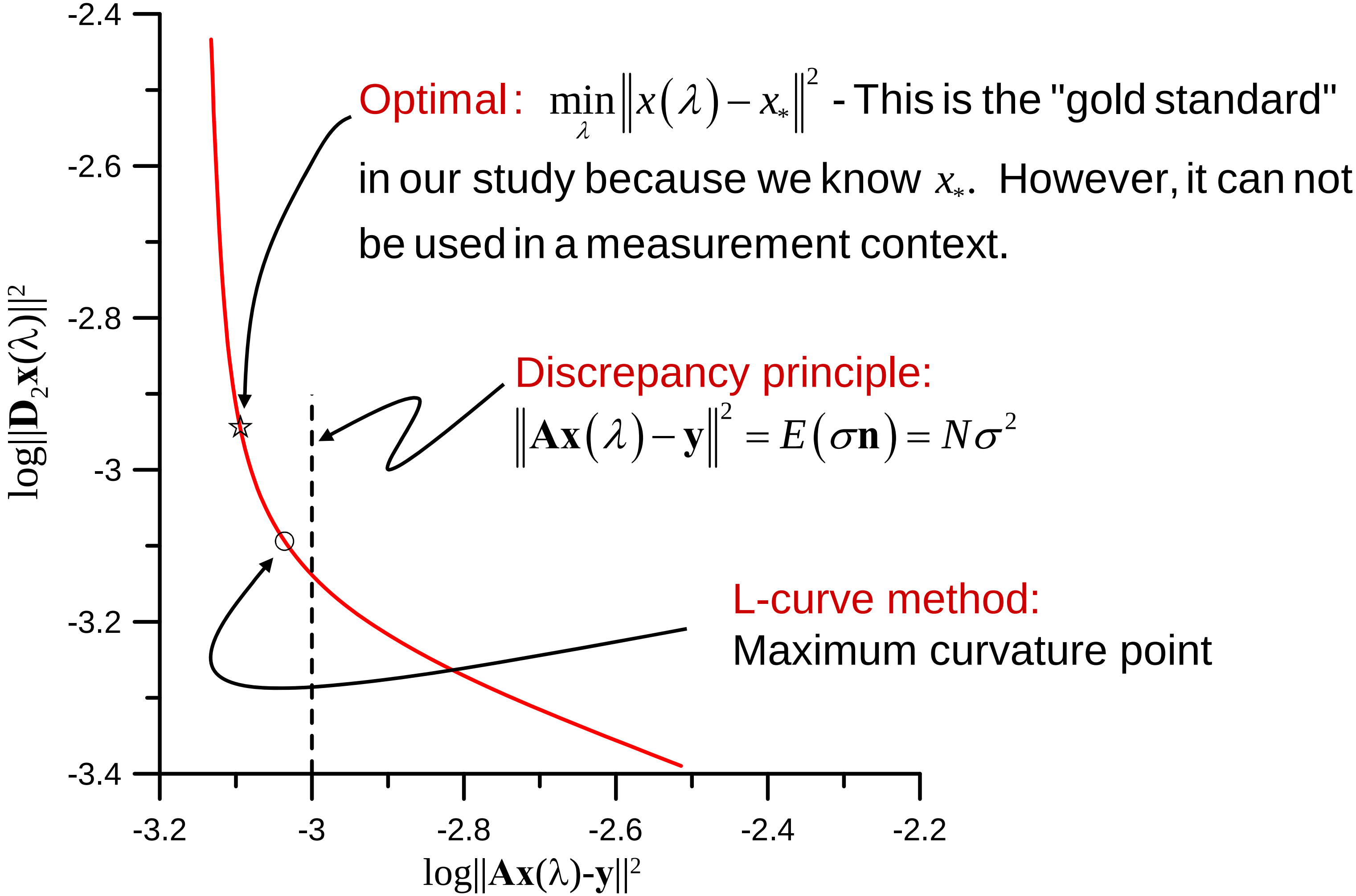
These terms serve as a low-pass noise filter.
Choice of λ determines effective cut-off.

L-curve



$\|\mathbf{Ax}(\lambda)-\mathbf{y}\|^2 \to 0$
$\log\|\mathbf{D}_2\mathbf{A}^{-1}\mathbf{y}\|^2$
4
0
-4
-8
-12
$\log\|\mathbf{D}_2\mathbf{x}(\lambda)\|^2$
λ increasing
$\log\|\mathbf{y}\|^2$
$\min_{\{\mathbf{x}\in\Re^n\}}\left\{\|\mathbf{Ax}-\mathbf{y}\|^2+\lambda^2\|\mathbf{D}_2\mathbf{x}\|^2\right\}$
-6
-4
-2
0
$\log\|\mathbf{Ax}(\lambda)\text{-}\mathbf{y}\|^2$
$\log\|\mathbf{D}_2\mathbf{x}(\lambda)\|^2 \to 0$
Balance achieved in this region

3 methods for λ selection



Simulation space

Vary ratio of durations: $\mathcal{T} = \dfrac{\tau(\mathbf{x}_*)}{\tau(\mathbf{a})}$

Vary signal to noise ratio: $\mathcal{S} = \dfrac{\|\mathbf{y}_*\|^2}{E\left(\|\sigma\mathbf{n}\|^2\right)} = \dfrac{\|\mathbf{y}_*\|^2}{N\sigma^2}$

- Scales with $\tau(\mathbf{x}_*)$ for fixed amplitude $\mathbf{x}_*$
- Scale $\mathbf{x}_*$ to keep $\|\mathbf{y}_*\|^2$ for every $\mathcal{T}$

$$\Rightarrow \mathcal{S} = \frac{1}{N\sigma^2}$$

Simulation space

- Want to stress deconvolution algorithms
  - System response = 4^{th}-order Butterworth
  - Input 2^{nd}-order Bessel-Thompson
- Sample rate fixed, although this might also be an interesting parameter
- Simulated 1000 waveform instances for each ($\mathcal{T}$, $\mathcal{S}$)

Conclusions from simulations

- RSS is a bad idea
  - Measurement can ‘speed up’ waveform
  - Pulse duration relationships <u>cannot be basis</u> for accuracy claims
- Tikhonov framework with these selectors is stable
- Two selectors appear roughly equivalent
  - L-curve slightly better, but more waveform shapes need to be studied
  - Problems with L-curve not observed

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The calibrated waveform

- A waveform is a set of ordered pairs $W=\{(t_j, x_j) \mid j=1,\ldots, J\}$
- t_j and x_j are subject to error

- For numerical analysis, the waveform is interpolated to a vector $\mathbf{X}=(x_1, \ldots , x_N)^T$, where $x_n = x(t_n)$ and $t_n = t_1 + n\Delta t$
- Each element of $\mathbf{X}$ is considered a random variable
- Correlations may exist between some or all elements of $\mathbf{X}$
- The calibrated waveform is a vector $\mathbf{Y} = f(\mathbf{X})$

- How do we express the uncertainty in vector $\mathbf{X}$**?**
- How do we propagate uncertainty from vector $\mathbf{X}$ to vector $\mathbf{Y}$?

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Transformation to Σ_Y

What is the uncertainty in $\mathbf{Y} = \left(y_1, \cdots y_N\right)^T = f(\mathbf{X})$?

$$\mathbf{Y} = f\left(\mathbf{X}\right) \approx \mathbf{X_0} + \mathbf{J}\left(\mathbf{X} - \mathbf{X_0}\right) + \ldots$$

Jacobian $$J_{ij} = \left.\frac{\partial f_i\left(\mathbf{X}\right)}{\partial x_j}\right|_{\mathbf{X_0}} = \left.\frac{\partial y_i}{\partial x_j}\right|_{\mathbf{X_0}}$$

Covariance $$\begin{aligned}\mathbf{\Sigma_Y} &= E\left(\left(\mathbf{Y} - \mathbf{Y_0}\right)\left(\mathbf{Y} - \mathbf{Y_0}\right)^T\right) \\ &\approx E\left(\left(\mathbf{J}\left(\mathbf{X} - \mathbf{X_0}\right)\right)\left(\mathbf{J}\left(\mathbf{X} - \mathbf{X_0}\right)\right)^T\right) \\ &\approx \mathbf{J\Sigma_X J}^T\end{aligned}$$

$$\boxed{\sigma_y^2 = \left(\frac{df}{dx}\right)^2 \sigma_x^2 \rightarrow \quad \mathbf{\Sigma_Y} = \mathbf{J\Sigma_X J}^{\mathrm{T}}}$$

- $f(\mathbf{X})$ can be a linear transformation, some quasilinear function, or a complicated algorithm

Covariance matrix Σ

$$\Sigma_{\mathbf{x}} = \begin{bmatrix} E\left(\left(x_1 - E(x_1)\right)^2\right) & E\left(\left(x_1 - E(x_1)\right)\left(x_2 - E(x_2)\right)\right) & \cdots \\ E\left(\left(x_2 - E(x_2)\right)\left(x_1 - E(x_1)\right)\right) & E\left(\left(x_2 - E(x_2)\right)^2\right) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$\operatorname{var}(x_i) = E\left(\left(x_i - E(x_i)\right)^2\right) = \sigma_i^2$$

$$\operatorname{cov}(x_i, x_j) = E\left(\left(x_i - E(x_i)\right)\left(x_j - E(x_j)\right)\right) = \sigma_i \sigma_j \rho_{ij}$$

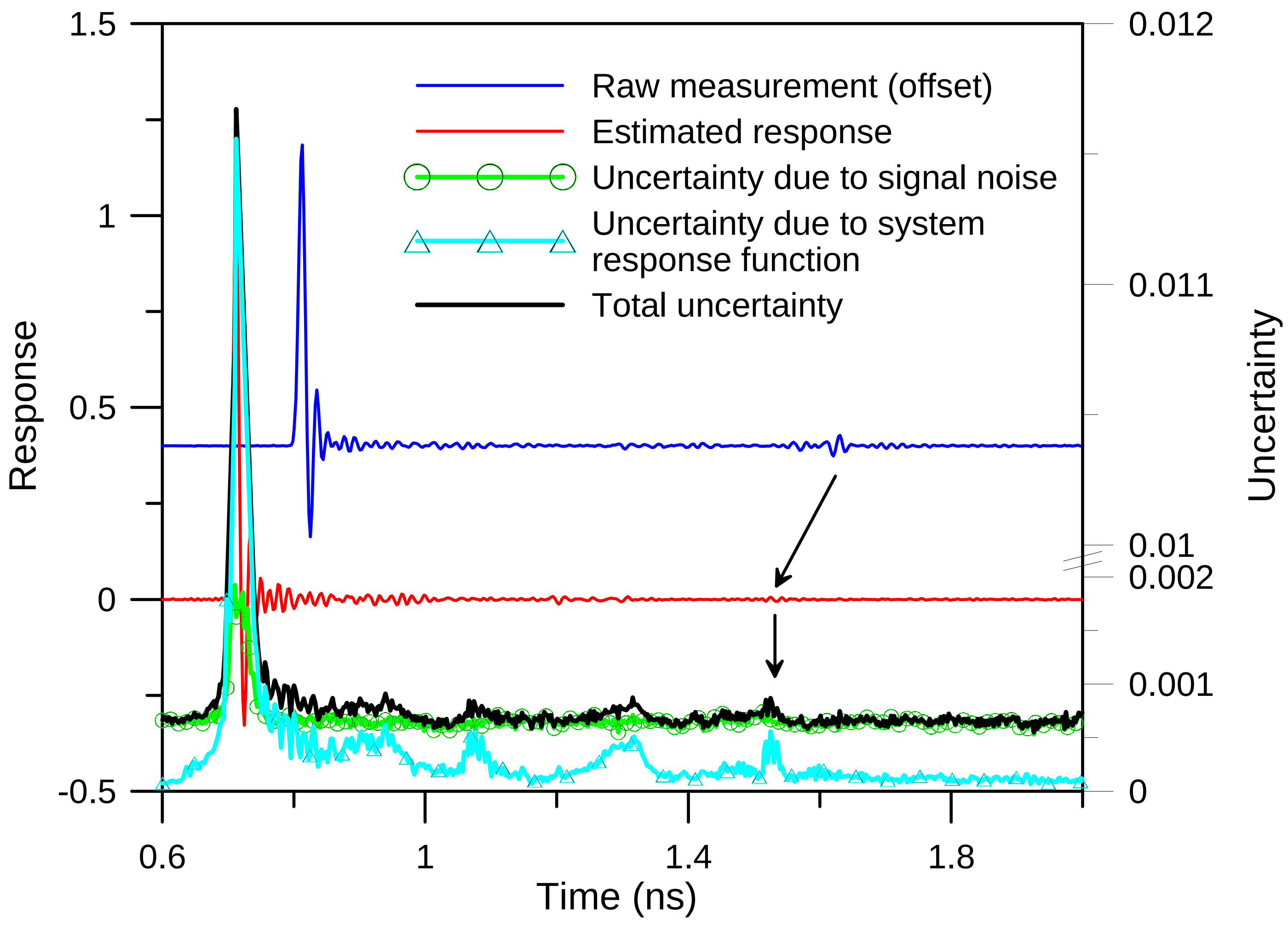
$$\text{where } E(x) = \int x p(x) dx \text{ and } p(x) = \text{pdf}$$

Oscilloscope calibration: Propagation of uncertainty

Stepping through oscilloscope calibration

- Timebase correction
- Interpolation to evenly spaced time grid
- Transformation of time-domain waveform to frequency-domain representation: $\mathbf{V}_S = \mathcal{F}\mathbf{v}_S$
- Deconvolve system response: $\mathbf{A} = \left(\dfrac{b_{PD}}{1\text{-}\Gamma_{PD}\Gamma_S} \right)^{-1} \mathbf{V}_S$
- Note: Regularization may be required for inversion
- Transformation back to time domain: $\mathbf{a} = \mathcal{F}^{-1}\mathbf{A}$
- Scalar pulse parameters, i.e., pulse transition duration, pulse amplitude, etc. $\sigma^2_{\mathcal{P}} = \mathbf{J}_{\mathcal{P}}\mathbf{\Sigma}_{\mathbf{A}}\mathbf{J}_{\mathcal{P}}^{T}$

Correlations distribute uncertainty where we expect it



Full waveform metrology includes:

1. Traceability to fundamental physics (EOS)
2. Calibrated time t
3. Calibrated voltage $v(t)$
   - Account for reflections and loss
   - Dynamic instrument response
4. Covariance matrix-based uncertainty analysis
   - Analysis of the waveform at each point in the measured epoch, along with uncertainty at each point
   - Allows propagation of uncertainty through a linear transformation
   - Fourier transforms, pulse parameters, etc.

References

NIST publications are available at the searchable web site:

- http://www.eeel.nist.gov/publications/publications.cgi

Other useful web pages include:

- http://www.nist.gov/eeel/optoelectronics/sources_detectors/measurements.cfm
- http://www.nist.gov/eeel/electromagnetics/rf_electronics/high_speed_electronics.cfm

Oscilloscope calibration

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