2010 Conference on Precision Electromagnetic Measurements
June 13-18, 2010, Daejeon Convention Center, Daejeon, Korea

Behavior of $\epsilon'(\omega)$ and $\tan\delta$ for a Class of Low-Loss Materials

James Baker-Jarvis, Michael D. Janezic, Bill Riddle, Sung Kim
National Institute of Standards and Technology 325 Broadway, Boulder, CO 80305*

Abstract

We report a study on the relaxation behavior of the real part of the permittivity. We also discuss the loss tangent of a class of materials in the microwave to millimeter band of frequencies. For relaxation response we show that the permittivity is a monotonic decreasing function of frequency. Also, for many low-loss ceramics, glasses, crystals, and solid polymers we found that the loss tangent increases nearly linearly with frequency. This linearity is explained in terms of the pulse-response function and the Sparks-King-Mills model. We show that the linearity may be used to extrapolate the loss tangent beyond the measurement band.

Permittivity in Relaxation

The permittivity is related to the pulse-response function $f(t)$[1] by

$$\epsilon(\omega) = \epsilon_\infty + [\epsilon_s - \epsilon_\infty] \int_0^\infty e^{-i\omega\tau} f(\tau) d\tau. \quad (1)$$

We can expand the exponential when $\omega\tau < 1$ to express the permittivity $\epsilon(\omega) = \epsilon_0(\epsilon'(\omega) - j\epsilon''(\omega))$ in terms of moments of the pulse-response function

$$\epsilon(\omega) = \epsilon_\infty + [\epsilon_s - \epsilon_\infty] \sum_{n=0,2..}^{\infty} (-1)^{n/2} < x^n > \omega^n$$
$$-i[\epsilon_s - \epsilon_\infty] \sum_{n=1,3,...}^{\infty} (-1)^{(n-1)/2} < x^n > \omega^n, \quad (2)$$

where $< x^0 >= 1$, ϵ_s is the static permittivity, ϵ_∞ is the optical permittivity, ϵ_0 is the permittivity of vacuum, and

$$< x^n >= \frac{1}{n!} \int_0^\infty \tau^n f(\tau) d\tau. \quad (3)$$

The real part of the permittivity is an expansion in terms of even moments and the odd part of the permittivity is an expansion in terms of odd moments. We see that at low frequencies, $\epsilon'(\omega)$ is approximated by $< x^0 >$ and $< x^2 >$. In this approximation, the permittivity will decrease.

*U.S. Government work, not subject to U.S. copyright

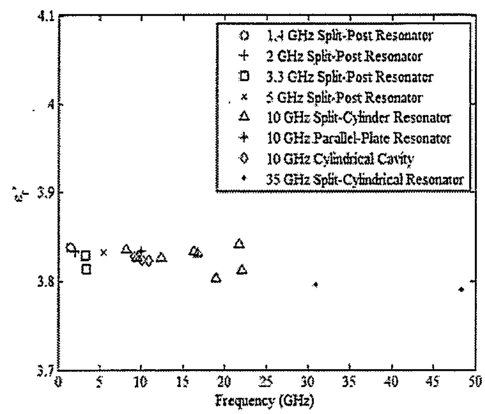

Figure 1: ϵ_r' of fused silica measured by various fixtures at different frequencies.

Additional evidence to support this behavior is obtained by considering $d\epsilon'(\omega)/d\omega$[2]. The distribution of relaxation times (DRT) model[1] has been used extensively to model dielectric relaxation in terms of the positive definite distribution function $y(t)$, where $\int_0^\infty y(\tau)d\tau = 1$, and

$$\epsilon'(\omega) = \epsilon_\infty + (\epsilon_s - \epsilon_\infty) \int_0^\infty \frac{y(\tau)}{1+\omega^2\tau^2} d\tau, \quad (4)$$

so that

$$d\epsilon'(\omega)/d\omega = -2(\epsilon_s - \epsilon_\infty)\omega \int_0^\infty \frac{\tau^2 y(\tau)}{(1+\omega^2\tau^2)^2} d\tau. \quad (5)$$

This shows that $d\epsilon'(\omega)/d\omega < 0$, with a maximum at $\omega = 0$. The DRT model reduces to eq.(2) when $< x^n >= \int y(\tau)\tau^n d\tau$. Therefore DRT is a low-frequency approximation to eq.(2). One example for a material with low loss is plotted in Fig. 1, where we see a slow decrease in permittivity as a function of frequency. To summarize, linear, homogeneous materials that exhibit only relaxation behavior in the the radio frequency (RF) through millimeter bands, the real part of the permittivity decreases as the frequency increases. This information is useful when interpreting measurement results. The permittivity increases only near material resonances and these occur only at higher frequencies.

The Loss Tangent of Some Low-Loss Materials

978-1-4244-6794-5/10/$26.00 ©2010 IEEE

Measurements show that the loss tangent in the microwave and millimeter bands of many low-loss ceramics, fused silica, and many solid polymers and glasses, increases nearly linearly as frequency increases[3, 4]. The origin of loss in this regime can be modeled by the two-phonon difference model of Sparks-King-Mills[4]. Typical examples of this behavior are shown in Figs. 2 and 3.

This linear behavior can be used to estimate $\tan\delta$ at frequencies beyond the measurement band. This was also noted by Zuccaro et al. [4] for lanthanum aluminate, where they extrapolated two orders of magnitude of frequency out of band. As an example, our specimen of fused silica was measured at 60 GHz with a Fabry-Perot resonator to obtain $\tan\delta = 5.75 \times 10^{-4}$, and an extrapolation of the data in Fig.2 predicts 5.80×10^{-4}.

We can obtain a linear frequency increase for $\tan\delta$ by use of the first term in the moment expansion in eq.(2): $\tan\delta \approx \omega(1 - \frac{\epsilon_\infty}{\epsilon_s}) < x^1 >$. We see that, in a linear expansion, the coefficient of ω is a constant.

Jonscher [5] argued that the susceptibility χ, defined through $\epsilon(\omega) = \epsilon_\infty + \chi(\omega)$ yields $\chi''/\chi' = C$ that is constant with frequency and is a universal behavior in disordered solids. Because in materials with low loss χ' decreases monotonically with increasing frequency, the Jonscher model would predict that $\tan\delta = \epsilon''/\epsilon' = C/(\epsilon_\infty/\chi' + 1)$, would be nearly constant or would decrease with increasing frequency. Although the Jonscher model applies to disordered high-loss materials at lower frequencies, it does not apply to the class of low-loss materials studied in this paper.

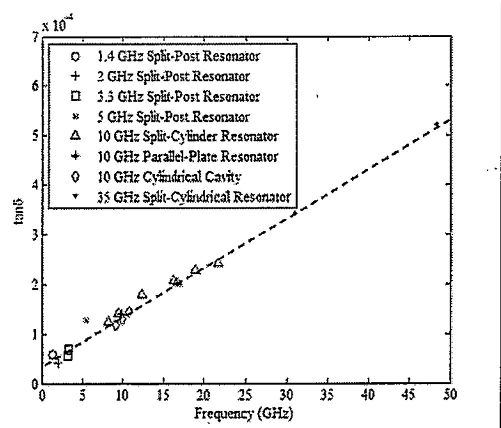

Figure 2: The loss tangent of fused silica measured by various fixtures at different frequencies.

Discussion

$\epsilon_r'(\omega)$ in the RF through millimeter bands must decrease as frequency increases. Measurements of many ceramics, glasses, and polymers exhibit

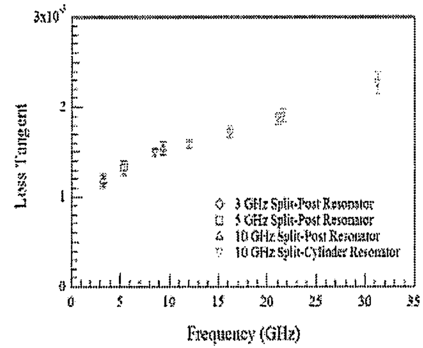

Figure 3: The loss tangent of a ceramic measured by various fixtures at different frequencies.

a loss tangent that increases approximately linearly with frequency, as shown in Fig.2. For many dielectric low-loss materials, Gurevich[6] showed that a universal frequency and temperature response is of the form $\tan\delta \propto \omega T^2$. The Sparks-King-Mills model also predicts this linear increase. However, the Jonscher model predicts that $\chi'(\omega) \propto \chi''(\omega) \propto \omega^{n-1}$, which implies χ''/χ' is independent of frequency. Therefore, the Jonscher model does not apply to many low-loss materials.

References

[1] C. J. F. Bottcher and P. Bordewijk, *Theory of Electric Polarization*, vol. I and II. New York: Elsevier, 1978.

[2] M. Coffey, "On the generic behavior of the electrical permittivity at low frequencies," *Phys. Lett. A.*, vol. 355, pp. 255–258, 2009.

[3] J. Baker-Jarvis, M. D. Janezic, B. Riddle, C. L. Holloway, N. G. Paulter, and J. E. Blendell, *Dielectric and conductor-loss characterization and measurements on electronic packaging materials.* July 2001. NIST Tech. Note 1520.

[4] C. Zuccaro, M. Winter, N. Klein, and K. Urban, "Microwave absorption in single crystals of lanthanum aluminate," *J. Appl. Phys.*, vol. 82, p. 5625, 1997.

[5] A. K. Jonscher, *Dielectric Relaxation in Solids.* London: Chelsea Dielectrics Press, 1983.

[6] V. L. Gurevich and A. K. Tagantsev, "Intrinsic dielectric loss in crystals," *Advances in Physics*, vol. 40, no. 6, pp. 719–767, 1991.

2010 Conference on Precision Electromagnetic Measurements

DIGEST

June 13-18, 2010
Daejeon, Korea

Editor: Yang Sup Song
Associate Editor: Jin-Seob Kang