Multi-Frequency Approach to Vector-Network-Analyzer Scattering-Parameter Measurements

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***Abstract*—We present a multi-frequency approach to vector-network-analyzer scattering-parameter measurements. This novel approach accounts for the relationships between the measurements at different frequencies, and thus breaks with the traditional paradigm for vector-network-analyzer scattering-parameter measurements, in which the measurements are carried out independently at each frequency. We review the theoretical foundations of the multi-frequency approach, and show that it leads to a significant reduction of the measurement uncertainty and to its more complete description.**

I. Introduction

We present the theoretical foundations and the most important applications of the multi-frequency approach to vector-network-analyzer (VNA) scattering-parameter (*S*-parameter) measurements [1]. The principle of this approach is to account for the relationships between VNA measurements at different frequencies.

Our multi-frequency approach stems from the observation that *S*-parameter measurement errors at different frequencies are related to each other [2] due to some common physical mechanisms underlying them. These mechanisms include the tolerances of the dimensional parameters of the calibration standards, cable length instability, nonrepeatability of the connector interface, and the test-set drift.

The simplest mathematical representation for the relationships between measurement errors at different frequencies is given by the covariance matrix [2]. The diagonal terms of this matrix describe the variances of the measurement errors, while the off-diagonal terms capture the statistical correlations between those errors, including the correlations between errors at different frequencies. Reference [2] develops a VNA-measurement uncertainty analysis approach that allows one to determine these correlations, and demonstrates further that they are essential in the uncertainty analysis of calibrated time-domain measurement systems that employ VNA measured *S*-parameters, such as when correcting for the impedance mismatch in oscilloscope measurements.

On the other hand, the relationships between measurement errors at different frequencies are also accounted for in the VNA calibration approaches [3], [4]. These approaches use a physical model for the transmission-line propagation constant to determine the unknown lengths of the calibration standards in the sliding load and offset shorts calibration, respectively.

In this work, we generalize the results presented in [2]–[5] by developing a comprehensive multi-frequency approach to VNA *S*-parameter measurements, that is, an approach accounting for the relationships between the measurements at different frequencies throughout the entire VNA measurement procedure. We review the theoretical foundations of the multi-frequency approach and discuss its practical applications, putting particular emphasis on the VNA calibration, VNA-measurement uncertainty analysis, and device-modeling based on VNA *S*-parameter measurements.

II. Physical Error Mechanisms

The use of the covariance matrix proposed in [2], although seemingly intuitive, leads to some difficulties. The vector quantity that underlies this matrix, which we refer to as the multi-frequency *S*-parameter-measurement error, is constructed out of real and imaginary parts of *S*-parameter-measurement errors at all frequencies. Thus, in most practical cases, the columns and rows of this matrix are linearly dependent as it characterizes the variability of a large number of random variables contained in the multi-frequency *S*-parameter-measurement error, which are dependent on a much smaller number of other random variables corresponding to the fundamental causes of the measurement errors [2]. Consequently, the covariance matrix is rank deficient and cannot be inverted; hence the conventional form of the multivariate Gaussian

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probability density function (see [6]) cannot be used with the multi-frequency *S*-parameter-measurement error. As a result, applications where the knowledge of the probability density function is required, such as statistical estimation procedures based on the maximum-likelihood criterion, commonly used in VNA calibration methods (*e.g.*, [7]–[9]) and measurement-based device modeling (*e.g.*, [10], [11]), cannot be directly extended to use the covariance matrix of [2].

Therefore, in order to remedy this mathematical difficulty, we followed a different approach. Instead of directly employing the covariance matrix of the multi-frequency *S*-parameter measurement error, we focused on the fundamental causes of the measurement errors which determine the structure of this matrix. A mathematical model for these causes is given by the notion of the physical error mechanism [2]. We define a single physical error mechanism as a scalar random variable corresponding to a physical parameter that is affected by errors, and a corresponding frequency-dependent function which characterizes the relationships between this parameter and the multi-frequency *S*-parameter measurement error. In the following, we show how to incorporate this concept into the description of the VNA measurement procedure.

III. Multi-Frequency VNA Calibration

As the physical error mechanisms contribute simultaneously to the measurement errors at all frequencies, the VNA calibration problem needs to be, in general, solved jointly at all measurement frequencies. In such a formulation of the VNA calibration problem, the inconsistency of the calibration equations is characterized in terms of the physical error mechanisms, instead of errors in *S*-parameters. We refer to this generalized physics-based formulation of the VNA calibration problem as the error-mechanism-based VNA calibration [1].

Practical implementation of the error-mechanism-based VNA calibration leads, however, to some difficulties. It requires, on one hand, detailed modeling and characterization of all of the physical error mechanisms responsible for the VNA measurement errors. On the other hand, the error-mechanism-based VNA calibration results in an optimization task that is difficult to solve due to its nonlinear and ill-posed character, and large scale.

The development of an error-mechanism-based description of VNA measurement errors requires modeling of the error mechanisms affecting the calibration standards and the VNA itself. Here, we focus only on the error mechanisms affecting the calibration standard. In the case of coaxial transmission lines, employed as calibration standards in this work, we model those mechanisms based on a detailed analysis of possible errors in the description of the standards. This analysis results in extended models of the lines that account for phenomena such as errors in the determination of the inner and outer conductor length, variation of conductor diameters and conductor loss among the lines, nonuniformity of the conductor diameters, and nonreproducibility of the center conductor gap [1], [5].

The large scale of this optimization task results from the fact that the VNA calibration coefficients are sought simultaneously at all measurement frequencies. Consequently, direct solution of the error-mechanism-based VNA calibration problem is very time-consuming and may be ill-conditioned. Hence, we develop a robust iterative numerical approach that exploits the relationships between the sought parameters in order to reduce the dimensionality of the optimization task. Our approach is based on a modified version of the classical Levenberg-Marquardt algorithm [12], in which we account for the sparse structure of the Jacobians of the residuals [1].

The ill-posed character of the error-mechanism-based VNA calibration, manifesting itself with difficulties in obtaining a unique solution, results from the fact that some of the estimated parameters are related to each other. We analyze the origins of those relationships and devise a general methodology for assuring the identifiability of the solution. This methodology relies on restricting the space of possible solutions with a set of linear equality constraints, based on some intuitive statistical properties of the physical error mechanisms [1], [5].

We tested the resulting multi-frequency calibration algorithm in the context of the coaxial multiline TRL calibration (see [13]) with the 1.85 mm coaxial transmission-line standards. The resulting algorithm, however, is generic and can easily be adapted to other redundant calibration schemes (such as [3], [4]), and other waveguide types. In our implementation, we account only for the error mechanisms affecting the calibration standards and use a conventional description of the VNA instrumentation errors. As an example, in Figure 1 we show the corrected measurement of the reflection-coefficient magnitude for a 5.4 mm long female offset-open along with the standard uncertainties, as obtained from our method and from [13]. The standard uncertainties were estimated by use of the residual analysis [14]. We see that both calibration methods deliver comparable results; however, the uncertainties of our new method are at least twice as small. This result clearly demonstrates the benefit of accounting for the relationships between measurements at different frequencies in the VNA calibration.

IV. Multi-Frequency VNA-Measurement Uncertainty Analysis

We introduced the multi-frequency uncertainty analysis for VNA *S*-parameter measurements in [2], and here we shall only summarize its key concepts. The principle of this analysis is to characterize all of the measurement uncertainties and possible statistical correlations between them, including the correlations between uncertainties at different frequencies. To this end, we represent the *S*-parameter measurement uncertainty with a multi-frequency covariance matrix as discussed earlier on. In order to determine this matrix, we identify and characterize the physical error mechanism underlying the VNA *S*-parameter measurement errors. We then evaluate the impact of each individual error mechanism on the measurement uncertainty

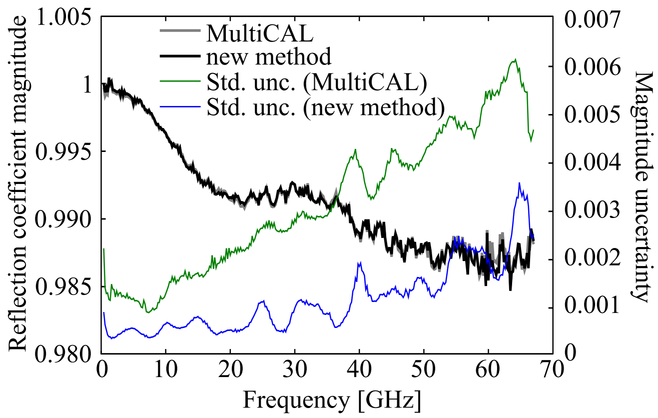

Fig. 1. Corrected reflection-coefficient magnitude for a 1.85 mm coaxial 5.4 mm long female offset-open along with the standard uncertainty from the residual analysis.

by use of linear error propagation, and from that construct the covariance matrix [2].

We divide the physical error mechanisms responsible for the VNA *S*-parameter-measurement errors into the error mechanisms affecting the calibration standards and the VNA itself. In the case of coaxial transmission lines, employed as calibration standards in this work, we used models of those mechanisms developed in the context of the multi-frequency VNA calibration (see Section III).

Regarding the modeling of error mechanisms affecting the VNA itself, we focus on the VNA-nonstationarity errors, that is, the connector nonrepeatability, cable instability, and test-set drift, which are the primary sources of VNA instrumentation errors. We neglect the VNA receiver errors, which for most modern VNAs are much smaller then VNA-nonstationarity errors. To describe the VNA-nonstationarity errors, we propose a novel approach that uses a stochastic model whose parameters are identified from repeated *S*-parameter measurements. The core of this model is a generic description for the frequency dependence of the VNA-nonstationarity errors, which uses a set of lumped element perturbations, added to the two-port describing the VNA calibration coefficients [15]. The stochastic models for the VNA-nonstationarity errors are then established by allowing the parameters of the lumped elements to vary randomly.

In the case of the connector repeatability and cable instability errors, we assume that the parameters of these elements vary according to the multivariate Gaussian probability density function [16]. This assumption has a simple physical justification. The lumped elements model the impact of changes in some dimensional parameters on the electrical properties of the connector interface or the cable, and we can reasonably assume those changes to be small and to follow the multivariate Gaussian probability density function. The electrical parameters are, therefore, approximately linear combinations of the dimensional parameters, and also posses the multivariate Gaussian probability density function.

As for the test-set drift, we follow the approach used in the drift modeling of scalar measurement instruments, and assume that it can be described as random walk phenomenon, referred to also as Brownian motion [17], [18]. The mathematical description of this phenomenon for the scalar case is given by the stochastic Wiener process [19]. We generalize this description to the multidimensional case and then apply it to model the variability of the lumped elements in the perturbations [1], [2].

We further show in [2] that the statistical correlations between measurement errors at different frequencies are essential when evaluating the uncertainties in calibrated time-domain measurements that employ the VNA *S*-parameter measurements. Examples of such time-domain measurements are high-speed oscilloscope measurements with a correction for the impedance mismatch or for the influence of an adapter.

In order to better illustrate the importance of these correlations, we show results of a simple analysis. We simulated the uncertainty analysis of the propagation of a 1 ns wide Gaussian pulse through a 1.85 mm coaxial adapter. The adapter was characterized by use of a 1.85 mm coaxial multiline TRL calibration. The covariance matrix of uncertainties in adapter *S*-parameters was determined with the use of methods discussed in [2]. We simulated two cases: when the uncertainties in the output pulse are determined accounting for the correlations between uncertainties for different frequencies, and when these correlations are neglected. Results of our simulation are shown in Figure 2. We see that when accounting for the statistical correlations between the uncertainties, the standard uncertainty (solid gray line) in the corrected waveform (solid black line) closely follows the shape of the waveform. However, when neglecting these correlations, the uncertainties (dashed gray line) are uniformly spread over the duration of the pulse. Consequently, the uncertainties away from the pulse peak are overestimated, while the uncertainties around the pulse peak are underestimated. Experimental results demonstrating the same behavior and involving oscilloscope measurements of short pulses generated by a photodiode are presented in [2].

V. Multi-Frequency Measurement-Based Device-Modeling

In measurement-based modeling, we determine an electrical model of a device based on a measurement of its electrical characteristics, such as a wideband VNA *S*-parameter measurement. Examples include modeling of active devices, such as microwave transistors, and passive devices, such as transmission line discontinuities [11].

The most common approach used in the device modeling is based on the maximum-likelihood estimation technique [10]. The key concept of this approach is the likelihood function which is determined based on the probability density function of the measurement errors. This function is used to assign a single scalar metric to the misfit between the model and the measurements. By maximizing this metric, the optimal set of model parameters is then found [10].

In order to account for the relationships between the measurement errors at different frequencies in this approach, we need therefore to first formulate the probability density

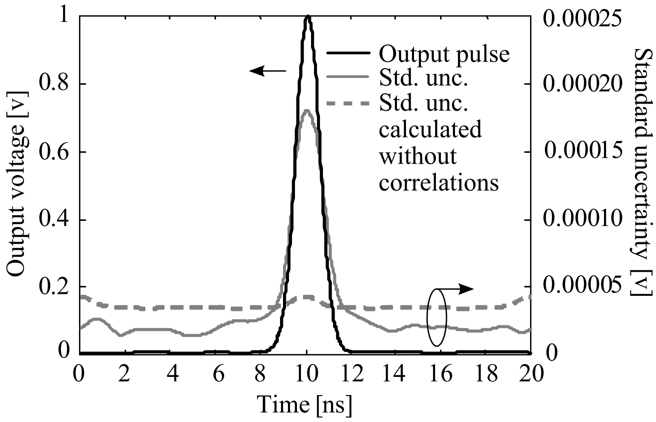

Fig. 2. Propagation of a 1 ns Gaussian pulse through a 1.85 mm coaxial adapter.

function for the multi-frequency *S*-parameter measurement error. In [1] it is shown that such a probability density function can be constructed with the use of the physical-error-mechanism concept and by applying the generalized matrix-pseudo-inverse (see [20]). This function turns out to have an intuitive form in which the probability for a given region in the domain of *S*-parameters is expressed in terms of the corresponding region in the domain of the physical error mechanisms [1].

Equipped with this error-mechanism-based representation of the multi-frequency *S*-parameter-measurement-error, we further reformulate the maximum-likelihood estimation technique. The key conclusion of our analysis concerns the quantification of the misfit between the measurement and the model. In standard methods, which do not account for the statistical correlations between *S*-parameter measurement errors at different frequencies, this misfit is quantified in terms of *S*-parameters. We showed that when accounting for these correlations, the misfit needs to be expressed in terms of the physical error mechanisms [1].

VI. Conclusions

In this work, we discussed the theoretical foundations and practical application of the multi-frequency approach to VNA *S*-parameter measurements. This approach introduces a new paradigm into VNA *S*-parameter measurements by accounting for the relationships between measurements at different frequencies. We demonstrated that exploiting these relationships significantly improves the accuracy of the VNA calibration. Furthermore, we showed that these relationships are essential when propagating the VNA measurement uncertainties into the time-domain, such as in the case of calibrated time-domain measurement systems. Finally, we introduced a generalized procedure for device-modeling based on *S*-parameter measurements that accounts for these relationships.

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