How often and how long should a cognitive radio sense the spectrum?

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***Abstract*—This paper presents a Bayesian framework and a pricing structure for a secondary wireless user that opportunistically uses a RF channel licensed to a network of N primary users. The secondary user operates in a time-slotted fashion, where each time slot consists of observing the channel for D seconds followed by possibly using it for W seconds depending on the decision the user makes after observing the channel. The paper assumes the secondary user observes the on-off Markov process modeling the primary user activity corrupted by additive white Gaussian noise, and it employs a decision rule that is a time-averager followed by a threshold device. The pricing structure includes rewards for the secondary user when it uses the channel without interfering with the primary users and penalties when it does so and it interferes. The paper derives a formula for the average per unit time net profit of the secondary user. Numerical results are presented that show the behavior of the maximum profit of the secondary user, its throughput, and the resulting level of interference to the primary users as functions of various network parameters.**

I. Introduction

The volume of wireless communications has dramatically increased over the past two decades and this trend is expected to continue with new and increasingly bandwidth-hungry mobile applications and services introduced every day. Wireless service providers typically argue that they need more radio frequency (RF) spectrum and lobby spectrum regulatory agencies to release more spectrum so that they can cope with rising data traffic demands. On the other hand, there is ample evidence [1] that suggests that much of the licensed spectrum is underutilized. Of course, the degree of underutilization varies greatly depending on the frequency band, location, and time, but it appears that at most locations at any given time opportunities exist to use some underutilized frequency bands. These observations and the increasing importance of software-defined radios led Mitola and Maguire [2] to propose the concept of a cognitive radio network (CRN) in 1999. A number of survey papers on the subject have been published since then ([3], [4], [5]) that cover the progress made in this field as well as open problems and challenges.

The license holders of any frequency band that might be regarded as a good candidate for opportunistic spectrum access (OSA) are understandably concerned that any opportunistic use of their spectrum would lead to a degradation of their radio communications or wireless services they provide over that band. Perhaps in certain circumstances, e.g. emergency response in the aftermath of large-scale natural or man-made disasters or when national security is at risk, these concerns are trumped by more important priorities and opportunistic use of any spectrum is justified. However, in the absence of such conditions, there has to be a balance between empowering unlicensed users to use an underutilized RF band and not noticeably degrading the radio communications of the licensed users. In fact, this is by far the most major obstacle to viability of OSA and deployment of CRNs.

There has been some work on developing techniques for unlicensed, henceforth called secondary, users to characterize, quantify, and manage interference to the licensed, henceforth called primary, users when a secondary user (SU) decides to opportunistically use the licensed spectrum. This problem is trivial in certain scenarios. For example, a SU could in principle use the spectrum allocated for television broadcasting when TV is off air, because TV broadcast schedules in a given geographic region are well known and widely advertised. This is an example of *spatial OSA*. Although technically more challenging, a SU might be allowed to transmit over the TV bands even when TV broadcasting is on, and this is the subject of a recent FCC ruling on the use of the so-called “white spaces” by unlicensed users [6]. The problem would then be that of detecting the presence of TV sets (receivers) in the vicinity of a prospective secondary transmitter. An early work on this topic [7] uses the RF energy leaked from TV local oscillators to detect their presence. *Temporal OSA* is the other type of OSA that arises when SUs try to exploit a licensed spectrum during very short periods of time when primary users (PUs) are not using the spectrum. This problem is in general quite challenging, because there is usually no coordination between PUs and SUs. At best, the latter have a statistical characterization of the primary system traffic. Hidden and exposed terminal problems, shadowing, fading, and other types of RF propagation effects [8] are some of the problems that have to be dealt with. Yet, a number of researchers have worked on this problem under various settings. [9] classifies classic detection problems that could be used for spectrum sensing into three categories: matched filter, energy detector, and feature detector. One example of spectrum sensing using feature detection is [10]. A more promising approach is collaborative / cooperative spectrum sensing where all users in a secondary network sense the licensed spectrum and arrive at a more reliable picture of spectrum occupancy through use of centralized or preferably distributed detection algorithms [11], [12]. Naturally, these techniques require more resources and have a higher overhead.

Of more relevance to our work are papers that consider not just the spectrum sensing aspect but also the benefit to the SU as a result of using the spectrum. Some work has been done on pricing, economic models, and applications of game theory to CRNs. For example, [13] proposes a game-theoretic model for analyzing the behavior and dynamics of a CRN. [14] uses game theory to analyze a scenario where multiple primary service providers compete with each other to offer spectrum opportunities to SUs with the goal of maximizing their profits subject to constraints on the quality of service (QoS) they provide to their own PUs. [15] is an application of stochastic control theory to the MAC layer protocol design for CRNs. Specifically, the paper considers N data channels, which both the PUs and SUs use in a time-slotted fashion. At the beginning of certain time slots, a SU selects one channel to sense and it transmits over that channel for L time slots if it finds it idle. The SU gets a reward of L units if the transmissions are successful and a reward of $-\alpha L$ if any of the transmissions fail due to a collision with the PUs. The paper designs a channel selection policy for the SUs by taking a stochastic control approach and maximizing a discounted sum of expected rewards over an infinite horizon.

Along similar lines, Markov decision theory has been used to optimize the SU spectrum sensing operation under the a periodic sensing assumption. Specifically, it is assumed that a SU senses one or more licensed channels at the beginning of a time slot and then may decide to use one of the licensed channels, not necessarily one of those that were sensed. The goal of the SU is to maximize its own throughput subject to limits on interference caused to the PUs. The papers described below assume error-free spectrum sensing and/or sensing and PU activity detection schemes that could suffer from errors. Whenever the latter is considered, the papers start from the assumption that the receiver operating curve (ROC) for the spectrum sensing operation and the associated decision rule are known. Therefore, the spectrum sensing operation is fine-tuned by selecting a point on the ROC curve according to the Neyman-Pearson (NP) criterion. [16] and [17] consider a time-slotted primary network with N channels and a Markov chain with $M = 2^N$ states and known transition probabilities that governs PU activity over these channels. At the beginning of each slot, a SU chooses a subset $\mathcal{A}_1$ of channels to observe with $|\mathcal{A}_1| \leq L_1 \leq N$, and then it uses a subset $\mathcal{A}_2 \subseteq \mathcal{A}_1$ of size no more than $L_2 \leq L_1$ channels to use. This leads to a finite-horizon, constrained partially observable Markov decision process (POMDP) problem, for which an optimal as well as a greedy suboptimal solution are found. These papers consider both cases of error-free spectrum sensing and sensing that could suffer from errors. The model considered in [18] is similar to the previous two references. However, in this paper the authors study the interaction between the PHY layer sensor operating characteristics and the MAC layer access strategy. The paper establishes a separation principle in the single-channel sensing case. It shows that the optimal solution consists of choosing in the first step a spectrum sensor (operating point on ROC curves for various channel sensors) and an access strategy (probabilities of use for various channels depending on sensing operation recommendations) to maximize the instantaneous SU throughput subject to a collision constraint (limit on interference caused to the PUs) and in the second step a sensing strategy (which channels to sense) to maximize the overall throughput. It turns out that a general separation principle does not exist in the case where the secondary user is allowed to sense multiple channels in each time slot. As an example, the paper considers a case where the PU signal is assumed to be a white Gaussian noise process that is corrupted by an additive white Gaussian noise (AWGN) process, in which case an energy detector is optimal under the NP criterion. In a departure from the time-slotted operation assumption for the PUs, [19] considers a continuous-time Markov chain (CTMC) model for PU activity. Specifically, the PU occupancy of the i'th channel, $i = 0, 1, \cdots, N-1$, is modeled by a homogeneous CTMC with exponential sojourn times with parameters λ_i and μ_i for the idle and busy states, respectively. The SU senses a single channel at the beginning of each time slot and decides if and over which channel, again not necessarily the one that was sensed, to transmit. This leads to a constrained Markov decision process (CMDP) problem in which the SU average throughput over an infinite horizon is maximized subject to an upper bound on the average interference caused for the PUs. The paper solves for the optimal policy via a linear program. Finally, [20] extends the results of [18] to the case of an unslotted primary system under certain conditions on the false alarm probability of the spectrum sensor.

There are some other papers on OSA with periodic sensing that are relevant to our paper. [21] considers a CTMC model for PU activity. In each cycle, an SU monitors the channel for $T_{monitor}$ seconds and uses an energy detector to decide whether there is PU activity on the channel sensed. If it does not detect PU activity, it uses the channel for T_{data} seconds. Otherwise, it initiates a search for other PU channels that lasts a random T_{search} seconds. The paper uses approximate formulae for the false-alarm and detection probabilities of the detector based on the central limit theorem (CLT) for detection of signals with large time-bandwidth products. This is in contrast with our paper, where exact formulae for these probabilities have been derived. [21] optimizes both the channel monitoring time and the channel search time to maximize the SU throughput while limiting the interference to the PUs. [22] is another paper that optimizes channel sensing time of an energy detector to maximize SU throughput subject to bounds on the interference to the PUs. This paper assumes a discrete-time signal model and uses CLT arguments to derive expressions for probabilities of false-alarm and detection. The paper also considers cooperative sensing using multiple mini-slots and multiple SUs. The former means the SU is allowed to divide up its sensing time in each cycle into M discontiguous mini-slots.

This paper considers a Bayesian framework for optimizing the average per unit time net profit of a SU in a network of primary users. It proposes a pricing structure that rewards the SU for the durations of time it successfully uses the channel and penalizes it during times that it interferes with the PUs. The paper considers an idealized model for channel sensing, an energy-detector "type" of decision rule for the SU, and a simple network model for the PUs and the SU. It derives an analytical formula for the average per unit time net profit of the SU and uses numerical methods to maximize the profit as a function of the sensing and channel use parameters of the SU. The analytical formulae involve, among others, computing the probability distribution of the time an on-off Markov process spends in the on state during an observation interval of length D. We should point out that [23] computes the probability distribution of "opportunity time" in OSA in a system consisting of N channels, where the PU activity over the channels is modeled by independent homogeneous CTMCs and the SU is allowed to switch from one channel to another as long as one is unoccupied. The opportunity time is defined as the duration of time from the first instance a channel is available until the time all channels become unavailable.

The organization of the paper is as follows. In Section II we present the model for the network consisting of the SU and the PUs, the pricing structure, the channel sensing and PU detection mechanism, and formulate the SU optimization problem. In Section IV, we present the analysis that leads to a formula for the SU average net profit. In Section V, we use numerical methods to maximize the net profit and present numerical results of this optimization as functions of various network parameters. Section VI concludes the paper.

II. Network Model

Consider a communication network consisting of N primary and a single secondary wireless users, all sharing a single communication channel. While the PUs are licensed to use the channel, the sole SU tries to sneak in and use the channel whenever it believes the channel is not in use by the PUs. The operation of the PUs is governed by a network administrator that allows at most one PU to use the channel at any given time. The time a PU holds the channel and uses it is characterized by an exponential random variable (r.v.) with parameter μ. These channel holding times are independent identically distributed (i.i.d.) across PUs and over time. Once a PU finishes using the channel, it releases it and waits until it needs the channel again. This interarrival time is characterized by an exponential r.v. with parameter λ, and such r.v.'s are once again i.i.d. across PUs and over time and are independent of the channel holding times. If a PU needs to use the channel but finds it in use, it gives up and waits until it needs the channel again. This leads to a continuous-time, on-off Markov process $\{X(t); t \geq 0\}$ model for channel occupancy, as depicted in Figure 1, where states '0' and '1' denote the channel being idle and busy, respectively, and $\bar{\lambda} = N\lambda$. The steady-state probability distribution of this Markov process is given by $\pi_0 = P(X(t) = 0) = \mu/(\bar{\lambda} + \mu)$ and $\pi_1 = P(X(t) = 1) = \bar{\lambda}/(\bar{\lambda} + \mu)$. We assume that $P(X(0) = 0) = \pi_0$, and hence $\{X(t); t \geq 0\}$ is stationary.

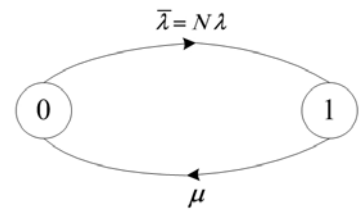


Fig. 1. Markov process model for channel occupancy $\{X(t); t \geq 0\}$.

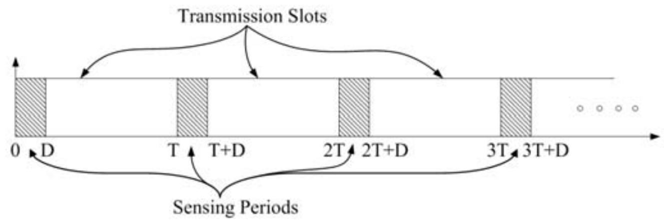


Fig. 2. The SU behaves in a periodic manner. It senses the channel for D seconds and then it may use it for $W = T - D$ seconds or wait for W seconds.

The SU operates in periodic fashion. It starts each period or cycle by sensing the spectrum for D seconds, based on which it decides what to do in the next W seconds: use the channel and transmit data or wait and back off. If the SU ends up using the channel, it may interfere with a PU even if the channel is idle at the beginning of the W-second interval. We assume the behavior of the SU does not affect $\{X(t)\}$ at all. Specifically, the PUs and their network administrator do not sense the channel before using it. If the SU begins using the channel while a PU is using it due to the SU's incorrect decision about PU activity on the channel, the PU will be unaware of this event and will continue to use the channel. The operation of the SU is depicted in Figure 2. We assume the SU has unlimited amount of data to send and will use the channel for the entire W-second transmission slot if it decides the channel is idle after observing it during the preceding D-second sensing period. Let $T = D + W$ denote the length of a SU cycle.

It is assumed that the SU observes an attenuated version of $\{X(t)\}$ corrupted by additive white Gaussian noise (AWGN). Specifically, the SU observes:

$$Y(t) = SX(t) + N(t) \qquad ; \qquad t \in [0, D],$$

where S is a constant attenuation factor known to the SU and $\{N(t); t \geq 0\}$ is a zero-mean, Gaussian random process with power spectral density $S_N(f) = N_0/2$ that is independent of $\{X(t); t \geq 0\}$. We realize that S is typically random and time-varying and not known to the SU. We leave consideration of more realistic channel propagation models to future work.

The SU needs to decide whether $X(D) = 0$ or $X(D) = 1$ based on observing $Y(t)$ during $[0, D]$. We do not address the problem of finding the optimum decision rule for this problem in this paper. Rather, we consider the following decision rule:

$$\hat{X}(D) = \begin{cases} 1 & ; \quad \text{if } \frac{1}{SD}\int_0^D Y(t)dt > \tau \\ 0 & ; \quad \text{otherwise} \end{cases} \tag{1}$$

Note that

$$\frac{1}{SD}\int_0^D Y(t)dt = \frac{1}{D}\int_0^D X(t)dt + \frac{1}{SD}\int_0^D N(t)dt \triangleq Z + \tilde{N}, \tag{2}$$

where Z is a continuous r.v. distributed over $[0,1]$ and $\tilde{N} \sim \mathcal{N}(0,\sigma^2)$ with $\sigma = \sqrt{\frac{N_0}{2DS^2}}$. If Z were a binary r.v. taking the values 0 and 1, then the decision rule given by Equation (1) would be optimal with a proper choice of the threshold τ. Its performance would depend on D and the signal-to-noise ratio (SNR)

$$\text{SNR in dB} = 10\, log_{10}\, \frac{S^2}{N_0/2}$$

Even under the present circumstances where Z is not a binary r.v., this decision rule would be a reasonable choice as long as the underlying process $\{X(t)\}$ does not change states too fast. In other words, the SNR determines the minimum D needed for averaging out the noise and for the decision rule to meet any required performance level, if Z were a binary r.v. The decision rule would still perform well if $\min(1/\lambda, 1/\mu)$ is well above this minimum D. Its performance is characterized by $p_{j|i} = P(\hat{X}(D) = j | X(D) = i)$, for $(i,j) \in \{0,1\}^2$. The SU will transmit over the channel for W seconds, if at the end of the D-second sensing period it decides the channel is idle. Otherwise, it will not use the channel.

III. Reward-Penalty Model & Bayesian Framework

We assume the SU earns \$$r$ per second while using the channel without causing interference for the PUs and has to pay a fine of \$$f$ per second while using the channel and interfering with the PUs. In other words, when the SU transmits over the channel, it earns \$$r$ per second while $X(t) = 0$ and has to pay a fine of \$$f$ per second while $X(t) = 1$. In addition, the SU will pay a per-incident fine of \$$F$ for each instance of interfering with the PUs. Referring to Figure 2, naturally the SU would neither have any revenue nor pay any fine, if as a result of sensing the channel it decides a PU is using it, i.e. $\hat{X}(D) = 1$, and hence does not use the channel during the ensuing transmission slot.

The idea behind the first fine is as follows. Suppose the SU starts transmitting over the channel and after a while a PU starts to use the channel also. If the SU somehow had the capability to detect the arrival of the PU and would stop using the channel, then it ought not to be fined as much as if it didn't have that capability and kept using the channel and interfering with the PU. One way for the SU to detect the arrival of a PU after starting to use the channel is through use of a control channel over which the SU is informed of such important events. This would probably require the presence of other SUs in the network that help each other with distributed spectrum sensing, a topic that is beyond the scope of the present paper.

We assume that $r > 0$, $f \geq 0$, and $F \geq 0$. We further assume that f and F cannot be simultaneously set to zero, because in that case the optimal strategy for the SU is to use the channel all the time without sensing it at all, and that makes the problem trivial. One may wonder why we introduced two types of penalties. The answer is that there are no OSA or cognitive radio networks deployed yet. Hence, the issue of what reward / penalty structure is appropriate needs to be discussed. We have provided flexibility in this choice by offering a formulation that handles both penalties. Two special cases of interest are $f = 0$ and $F = 0$, which correspond to the scenarios where only one of the penalties survives and is used.

We propose a Bayesian framework for maximizing the average per unit time net profit of the SU. Specifically, the goal of this paper is to maximize

$$\Lambda = E[R] = \frac{1}{T}\sum_{i=0}^{1} \pi_i\, p_{0|i}\, E\left[R | X(D) = i, \hat{X}(D) = 0\right] \tag{3}$$

where R is the net profit of the SU based on the reward-penalty model described above. Note that there are no terms corresponding to $p_{1|0}$ and $p_{1|1}$ in Equation (3), because the SU will simply not use the channel when it decides that $\hat{X}(D) = 1$, and hence its net profit would be zero in such instances. Λ is a function of several variables, namely, N, λ, μ, S, N_0, r, f, F, D, W, and τ. It actually turns out that the dependence of Λ on N and λ is through $\bar{\lambda} = N\lambda$. Similarly, Λ depends on S and N_0 through the SNR. Hence, Λ is a function of nine variables instead of eleven!

The primary goal of this paper is to maximize Λ as a function of (D, W, τ), given the other six independent variables ($\bar{\lambda}$, μ, SNR, r, f, and F). This essentially determines the optimal strategy for the SU in the sense of maximizing its average per unit time net profit. It may turn out that the optimal strategy is not to use the channel under any circumstances. This may happen for a number of reasons, e.g. if the penalties (f and F) are large compared to the reward rate r. It would be interesting to determine in which regions of the three-dimensional rfF-space the SU is better off not using the channel at all. That is, what price/penalties would make an opportunistic use of the channel unprofitable even with an optimal strategy for the SU?

IV. Analysis

In this section we derive an expression for Λ and make some remarks about maximizing it. As shown in Equation (3), we need analytic expressions for four terms in order to compute Λ, namely, $p_{0|0}$, $p_{0|1}$, $E\left[R | X(D) = 0, \hat{X}(D) = 0\right]$, and $E\left[R | X(D) = 1, \hat{X}(D) = 0\right]$. We first address the problem of deriving expressions for the conditional expected net profits and then the two conditional probabilities. In order to do the former, we decompose R into two terms, the contribution $\hat{R}$ of the per-second reward r / penalty f and the contribution $\tilde{R}$ of the per-incident penalty F. It is clear that $R = \hat{R} - \tilde{R}$.

A. Contribution of Per-Second Components to Net Profit

If the SU decides to use the channel during the transmission slot $[D,T]$, then we can write

$$\hat{R} = \int_D^T \left[r - (r+f)X(t)\right] dt = rW - (r+f)\int_D^T X(t)dt,$$

and hence, for $i = 0, 1$, we can write

$$\begin{aligned}
& E\left[\hat{R}|X(D) = i, \hat{X}(D) = 0\right] \\
= \;& rW - (r+f)E\left[\int_D^T X(t)dt \middle| X(D) = i, \hat{X}(D) = 0\right] \\
= \;& rW - (r+f)\int_D^T E\left[X(t)|X(D) = i, \hat{X}(D) = 0\right] dt \\
= \;& rW - (r+f)\int_D^T E\left[X(t)|X(D) = i\right] dt \\
= \;& rW - (r+f)\int_0^W E\left[X(t+D)|X(D) = i\right] dt \\
= \;& rW - (r+f)\int_0^W E\left[X(t)|X(0) = i\right] dt,
\end{aligned}$$

where in the third line we have interchanged the order of the integral and the expectation, the Markov property has been used in the fourth line, the fifth line is the result of a change of variable in the integral, and the last line is due to the time-homogeneity of the transitions in the Markov process $\{X(t)\}$. Since $X(t)$ is a 0-1 binary r.v., $E[X(t)] = P(X(t) = 1)$. Hence,

$$\begin{aligned}
& E\left[\hat{R}|X(D) = 0, \hat{X}(D) = 0\right] \\
= \;& rW - (r+f)\int_0^W P(X(t) = 1|X(0) = i)dt \quad (4)
\end{aligned}$$

The integrand in the last equation is a transition probability of a birth-death random process. It is given by (see e.g. [24], Page 150)

$$P(X(t) = 1|X(0) = i) = \frac{\bar{\lambda}}{\bar{\lambda}+\mu}\Big(1 - e^{-(\bar{\lambda}+\mu)t}\Big) + ie^{-(\bar{\lambda}+\mu)t}$$

Using this result in Equation (4) leads to

$$\begin{aligned}
& E\left[\hat{R}|X(D) = i, \hat{X}(D) = 0\right] \quad (5) \\
= \;& \frac{\mu r - \bar{\lambda} f}{\bar{\lambda}+\mu}W + \frac{((1-i)\bar{\lambda} - i\mu)(r+f)}{(\bar{\lambda}+\mu)^2}\left[1 - e^{-(\bar{\lambda}+\mu)W}\right]
\end{aligned}$$

B. Contribution of Per-Incident Penalty to Net Profit

Let $L(t)$ denote the number of times the PUs use the channel during the interval $[D, D+t)$. Then

$$\begin{aligned}
& E\left[\tilde{R}|X(D) = 0, \hat{X}(D) = 0\right] \\
= \;& FE\left[L(t)|X(D) = 0\right] \\
= \;& F\sum_{n=1}^{\infty} nP(L(t) = n|X(D) = 0) \\
\triangleq \;& Fg(t)
\end{aligned}$$

To compute this expectation, we introduce the following conditional probabilities for $t \geq 0$ and $n = 0, 1, 2, \ldots$:

$$\begin{aligned}
a_n(t) &\triangleq P(L(t) = n, X(D+t) = 1|X(D) = 0) \\
b_n(t) &\triangleq P(L(t) = n, X(D+t) = 0|X(D) = 0)
\end{aligned}$$

It is obvious that $a_0(t) = 0$ and $b_0(t) = e^{-\bar{\lambda}t}$. Now consider the following Kolmogorov's forward equations:

$$\begin{aligned}
\dot{a}_n(t) &= -\mu a_n(t) + \bar{\lambda} b_{n-1}(t) \quad ; \quad n = 1, 2, 3, \ldots \\
\dot{b}_n(t) &= +\mu a_n(t) - \bar{\lambda} b_n(t) \quad ; \quad n = 0, 1, 2, \ldots,
\end{aligned}$$

where $\dot{a}_n(t)$ and $\dot{b}_n(t)$ denote the derivatives of $a_n(t)$ and $b_n(t)$, respectively. Taking Laplace transform of the above equations, we get:

$$\begin{aligned}
sA_n(s) &= -\mu A_n(s) + \bar{\lambda} B_{n-1}(s) \quad ; \quad n = 1, 2, 3, \ldots \\
sB_n(s) &= +\mu A_n(s) - \bar{\lambda} B_n(s) \quad ; \quad n = 0, 1, 2, \ldots
\end{aligned}$$

The second equation implies that

$$A_n(s) = \frac{s+\bar{\lambda}}{\mu}B_n(s) \quad ; \quad n = 1, 2, 3, \ldots$$

Plugging this into the first equation leads to

$$B_n(s) = \frac{\bar{\lambda}\mu}{(s+\bar{\lambda})(s+\mu)}B_{n-1}(s) \quad ; \quad n = 1, 2, 3, \ldots$$

Given that $B_0(s) = 1/(s+\bar{\lambda})$, we get, for $n = 1, 2, 3, \ldots$,

$$\begin{aligned}
B_n(s) &= \frac{(\bar{\lambda}\mu)^n}{(s+\bar{\lambda})^{n+1}(s+\mu)^n} \\
A_n(s) &= \frac{(\bar{\lambda}\mu)^n}{\mu(s+\bar{\lambda})^n(s+\mu)^n}
\end{aligned}$$

Note that $g(t) = \sum_{n=1}^{\infty} n\left[a_n(t) + b_n(t)\right]$. Taking Laplace transform of this identity yields

$$\begin{aligned}
G(s) &= \sum_{n=1}^{\infty} n\left[A_n(s) + B_n(s)\right] \\
&= \frac{s+(\bar{\lambda}+\mu)}{\mu(s+\bar{\lambda})}\sum_{n=1}^{\infty} n\left[\frac{\bar{\lambda}\mu}{(s+\bar{\lambda})(s+\mu)}\right]^n
\end{aligned}$$

The infinite series in the last equation converges to $\frac{\bar{\lambda}\mu(s+\bar{\lambda})(s+\mu)}{s^2(s+(\bar{\lambda}+\mu))^2}$, if $||(s+\bar{\lambda})(s+\mu)|| > \bar{\lambda}\mu$, where $||.||$ denotes the magnitude of a complex number. Therefore,

$$G(s) = \frac{\bar{\lambda}(s+\mu)}{s^2(s+(\bar{\lambda}+\mu))},$$

and taking inverse Laplace transform of this equation we get

$$g(t) = \frac{\bar{\lambda}\mu t}{\bar{\lambda}+\mu} + \frac{\bar{\lambda}^2}{(\bar{\lambda}+\mu)^2}\left[1-e^{-(\bar{\lambda}+\mu)t}\right] \quad ; \quad t \geq 0$$

and hence,

$$\begin{aligned} & E\left[\tilde{R}|X(D)=0,\hat{X}(D)=0\right] \quad (6) \\ = \quad & F\left\{\frac{\bar{\lambda}\mu W}{\bar{\lambda}+\mu} + \frac{\bar{\lambda}^2}{(\bar{\lambda}+\mu)^2}\left[1-e^{-(\bar{\lambda}+\mu)W}\right]\right\} \end{aligned}$$

Using a similar technique, it can be shown that

$$\begin{aligned} & E\left[\tilde{R}|X(D)=1,\hat{X}(D)=0\right] \quad (7) \\ = \quad & F\left\{\frac{\bar{\lambda}\mu W}{\bar{\lambda}+\mu} + 1 - \frac{\bar{\lambda}\mu}{(\bar{\lambda}+\mu)^2}\left[1-e^{-(\bar{\lambda}+\mu)W}\right]\right\} \end{aligned}$$

Finally, since $R = \hat{R} - \tilde{R}$, combining Equations (5) (with $i=0$) and (6), we get

$$\begin{aligned} & E\left[R|X(D)=0,\hat{X}(D)=0\right] \quad (8) \\ = \; & \frac{\mu r - \bar{\lambda}(f+\mu F)}{\bar{\lambda}+\mu}W + \frac{\bar{\lambda}(r+f-\bar{\lambda}F)}{(\bar{\lambda}+\mu)^2}\left[1-e^{-(\bar{\lambda}+\mu)W}\right], \end{aligned}$$

and combining Equations (5) (with $(i=1)$ and (7), we get

$$\begin{aligned} & E\left[R|X(D)=1,\hat{X}(D)=0\right] \quad (9) \\ = \; & \frac{\mu r - \bar{\lambda}(f+\mu F)}{\bar{\lambda}+\mu}W - \frac{\mu(r+f-\bar{\lambda}F)}{(\bar{\lambda}+\mu)^2}\left[1-e^{-(\bar{\lambda}+\mu)W}\right] - F \end{aligned}$$

C. *Computation of* $p_{0|0}$ *and* $p_{0|1}$

The r.v.'s Z and $\tilde{N}$, as defined in Equation (2), are conditionally independent given $X(0)=i$, for $i=0,1$. In addition, $\tilde{N}$ and $X(D)$ are independent. Hence, for $i=0,1$,

$$\begin{aligned} p_{0|i} &= P(Z+\tilde{N} \leq \tau | X(D)=i) \\ &= \int_{-\infty}^{\infty} f_{\tilde{N}}(\tau - z) F_Z(z|X(D)=i)dz \\ &= Q\left(\frac{1-\tau}{\sigma}\right) + \int_0^1 \frac{e^{-\frac{(z-\tau)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} F_Z(z|X(D)=i)dz, \quad (10) \end{aligned}$$

where the last line is due to the fact that $F_Z(z|X(D)=i)$ is zero if $z<0$ and one if $z \geq 1$, and

$$Q(x) \triangleq \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du \quad ; \quad x \in \mathbb{R}$$

In order to compute $F_Z(z|X(D)=i)$, we use the fact that $\{X(t)\}$ is time-reversible. Therefore, $F_Z(z|X(D)=i) = F_Z(z|X(0)=i)$. Takács's work on sojourn times in stochastic processes [25] is exactly what's needed here. In fact, he computes the cumulative distribution function (CDF) of the sojourn time in the case where the two r.v.'s characterizing sojourn to states '0' and '1' have any general probability distributions. As a special case, he considers the case where the sojourn times are exponentially distributed, as in this paper. The results for $0 \leq z < 1$ are as follows:

$$\begin{aligned} F_Z(z|X(D)=0) = e^{-\bar{\lambda}D(1-z)} \Bigg[1 + \sqrt{\bar{\lambda}\mu D(1-z)} \times \quad (11) \\ \int_0^{Dz} \frac{e^{-\mu y}}{\sqrt{y}} I_1\left(2\sqrt{\bar{\lambda}\mu D(1-z)y}\right) dy \Bigg] \end{aligned}$$

$$\begin{aligned} F_Z(z|X(D)=1) = 1 - e^{-\mu D z} \Bigg[1 + \sqrt{\bar{\lambda}\mu D z} \times \quad (12) \\ \int_0^{D(1-z)} \frac{e^{-\bar{\lambda} y}}{\sqrt{y}} I_1\left(2\sqrt{\bar{\lambda}\mu D z y}\right) dy \Bigg] \end{aligned}$$

where $I_1(.)$ is the Bessel function of order 1 for imaginary arguments. Note that the first CDF has a jump discontinuity of size $e^{-\bar{\lambda}D}$ at $z=0$ and the second one a jump discontinuity of size $e^{-\mu D}$ at $z=1$. Also note that

$$F_Z(z|X(D)=0) \geq F_Z(z|X(D)=1) \quad ; \quad z \in \mathbb{R} \quad (13)$$

Since this fact is rather intuitive, a formal proof is not provided.

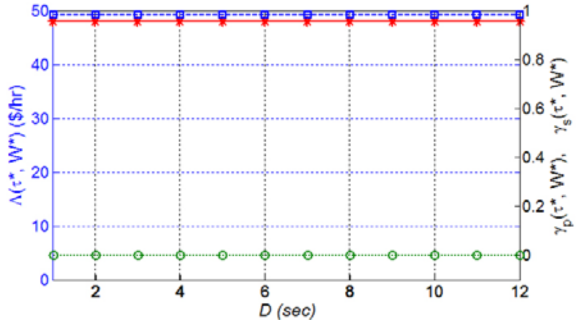
Let $\Delta(\tau) \triangleq p_{0|0}(\tau) - p_{0|1}(\tau)$. It is obvious that $-1 \leq \Delta(\tau) \leq 1$, for all $\tau \in \mathbb{R}$. Furthermore,

$$\begin{aligned} \Delta(\tau) \;=\; & \int_0^1 \frac{e^{-\frac{(z-\tau)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \\ & [F_Z(z|X(D)=0) - F_Z(z|X(D)=1)]\, dz \end{aligned}$$

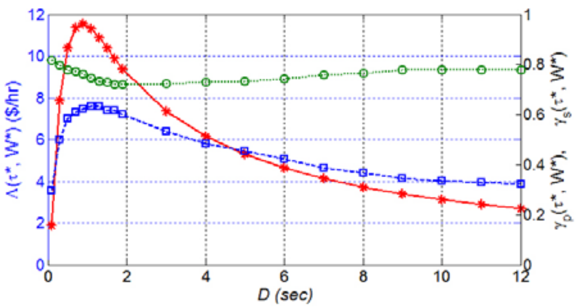
Using Equation (13) in the latter implies that $\Delta(\tau) \geq 0$, for all $\tau \in \mathbb{R}$. Hence, $0 \leq \Delta(\tau) \leq 1$, for all $\tau \in \mathbb{R}$. It is also true that $\Delta(\tau)$ tends to zero as $\tau \to \infty$ or $\tau \to -\infty$. Finally, we show that $\Delta(\tau)$ achieves its maximum value at some $\tau_M \in (0,1)$. The existence of the maximum is obvious. We show that it lies in the interval $(0,1)$ by setting the derivative of $\Delta(\tau)$ to zero, which leads to

$$\begin{aligned} 0 \;=\; & \int_0^1 (z-\tau) e^{-\frac{(z-\tau)^2}{2\sigma^2}} \\ & [F_Z(z|X(D)=0) - F_Z(z|X(D)=1)]\, dz \end{aligned}$$

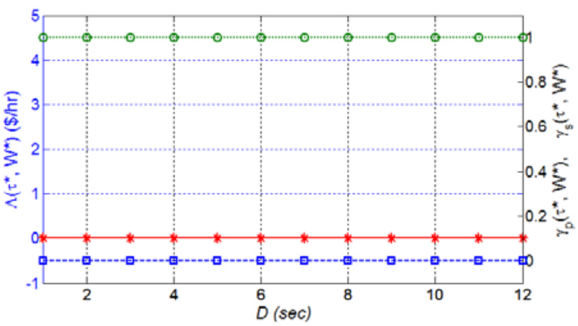
This equation cannot have a root $\tau \leq 0$, because the integrand would be nonnegative for all $z \in [0,1]$ and positive for some $z \in (0,1]$. Similarly, it cannot have a root $\tau \geq 1$, because the integrand would be nonpositive for all $z \in [0,1]$ and negative for some $z \in [0,1)$. These facts about $\Delta(\tau)$ are used in maximization of Λ.



(a) $D^* = 0$, $\tau^* = +\infty$, $W^* = +\infty$ (f=0.02, F=0.5)



(b) $D^* = 0.9$, $\tau^* = 0.4$, $W^* = 4.12$ (f=0.2, F=1)



(c) $D^* = 0$, $\tau^* = -\infty$, $W^* = 0$ (f=0.5, F=2)

Fig. 3. SU profit with optimal values for τ and W for a given D

TABLE I
NETWORK PARAMETERS

Parameters	Value
N	5
λ	0.002/sec
μ	0.2/sec
SNR	10 dB
r	\$0.02/sec
f	\$0.2/sec
F	\$1.0

V. NUMERICAL RESULTS

We used MATLAB[1] to compute the Λ. Given a set of values for the parameters r, f, F, λ, μ, and SNR, we can find the values for D, τ, and W that maximize the average per unit time profit for the SU and the corresponding profit. This provides insight into how the network behaves and performs as a function of various network parameters. It also allows the SU to adjust the parameters that are under its control, namely D, τ, and W. We also use two other metrics to assess the performance of the network considered in this paper with the strategy used by the SUs. The first one quantifies the degradation in the PU that has been used in some papers, such as [26]. Basically, it is a measure of the fraction of time the SU interferes with the PUs while the latter are using the channel. Let the r.v. I denote the time duration over one SU sensing and transmission cycle T that the SU inteferes with the PUs. I is given by

$$I = \begin{cases} \int_D^T X(t)dt & ; \text{ if } \hat{X}(D) = 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

and its expectation is given by

$$E[I] = \sum_{i=0}^{1} \pi_i \, p_{0|i} \, E\left[I | X(D) = i, \hat{X}(D) = 0\right],$$

Following a procedure similar to the calculation given in Section IV.A, it can be shown that

$$E\left[I | X(D) = i, \hat{X}(D) = 0\right] = \frac{\bar{\lambda}}{\bar{\lambda} + \mu} W - \frac{(1-i)\bar{\lambda} - i\mu}{(\bar{\lambda} + \mu)^2} \left[1 - e^{-(\bar{\lambda}+\mu)W}\right]$$

Note that on the average the PUs would use the channel uninterfered $100\pi_1\%$ of the time, if the SU were not present. Therefore, we define the *throughput efficiency* of the PUs as

$$\gamma_P = 1 - \frac{E[I]/T}{\pi_1}$$

The second metric is the throughput efficiency γ_S for the SU. It is the fraction of time the SU manages to use the channel without causing interference to or being interfered by the PUs. Ignoring the effect of erroneous transmissions due to channel noise, if the data transmission rate of the SU is r_S bits per second (bps), then its throughput would be $\gamma_S r_S$ bps. The derivation for γ_S is similar to that used for γ_P earlier. Let the r.v. T_S denote the time duration over one SU cycle T that the SU manages to transmit data without any PUs on the channel. T_S is given by

$$T_S = \begin{cases} W - \int_D^T X(t)dt & ; \text{ if } \hat{X}(D) = 0 \\ 0 & ; \text{ otherwise} \end{cases}$$

and its expectation is given by

$$E[T_S] = \sum_{i=0}^{1} \pi_i \, p_{0|i} \, E\left[T_S | X(D) = i, \hat{X}(D) = 0\right]$$

However,

$$E\left[T_S | X(D) = i, \hat{X}(D) = 0\right] = W - E\left[I | X(D) = i, \hat{X}(D) = 0\right]$$

Finally, γ_S is defined by

[1]**Disclaimer:** The National Institute of Standrads and Technology does not endorse any commercial software product mentioned in this article.

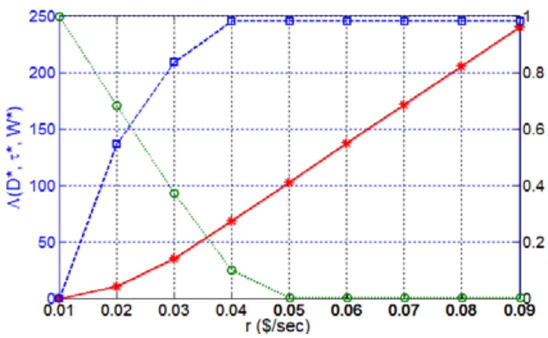


Fig. 4. Maximum SU profit vs. r

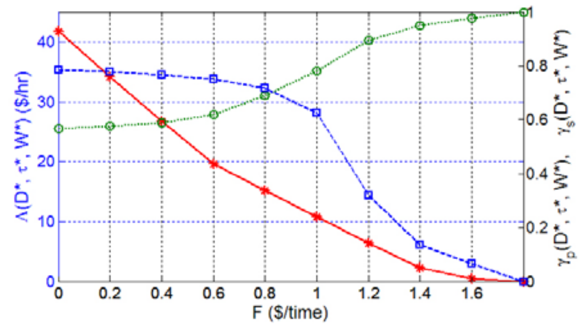


Fig. 6. Maximum SU profit vs. F

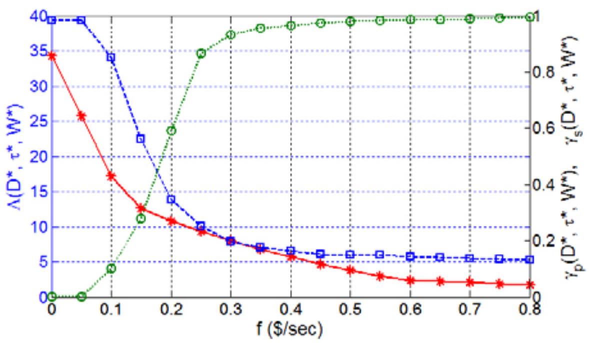


Fig. 5. Maximum SU profit vs. f

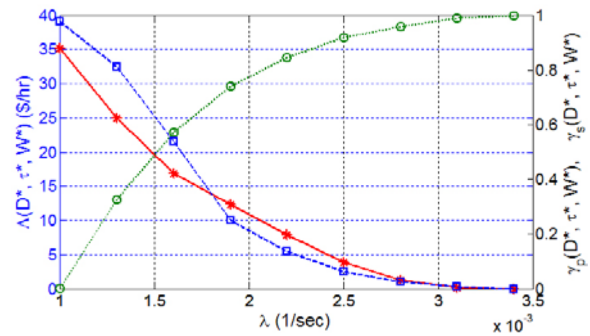


Fig. 7. Maximum SU profit vs. λ

$$\gamma_S = \frac{E[T_S]}{T}$$

Note that γ_P and γ_S both take values in $[0, 1]$, but the normalization for these metrics is different. In the case of γ_P the normalization is basically with respect to the fraction of time the PUs would ordinarily (i.e. in the absence of the SU) use the channel. On the other hand, $\gamma_S = 0.7$ means that the SU manages to use the channel 70% of the time without being interfered by the PUs.

Without getting into the details, our numerical procedures first maximize the SU average profit for a given sensing period D by appropriately choosing the threshold τ and the transmission period W. Let τ^* and W^* denote the optimum values for these SU-controllable parameters. Figure 3 presents $\Lambda(\tau^*, W^*)$, $\gamma_S(\tau^*, W^*)$, and $\gamma_P(\tau^*, W^*)$ as functions of D under three scenarios with common values for parameters N, λ, μ, SNR, and r set to, respectively, 5, 0.002/sec, 0.2/sec, 10dB, and \$0.02/sec. In this figure and all the figures that follow, Λ, γ_S, and γ_P have been plotted with, respectively, solid red, dashed blue, and dotted green lines. The difference between the three scenarios is that the values of the penalties f and F are increased from the first scenario to the thrid one, as shown in the captions for Figures 3a-3c. In Figure 3a, the reward to penalty ratio is large, and we can see that the maximum profit is always a positive constant as a function of D. The optimum values for τ and W are $\tau^* = +\infty$ and $W^* = +\infty$. This result indicates that the SU should use the channel forever without any time spent on channel sensing. This may sound unusual, because typically the PUs would not tolerate being interfered by SUs on a continuous basis. The fact that the SU can afford to behave in this manner is due to the fact that the reward for using the channel is too high compared to the penalties for interfering with the PUs. The problem of setting the reward rate and the penalties is beyond the scope of this paper. In the case of a moderate reward to penalty ratio, such as in the second scenario (Figure 3b), we can see that the maximum profit is \$$11.26$/hr and is achieved at just one value for D. The optimal values for D, τ, and W are given in the caption for the figure. One interesting conclusion from this figure is that it is possible for the SU to transmit data in an opportunstic manner and achieve $\gamma_S = 0.65$ without causing too much interference for the PUs ($\gamma_P = 0.7$). For a small reward to penalty ratio, Figure 3c shows that the maximum attainable profit is zero, and $\tau^* = -\infty$ and $W^* = 0$, which indicates that the SU would never use the channel since the reward rate is not commensurate with the penalties imposed. These three scenarios confirm that the pricing structure has a major effect on the SU strategy.

Next we investigate the impact of the parameters r, f, F, λ, μ and SNR on the maximum attainable SU profit and the corresponding throughput efficiencies for the SU and the PUs. Since the objective of our analysis is to find the optimal sensing period D^*, sensing threshold τ^*, and transmission

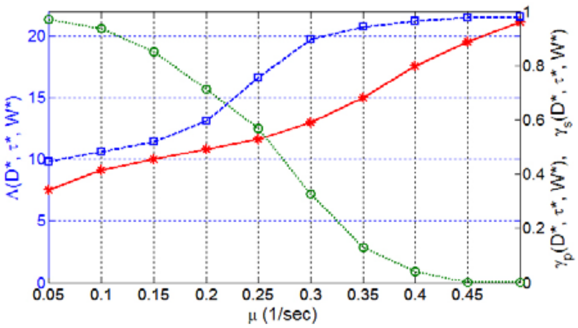


Fig. 8. Maximum SU profit vs. μ

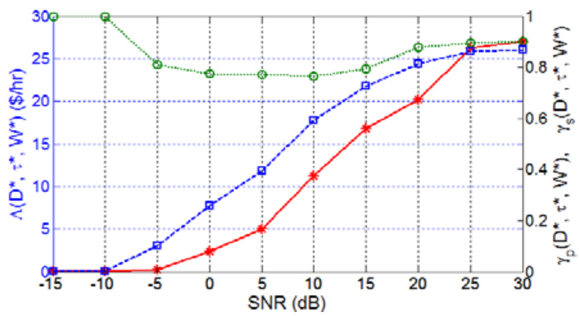


Fig. 9. Maximum SU profit vs. SNR

period W^* that maximize the SU average profit Λ, we plot $\Lambda^* = \Lambda(D^*, \tau^*, W^*)$ as a function of the six parameters mentioned above. We also plot the throughput efficiencies $\gamma_S^* = \gamma_S(D^*, \tau^*, W^*)$ and $\gamma_P^* = \gamma_S(D^*, \tau^*, W^*)$. Table I shows the default values of these parameters. In each of the Figures 4-9, we set five of these parameters to their default values and plot Λ^* (solid red line), γ_S^* (dashed blue line), and γ_P^* (dotted green line) as functions of the remaining parameter. Figure 4 shows Λ^* and γ^*'s as functions of r, and it shows that Λ^* increases almost linearly with r. Figures 5-6 plot Λ^* and γ^*'s as functions of f and F, respectively. We can see that Λ^* (γ_P^*) is maximized (minimized) when $f = 0$ or $F = 0$, and it decreases (increases) afterwards and drops to zero (saturates at one) when these penalties are sufficiently large. Figures 7-8 show Λ^* and γ^*'s as functions of λ and μ, respectively. We can see that Λ^* (γ_P^*) decreases (increases) with λ and increases (decreases) with μ. These are due to the facts that channel utilization by the PUs increases with λ and decreases with μ. Figure 9 shows Λ^* and γ^*'s as functions of SNR. The results demonstrate that Λ^* increases with SNR, because the SU is able to make a more reliable decision on channel occupancy by the PUs.

VI. Conclusions

In this paper we have proposed an economic model for OSA of a primary network by a SU. We have considered an idealized and simple model for SU spectrum sensing and detection of PU activity. We have optimized the net profit of the secondary user as a function of various network parameters. In particular, we have found the optimum values for duration D of sensing, the threshold τ the spectrum sensing decision rule uses, and the duration W of channel use. We have presented numerical results of the above optimization problem. The results show under the correct circumstances, the SU can coexist with the PUs. In other words, the SU can get a reasonable fraction of time to transmit over the channel opportunistically without causing undue interference to the PUs. It all depends on the network and pricing parameters

There are many ways in which this work can be extended. It would be nice to extend the results to the case of a PU network with K channels and to allow a multitude of SUs. Another direction is to find the optimum detector for the AWGN-corrupted signal model for the SU. Yet another desired improvement is a realistic propagation model from a PU transmitter to an SU receiver. It is also desirable to consider practical MAC protocols for the PUs, what queueing discipline they use if they don't get access to their network due to increased PU traffic or if their transmissions collide with SUs opportunistically using the channel.

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