# Power Spectral Density: Is it right?

**J. P. Lehan,[1,2] Ulf Griesmann,[3] and Jiyoung Chu[3,4]**
[1]*Department of Physics, University of Maryland, Baltimore County, 1000 Hilltop Circle, Baltimore MD 21250, USA*
[2]*NASA Goddard Space Flight Center, Code 661, Greenbelt MD 20771, USA*
[3]*National Institute of Standards and Technology, 100 Bureau Drive, Gaithersburg MD 20899, USA*
[4]*Current address: Samsung Electronics Co., 416, Maetan 3-dong, Yeongtong-gu, Suwon-si, Gyeonggi-do, South Korea*

*Corresponding author:* jolehan@umbc.edu

**Abstract:** We concentrate on the instrumental issues surrounding power spectral density (PSD) determination, using as an example, the most common optical shop QA tool, the Fizeau interferometer. We briefly discuss the properties of an ideal calibration method for PSD and some of the methods that have been used for this task. Finally, we discuss the method we have been using and some general rules for obtaining better PSD data.



**OCIS codes:** (120.0120) Instrumentation, measurement, and metrology;

Power spectral density (PSD) is a statistical measure of an optical surface that is useful in predicting the scattering behavior of a single optical surface. It also proves useful in the system engineering of optical systems, yielding budgetary information for flow-down of top-level optical performance to subsystem and component level. This practice is has gained acceptance in engineering space flight systems like JWST, IXO, and JDEM. A consequence of this is that many components are being specified to optical fabricators in terms of PSD. To respond to this, interferometer vendors have added PSD to their analysis menus. Although the calculation of PSD seems simple, getting accurate results is anything but.

Several groups have explored the spatial frequency response of interferometers. Early work was performed with transmission gratings in dichromated gelatin.[1,2] Later studies employed a single step [3,4] function. An improvement over the single step was a pseudo-random pattern square-step pattern yielding a flat PSD over some spectral band.[5,6] An alternate approach aimed at yielding a "picket-fence" PSD over a spectral band of interest.[7] Each of these methods operate under differing assumptions and have different strengths and weaknesses.

For a Fizeau interferometer in an optical shop, ideally, one would want to have a calibration method that meets all of the following criteria: 1. it has a PSD (Fourier-space signature) very different from the log-log linear signature of most optics; 2. it uniformly samples spatial frequency space (over some bandwidth of interest); 3. it uniformly samples object space (over the field of view of the system); 4. it is accurate and verifiable; 5. it is easily interpreted; 6. it is flexible (can be used in multiple testing configurations); and 7. it is fast.

None of the methods outlined above meet all of these criteria. As a result, the method one wants to use depends upon the application. Criteria 2 and 3 are nearly mutually exclusive and result in significant compromises. Criterion 7 also can result in significant compromise.

The various methods fall into two rough groups that depend upon a fundamental starting assumption. This is whether the system performance is field invariant or not. An ideal interferometric system is field invariant, however, this is rarely achieved. If the system’s spatial frequency performance where limited by a set, known, band-limited, transfer function, a single analytic correction could be applied to all the data. In the presence of optical aberrations but still in the field-invariant assumption, a single measurement is all that is required since the spatial frequency behavior is assumed to be the same over the field of view of the instrument. Such a case might be where only spherical aberration is present. Under the more-general field-variant assumption, one still has to assume a “field-invariant patch” size and measure various spatial frequency samples in this region.

In our experiments to develop a interferometric calibration procedure, we have adopted a field-varying assumption and a $\pi/6$ azimuthal patch size with the underlying assumption that the system under test is nominally rotationally-symmetric. With these assumptions, the data needed for calibration are gathered in three exposures with the calibration sample rotated by π/6 between exposures. The spatial frequencies vary radially and contain different fundamental frequencies in adjacent azimuthal sectors at a given radius.[7]

The calibration article has a binary pattern. This complicates the analysis somewhat but has the advantage that a larger number of spatial frequencies are covered for each region since the fundamental and harmonics are sampled simultaneously. In addition, the lithography is accurate and relatively simple since only a single mask is needed.

By employing the mask used to fabricate the calibration sample and the sample itself, we can measure the modulation transfer function (MTF) and the height transfer function (HTF) of our interferometer. The MTF is the image-contrast/object-contrast vs. spatial frequency. The HTF is the object-height/image-height vs. spatial frequency.[8] Unlike the MTF, however, the numerator and denominator are not self-normalized for the HTF.

Fig.1 shows the measured MTF and HTF along with the modeled ideal behavior for a research interferometer using our square wave targets. Note that the two quantities track each other but are not the same. They do, however, contain the same amount of information about the system spatial-frequency behavior. Thus an MTF measurement can be a substitute for an HTF measurement.

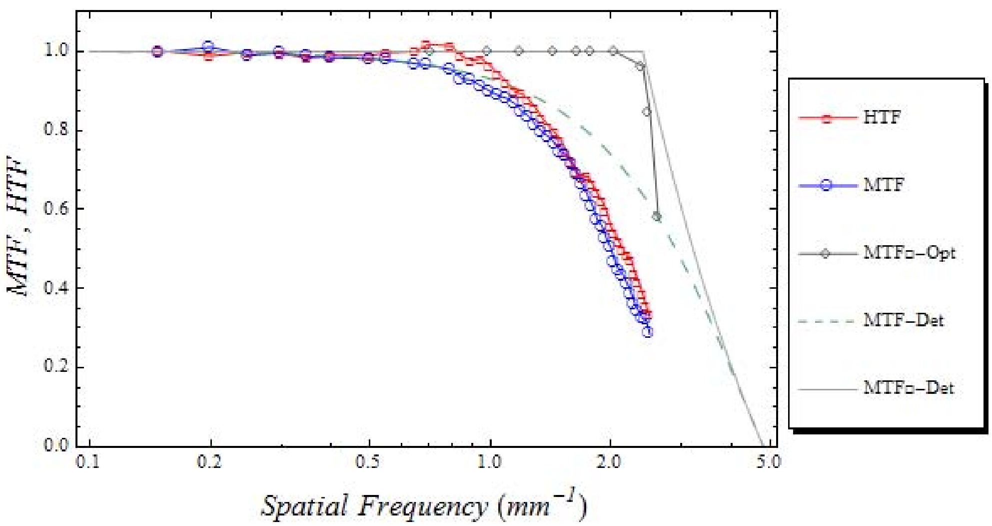


Figure 1: Measured meridonal (radial) MTF and HTF for a research interferometer near the optical axis (paraxial) along with the modeled optical square-wave MTF (MTF□-Opt), ideal detector square-wave MTF (MTF□-Det), and the ideal sine wave MTF (MTF-Det) for comparison.

As is well known in imaging systems, aberrations in the optical system very quickly degrade the MTF performance. The same is true for the HTF in interferometers. Most of the aberrations are controlled by the interferometer designer and manufacturer. The primary aberrations in the user's control are defocus and tip/tilt. Accurate pupil imaging is critical to good spatial frequency response of the measurement. This proves difficult in many instances because changes in the test set-up changes aberrations like field curvature. In addition, determining the best focus is complicated if spatially-coherent light is employed as in most modern phase-shifting Fizeau interferometers. Another complication is that the featureless objects of most optical tests do not facilitate focusing.

Unfortunately, the calibration is also only valid for a single configuration. Changes in zoom, test configuration, focus, and even tip and tilt will invalidate the calibration to more or less degree. Additionally, calibration cannot restore regions of the field where aberrations have resulted in nearly zero response for a spatial frequency region of interest.

# OWE1.pdf

Careful design of testing configurations is critical to assure good pupil imaging if spatial frequency information well beyond the lowest order form errors is desired. The use of MTF transparencies at the object or in a conjugate plane to assure the entire field is in focus is also important for insuring the best spatial-frequency performance of the measurement.

[1] E. Novak et al., "Optical resolution of phase measurements of laser Fizeau interferometers", Proc. SPIE **2870**, 545-552 (1996).
[2] E. Novak et al., "Transfer function characterization of laser Fizeau interferometer for high spatial frequency phase measurements", Proc. SPIE **3134**, 114-121 (1997).
[3] Dan Pantzer, "Step response and spatial resolution of an optical heterodyne profiling instrument," Appl. Opt. **26**, pp. 3915-3918 (1987).
[4] Takacs, P. Z., Li, M. X., Furenlid, K. and Church, E. L., "Step-height standard for surface-profiler calibration," Proc. of SPIE **1995**, 235-44 (1993).
[5] Yashchuk, V. V., McKinney, W. R. and Takacs, P. Z., "Binary Pseudorandom Grating as a Standard Test Surface for measurement of Modulation Transfer Function of Interferometric Microscopes," Proc. SPIE **6704**, 670408-1-12 (2007).
[6] Yashchuk, V. V., McKinney, W. R. and Takacs, P. Z., "Binary Pseudorandom Grating Standard for Calibration of Surface Profilometers," Opt. Eng. **47**, 073602-1-5 (2008).
[7] J. Chu, Q. Wang, J. P. Lehan, G. Gao, and U. Griesmann, "Measuring the phase transfer function of a phase-shifting interferometer," Proc. SPIE **7064**, 70640C-1-8 (2008).
[8] B. Dörband and J. Hetzler, "Characterizing lateral resolution of interferometers: the Height Transfer Function (HTF)," Proc. SPIE **5878**, 587806-1-12 (2005).